**MATH 1007G** **Term test #1 – solution** **27 January 2014**

This test was written in room: ________________ Total marks: 40 marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

**PLEASE PRINT** ______________________________ ____________

First name Last name Student number

**Please show your work where appropriate!** TA's have extra paper if you need it. Test duration: 50 minutes.

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**1.** [2 marks] **Simplify** as much as possible.

$$\frac{\frac{a^2}{b}-b}{\frac{a-b}{ab}} = \frac{\frac{a^2-b^2}{b}}{\frac{a-b}{ab}} = \frac{\cancel{(a-b)}(a+b)}{\cancel{b}} \cdot \frac{a\cancel{b}}{\cancel{a-b}} = a(a+b) = a^2+ab$$

**2.** [2] Determine the value of $x$ such that $\left(\frac{a}{b}\right)^x = \sqrt{\frac{a}{b}\sqrt{\frac{b}{a}}}$

$$= \sqrt{\frac{a}{b}\left(\frac{b}{a}\right)^{1/2}} = \sqrt{\left(\frac{a}{b}\right)^{1/2}} = \left(\frac{a}{b}\right)^{1/4} \Longrightarrow x = 1/4$$

**3.** Fill in the blanks:

a. [1] $\sin(x - \pi/2) = \cos x$ ... TRUE ___ FALSE X

b. [1] $\cos(-\pi/2) =$ 0

c. [1] If $f(x) = 2\tan(x - \pi/2)$ then the **range** of $f$ is $\mathbb{R}$

d. [1] The range of the function $f(x) = (1.1)^x + 5$ is $(5, \infty)$

e. [1] The function $f(x) = \sin(-x)$ is decreasing on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ TRUE X FALSE ___

**4.** Let $f(x) = \dfrac{3x+1}{2x-3}$ and $g(x) = \sqrt{3x+5}$

a. [2] What are the domains of each function, $f$ and $g$? (*answer only ok*)

$\text{dom}(f) = \mathbb{R} \setminus \{3/2\}$ ; $\text{dom}(g) = [-5/3, \infty)$

b. [2] What is the domain $f + g$ ? (*answer only ok*)

$[-5/3, 3/2) \cup (3/2, \infty) = \{x \in \mathbb{R} \mid x \geq -5/3 \text{ and } x \neq 3/2\}$

c. [3] What is the rule of $f \circ f\,(x)$ ? (*simplify as much as possible and show your work*)

$$f\circ f(x) = f(f(x)) = \frac{3f(x)+1}{2f(x)-3} = \frac{3\left(\frac{3x+1}{2x-3}\right)+1}{2\left(\frac{3x+1}{2x-3}\right)-3} = \frac{9x+3+2x-3}{6x+2-6x+9} = \frac{11x}{11} = x \quad (x \neq 3/2)$$

d. [2] What is the domain of $f \circ f\,(x)$ ? (*explain your answer*)

$$\text{dom}(f\circ f) = \{x \in \text{dom}(f) \mid f(x) \in \text{dom}(f)\} = \mathbb{R} \setminus \{3/2\}$$

e. [3] What is the rule for the inverse of $f$ (denoted $f^{-1}$)?

$$y = f(x) = \frac{3x+1}{2x-3} \Longrightarrow$$

$$\ldots \Longrightarrow x = f(y) = \frac{3y+1}{2y-3} \Longrightarrow 2xy - 3x = 3y + 1$$

$$\ldots \Longrightarrow 2xy - 3y = 3x + 1$$

$$y(2x-3) = 3x+1 \Longrightarrow \boxed{y = f^{-1}(x) = \frac{3x+1}{2x-3}} \quad (x \neq 3/2)$$

**5.** [2.5] Determine the equation of the line that passes through the points (1, –1) and (–3, 1).

$m = \frac{\Delta y}{\Delta x} = \frac{1-(-1)}{-3-1} = -\frac{1}{2}$ ; and with $(1,-1)$ on the line:

$-1 = -\frac{1}{2}(1) + b \Rightarrow b = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$

**6.** Solve the following, expressing the solution in the form of an interval or a union of sub-intervals.

a. [2] $3x+5>10x+26$

$3x-10x > 26-5$

$-7x > 21 \Rightarrow x < -3$

$\therefore \; (-\infty, -3)$

b. [3] $|x-2| \geq 3 \Rightarrow x-2 \geq 3$ or $x-2 \leq -3$

INDEED:

CASE 1: $x-2 \geq 0 \Rightarrow x \geq 2$

$x-2 \geq 3$

$x \geq 5$ ✓ consistent with

CASE 2: $x-2<0 \Rightarrow x<2$

$-(x-2) \geq 3$

$x-2 \leq -3$

$x \leq -1$ ✓ consistent with

$\therefore \; (-\infty, -1] \cup [5, \infty)$

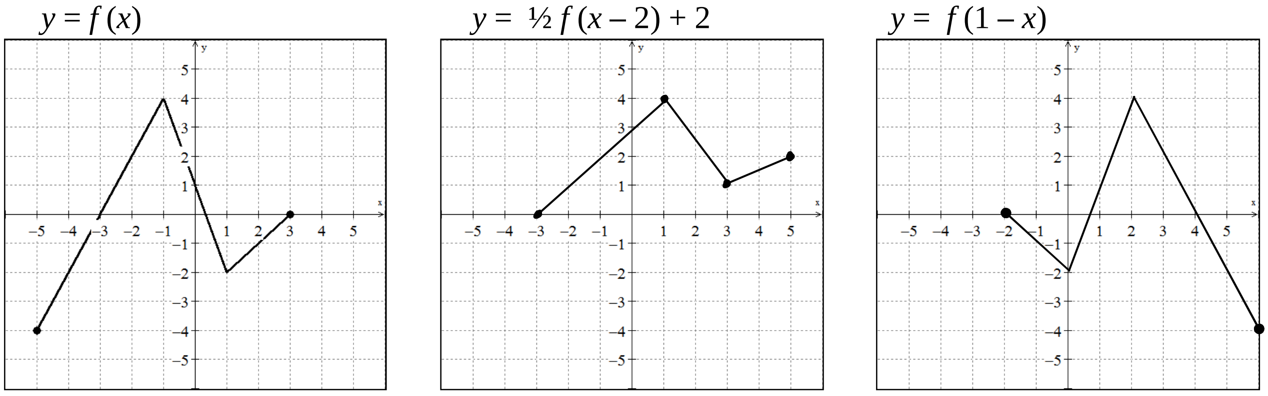
$f(x)$

① $g(x) = f(-x)$

$h(x) \; g(x-1) = f(-(x-1))$

**7.** [4] The graph of $f$ is plotted. Plot the graphs of $y = \frac{1}{2} f(x-2) + 2$ and $y = f(1-x)$

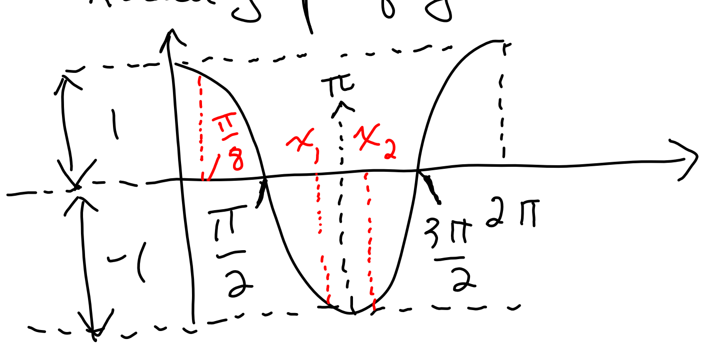
$y = f(x)$ $\qquad$ $y = \frac{1}{2} f(x-2) + 2$ $\qquad$ $y = f(1-x)$



**8.** [3] Determine the value of $x$, in the interval $0 \leq x \leq 2\pi$ $[0, 2\pi]$, for which $\cos x = -\cos(\pi/8)$

**HINT**: use the various symmetries of the $y = \cos x$ curve. (Even about the min./max. and odd about the zeros)

Recall graph of $y = \cos x$ on $[0, 2\pi]$



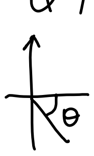

From graph $x_1 = \pi - \frac{\pi}{8} = \frac{7\pi}{8}$

$x_2 = \pi + \frac{\pi}{8} = \frac{9\pi}{8}$

**9.** Let $\cos\theta = 2/5$ and $3\pi/2 \leq \theta \leq 2\pi$. Determine the following two trigonometric ratios:

*(You may use a graphical technique or one involving identities)* $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

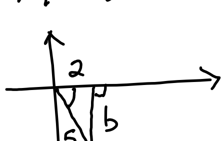
$[\frac{3\pi}{2}, 2\pi]$ ⇓ Q4



a. [2.5] $\sin\theta$ on $[3\pi/2, 2\pi]$, $\sin\theta < 0$

$\therefore \; \sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-(2/5)^2} = -\sqrt{\frac{21}{25}} = \frac{-\sqrt{21}}{5}$

Or :




$\Rightarrow b = -\sqrt{5^2-2^2} = -\sqrt{21} \Rightarrow \sin\theta = -\sqrt{21}/5$

b. [2] $\tan\theta =$

$\ldots = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{21}/5}{2/5} = -\frac{\sqrt{21}}{2}$