MATH 1007G **Term test #2 – solution** **10 February 2014**

This test was written in room: ________________ Total marks: **40** marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________ ____________

First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

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1. Fill in the blanks:

a. [1] $\arcsin(-1/2) =$ $-\pi/6$

b. [1] Let $f(x) = 2\arccos x$. The range of f is $[0, 2\pi]$

c. [1] Let $f(x) = \arctan(x-2)$. The domain of f is: $\mathbb{R}$

d. [1] Let $f(x) = \ln(x+1)$. The domain of f is $(-1, \infty)$

e. [1] The function $f(x) = \log_{1/3} x$ is decreasing on its domain. **TRUE** ✓ **FALSE**_____

f. [1] $\arctan x = \dfrac{\arcsin x}{\arccos x}$. **TRUE**_____ **FALSE** ✓

2. Let $f(x) = \dfrac{2x^2 - 3x - 5}{x+1}$. Determine:

a. [1] $\lim_{x\to 2} f(x) \xrightarrow{D.S.} \dfrac{2(2)^2 - 3(2) - 5}{2+1} = \dfrac{8-11}{3} = \boxed{-1}$

b. [1] $\lim_{x\to 0} f(x) \xrightarrow{D.S.} \boxed{-5}$

c. [3] $\lim_{x\to -1} f(x) \xrightarrow{D.S.} \dfrac{2(-1)^2 - 3(-1) - 5}{-1+1} = \dfrac{5-5}{0} = \dfrac{0}{0}$

$\ldots = \lim_{x\to -1}\left[\dfrac{\cancel{(x+1)}(2x-5)}{\cancel{x+1}}\right] = \lim_{x\to -1}[2x-5] \xrightarrow{D.S.} 2(-1)-5 = \boxed{-7}$

$(x \neq -1)$

3. Determine the following limits if they exist. If a limit does not exist, say so.

a. [3] $\lim_{x\to 0} \dfrac{-5(x-1)^2+5}{x} \xrightarrow{D.S.} \dfrac{-5(0-1)^2+5}{0} = \dfrac{0}{0}$

$\ldots = \lim_{x\to 0}\left[\dfrac{-5(x^2-2x+1)+5}{x}\right] = \lim_{x\to 0}\left[\dfrac{-5x^2+10x\cancel{-5+5}}{x}\right] =$

$\ldots = \lim_{x\to 0}\left[\dfrac{\cancel{x}(-5x+10)}{\cancel{x}}\right] = \lim_{x\to 0}[-5x+10] \xrightarrow{D.S.} \boxed{10}$

$(x \neq 0)$

b. [4] $\lim_{x\to 0} \dfrac{\sqrt{x^2+9}-3}{x^2} \xrightarrow{D.S.} \dfrac{\sqrt{0^2+9}-3}{0^2} = \dfrac{0}{0}$

(remember the trick used in class)

$\ldots = \lim_{x\to 0}\left[\dfrac{\sqrt{x^2+9}-3}{x^2}\cdot\dfrac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}\right] = \lim_{x\to 0}\left[\dfrac{x^2+\cancel{9-9}}{x^2}\cdot\dfrac{1}{\sqrt{x^2+9}-3}\right]$

$\ldots = \lim_{x\to 0}\left[\dfrac{1}{\sqrt{x^2+9}+3}\right] \xrightarrow{D.S.} \boxed{\dfrac{1}{6}}$

$(x \neq 0)$

c. [3] $\lim_{x\to 4^-} \dfrac{x^2-16}{|x-4|} \xrightarrow{D.S.} \dfrac{4^2-16}{4-4} = \dfrac{0}{0}$

$= \lim_{x\to 4} \dfrac{(x-4)(x+4)}{-(x-4)} = \lim_{x\to 4} -(x+4) \xrightarrow{D.S.} -(4+4) = \boxed{-8}$

d. [3] $\lim_{x\to 1} \frac{\sin(x-1)}{x^2-1}$ $\xrightarrow{D.S.}$ $\frac{\sin(1-1)}{1^2-1} = \frac{0}{0}$

$= \lim_{x\to 1} \frac{\sin(x-1)}{(x+1)(x-1)} = \lim_{x\to 1}\left(\frac{1}{x+1}\right) \cdot \lim_{\substack{x\to 1\\(x-1)\to 0}} \left[\frac{\sin(x-1)}{x-1}\right]$

$\ldots = \frac{1}{1+1} \cdot 1 = \boxed{\frac{1}{2}}$

e. [3] $\lim_{x\to 3} \frac{\frac{5}{x}-\frac{5}{3}}{x-3}$ $\xrightarrow{D.S.}$ $\frac{\frac{5}{3}-\frac{5}{3}}{3-3} = \frac{0}{0}$

$= \lim_{x\to 3}\left[\frac{\frac{15-5x}{3x}}{x-3}\right] = \lim_{x\to 3}\left[\frac{5\,(\overset{-1}{\cancel{3-x}})}{3x\,(\cancel{x-3})}\right]_{(x\neq 3)} = \lim_{x\to 3}\left[\frac{-5}{3x}\right] \xrightarrow{D.S.}$

$\frac{-5}{3(3)} = \boxed{\frac{-5}{9}}$

4. Simplify as much as possible.

a. [3] $\tan\left(\arccos(-3/4)\right) = \frac{\sin(\arccos(-3/4))}{\cos(\arccos(-3/4))} = \frac{\sqrt{1-\cos^2(\arccos\frac{-3}{4})}}{-3/4}$

$\ldots = \frac{\sqrt{1-9/16}}{-3/4} = \frac{\sqrt{7}/4}{-3/4} = \boxed{-\frac{\sqrt{7}}{3}}$

b. [3] $\left(\sqrt[3]{25}\right)^{\log_5 27} = \left(5^{2/3}\right)^{\log_5 27} = 5^{\left(\frac{2}{3}\cdot\log_5 27\right)} =$

$\ldots = 5^{\log_5 27^{2/3}} = 27^{2/3} = \boxed{9}$

5. [3] Let $\log_2(y-1) - 2\log_2 5 = x^2 - \log_2 x$. Express y as a function of x.

$\log_2(y-1) - \log_2 25 + \log_2 x = x^2$

$\log_2\left[\frac{x(y-1)}{25}\right] = x^2$

$\frac{x(y-1)}{25} = 2^{x^2}$

$\rightarrow y - 1 = \frac{25\cdot 2^{x^2}}{x}$

$\boxed{y = \frac{25\cdot 2^{x^2}}{x} + 1}$

6. Let $f(x) = \begin{cases} \sqrt{6-x}+3, & \text{if } x \le 2 \\ \dfrac{x^2-2x}{x-2}, & \text{if } x > 2 \end{cases}$

a. [3] Determine $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \sqrt{6-x}+3 \xrightarrow{D.S.} \sqrt{6-2}+3 = \boxed{5}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2-2x}{x-2} \xrightarrow{D.S.} \frac{0}{0} = \lim_{x\to 2^+}\left[\frac{x\,(\cancel{x-2})}{\cancel{x-2}}\right] =$

$\lim_{x\to 2^+}[x] = \boxed{2}$

b. [1] What is $\lim_{x\to 2} f(x)$? Justify your answer.

$\lim_{x\to 2} f(x)$ d.n.e. because $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$