MATH 1007G **Term test #4 – solution** **24 March 2014**

This test was written in room: ________________ Total marks: **35** marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________________ ____________

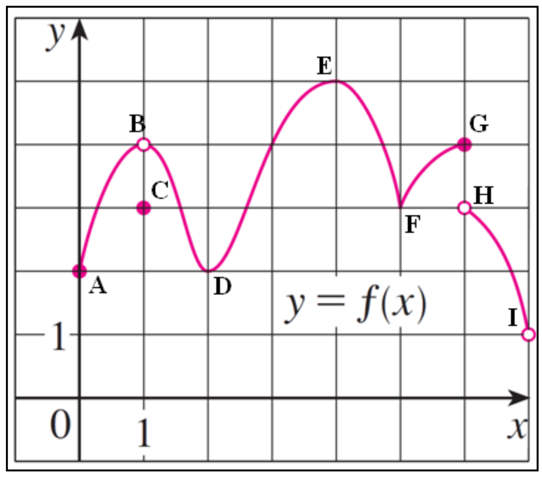
First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. [4 marks] Label each location on the graph according to the following list**:

1: local max. **2**: local min. **3**: absolute max. **4**: absolute min. **5**: none of the previous

Location:	Label number:
A	2
B	5
C	2
D	2
E	1 and 3
F	2
G	1
H	5
I	5



**Note: A particular location may be assigned more than one number.

2. [4] Using implicit differentiation, find the slope and equation of the line tangent to the curve $x^3 + y^2 = 2x^2 - xy^2 + x^2y$ at the point $(1,1)$.

$3x^2 + 2yy' = 4x - x\,2yy' - y^2 + x^2y' + 2xy$

$y'(2y + 2xy - x^2) = 4x - y^2 + 2xy - 3x^2$

$$y' = \frac{4x - y^2 + 2xy - 3x^2}{2y + 2xy - x^2}$$

$$y'\Big|_{(1,1)} = \frac{4 - 1 + 2 - 3}{2 + 2 - 1} = 2/3$$

So $y = \frac{2}{3}x + b$, $(1,1)$ on line

$\therefore 1 = \frac{2}{3}(1) + b \Rightarrow b = 1/3$

EQUATION

$y = \frac{2}{3}x + \frac{1}{3}$

3. [2] Let f be a one-to-one differentiable function, with $f(1) = 0$ and $f'(1) = 2/5$. Find $(f^{-1})'(0)$.

$f(1) = 0 \Rightarrow f^{-1}(0) = 1$

Also: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$\therefore (f^{-1})'(0) = \dfrac{1}{f'(f^{-1}(0))} = \dfrac{1}{f'(1)} = \ldots = \dfrac{1}{2/5} = \boxed{5/2}$

4. [2] Determine the differential of $y = x^4 - 3x^2$.

$\dfrac{dy}{dx} = 4x^3 - 6x \Rightarrow dy = (4x^3 - 6x)\,dx$

5. [4] Differentiate $y = x^{\tan x}$ using logarithmic differentiation (**DO NOT SIMPLIFY**).

$\ln y = \ln\left[x^{\tan x}\right] = \tan x \cdot \ln x = uv$

$u = \tan x$
$u' = \sec^2 x$
$v = \ln x$
$v' = 1/x$

$\dfrac{y'}{y} = \tan x\,(1/x) + \ln x\,(\sec^2 x)$

$y' = y\left[\dfrac{\tan x}{x} + \ln x\,(\sec^2 x)\right]$

$y' = \left[x^{\tan x}\right]\left(\dfrac{\tan x}{x} + \ln x\,(\sec^2 x)\right)$

6. [3x3] Differentiate the following functions (do **NOT** simplify):

a. $y = f(x) = (x^3 - \frac{2}{x})^4 \cdot (\sec x + \log_2 x) = uv$

$u = (x^3 - 2/x)^4$

$u' = 4(x^3 - 2/x)^3(3x^2 + 2/x^2)$

$v = \sec x + \log_2 x$

$v' = \sec x \tan x + \frac{1}{\ln 2}\left(\frac{1}{x}\right)$

$f'(x) = uv' + vu'$

$\ldots = (x^3 - 2/x)^4\left(\sec x \tan x + \frac{1}{x \ln 2}\right) + (\sec x + \log_2 x)\, 4(x^3 - 2/x)^3(3x^2 + 2/x^2)$

b. $y = f(x) = \arctan\left(\sqrt{x^3}\right) = \arctan w$, $w = \sqrt{x^3} = x^{3/2}$

$\frac{dw}{dx} = \frac{3}{2} x^{1/2} = \frac{3}{2}\sqrt{x}$

$f'(x) = \frac{1}{1+w^2} \cdot \frac{dw}{dx} =$

$\ldots = \frac{1}{1+x^3} \cdot \left(\frac{3}{2}\sqrt{x}\right)$

$(\sqrt{x^3})^2$

c. $y = f(x) = 3^{(x \sin x)} = 3^w$, $w = x \sin x$
$(u)\ (v)$

$\frac{dw}{dx} = uv' + vu'$

$u = x$
$u' = 1$
$v = \sin x$
$v' = \cos x$

$f'(x) = (\ln 3)\, 3^w \cdot \frac{dw}{dx}$

$\ldots = (\ln 3)\, 3^{(x \sin x)} \cdot (x \cos x + \sin x)$

7. [4] Write down the linearization of f at $x = 13$ for $f(x) = \sqrt{x+12}$. Use it to estimate $\sqrt{25.01}$.

$L(x) = f(a) + f'(a) \cdot (x-a)$, with $a = 13$

$f(13) = \sqrt{13+12} = 5$

$f'(x) = \frac{1}{2\sqrt{x+12}}$

$f'(13) = \frac{1}{2\sqrt{13+12}} = \frac{1}{10}$

$\therefore L(x) = 5 + \frac{1}{10}(x-13)$

and $\sqrt{25.01} = f(13.01) \approx L(13.01) =$

$\ldots = 5 + \frac{1}{10}(13.01 - 13)$

$= 5 + \frac{1}{1000} = \boxed{5.001}$

8. [3] Let $y = f(x) = 10 - 3x^4 + 24x^2$. Determine the critical points of the function.

$f'(x) = -12x^3 + 48x = -12x(x^2-4) = -12x(x-2)(x+2)$

$f'(x) = 0 \Rightarrow -12x(x-2)(x+2) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 2 \\ x = -2 \end{cases}$

9. [3] Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 10$. Determine the absolute max. and absolute min. of f on the interval [–1, 1].

$f(-1) = 3(-1)^4 - 4(-1)^3 - 12(-1)^2 + 10 = 3 + 4 - 12 + 10 = \boxed{5}$

$f(1) = 3 - 4 - 12 + 10 = \boxed{-3}$

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$

$= 12x(x-2)(x+1)$

$f'(x) = 0 \Rightarrow 12x(x-2)(x+1) = 0 \Rightarrow x = \begin{cases} -1 \checkmark \\ 0 \checkmark \\ 2 \to \text{IGNORE} \end{cases}$

$f(-1) = 5$ (already determined)

$f(0) = \boxed{10}$

$\therefore$ on $[-1, 1]$: abs. max. 10
" min. -3