

Student number: _____, Total marks: _____ out of XX

Problem	1	2	3	4	5	6
Marks						

Question 1. (a) Compute the derivative of the function $g(x) = \frac{1+x^2}{\sqrt{x}+x^{-1}}$.
[Do not simplify your result.]

$$g'(x) = \frac{2x(\sqrt{x} + x^{-1}) - (\frac{1}{2\sqrt{x}} - x^{-2})(1+x^2)}{(\sqrt{x} + x^{-1})^2}$$

(b) Let $f(x) = \frac{x^2 - 4}{|x - 2|}$.

$$\lim_{x \rightarrow 2^+} f(x) = 4$$

$$\lim_{x \rightarrow 2^-} f(x) = -4$$

Is f continuous at $x = 2$? Answer **NO**

Question 2. (a) Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{1}{2x-3}.$$

(b) Use the rules from class to find the derivative of the following function.

$$f(x) = \ln \left(e^{x^3} \cdot \frac{x^4 - 3x^2 + 17x - 8}{\ln(x)} \right)$$

$$f'(x) = 3x^2 + \frac{4x^3 - 6x + 17}{x^4 - 3x^2 + 17x - 8} - \frac{1}{x \ln x}$$

Question 3. The equation

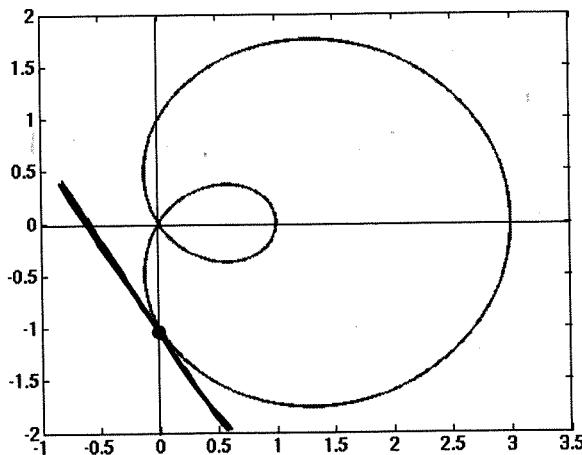
$$(x^2 + y^2 - 2x)^2 = x^2 + y^2$$

defines a curve in the x - y -plane. The image of this curve is plotted below.

- (a) Verify algebraically that the point $(0, -1)$ is on the curve.
- (b) Use implicit differentiation to find $\frac{dy}{dx}$.
- (c) Find the equation of the tangent line to the curve at the point $(0, -1)$.
- (d) Draw the tangent line into the plot below.

$$(b) \quad \frac{dy}{dx} = \frac{(x^2 + y^2 - 2x)(2x - 2) - x}{y - 2y(x^2 + y^2 - 2x)}$$

$$(c) \quad y = -2x - 1$$



Question 4.

Is the following function continuous at $x = 1$? Justify your answer in a short sentence.

$$f(x) = \cos(2x) + \frac{3x^2 - 5x}{x^2 - 2}$$

Answer: **Yes**

Question 5.

(a) Find all the points where the slope of the function $f(x) = x^3e^{-x}$ is zero.

Answer:

$$x = 0 \quad \text{or} \quad x = 3$$

(b) Find the intervals where the $f'(x)$ is positive / negative.

$f'(x)$ positive:

$$x < 3 \quad (\text{i.e. } (-\infty, 3))$$

$f'(x)$ negative:

$$x > 3 \quad (\text{i.e. } (3, \infty))$$

Question 6. Consider the discrete-time dynamical system (DTDS)

$$M_{t+1} = -0.8M_t + 6$$

(a) Find the updating function of the DTDS.

$$f(m) = -0.8m + 6$$

(b) Find the equilibrium point of the DTDS.

$$m^* = \frac{60}{18}$$

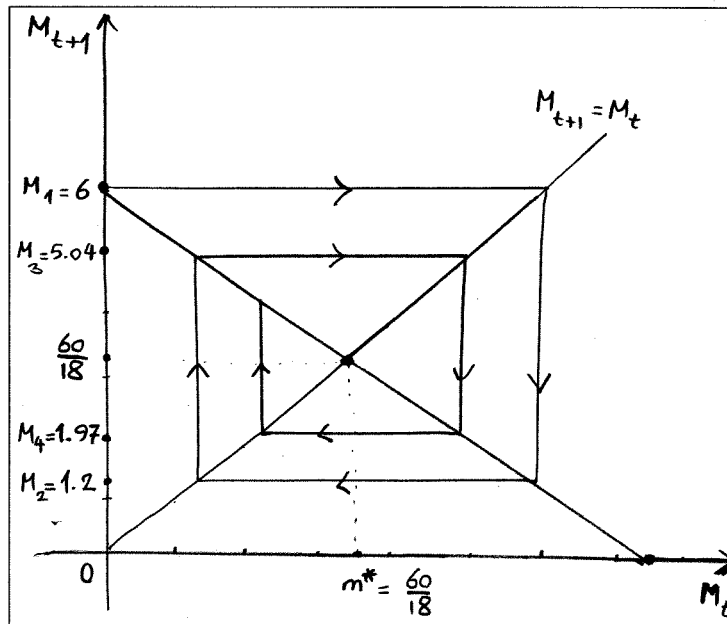
(c) Give the general solution formula for the DTDS:

$$M_t = (-0.8)^t \left(M_0 - \frac{60}{18} \right) + \frac{60}{18}$$

(d) Calculate M_{10} if $M_0 = 0$.

$$M_{10} = 2.9754$$

(e) Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) Is the equilibrium point stable or unstable?

Stable