

**CARLETON UNIVERSITY**  
**Department of Mechanical and Aerospace**  
**Engineering**

**MAAE3202 Mechanics of Solids II**

**COURSE MANUAL**

**2014-15**

**CARLETON UNIVERSITY**  
**Department of Mechanical and Aerospace Engineering**

**MAAE3202 Mechanics of Solids II**

**COURSE OUTLINE**

**1. Review of Basic Principles of Solution**

Equilibrium; stress-strain relations; compatibility. Examples of statically determinate and indeterminate systems.

**2. Stress and Strain Transformations**

Review of notations for stresses. Stresses on a plane inclined to the direction of loading. 2D general stress transformations; Mohr's stress circle; principal stresses; maximum shear stress; applications. 2D general strain transformations; Mohr's strain circle; principal strains. Use of electric resistance strain gauges; construction of Mohr's circle for rosette strains.

**3. Torsion**

Review of torsion of circular sections. Torsion of thin-walled non-circular sections.

**4. Elastic Beam Bending**

Review of simple beam bending; features of symmetric bending. Determination of centroidal axes; unsymmetric bending; shear stresses in thin-walled sections; shear centre.

**5. Energy Methods**

Strain and complementary energy methods; Castigliano's theorems for determining deflections and redundants in statically determinate and indeterminate systems.

**6. Criteria for Yielding and Brittle Rupture**

Review of elastic versus plastic deformation. Rankine's criterion, Tresca's criterion and von Mises' criterion for yielding; Mohr's theory for brittle rupture.

**7. Analysis of Variation of Stress and Strain**

Governing equations in two-dimensional elasticity: strain-displacement relations; equilibrium equations; compatibility equations in terms of strains and stresses.

Requirements for solution.

## 8. Axi-symmetric Deformations

Governing equations in axi-symmetric stress analysis. Stress distributions in a thick-walled cylinder: Lamé's solution and the Lamé line method; compound cylinders. Stresses in thin rotating discs.

### Assessment

Laboratory Experiments & Report	25%
Mid-term examination	15%
Final examination	60%
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Total	100%
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### Note:

- (a) A student who receives a **FAIL** grade in either the Final Examination or the Laboratory component will receive a **FAIL** grade in this course.
- (b) Absenteeism from any laboratory session and/or non-submission of the log-book for same without a valid reason will automatically result in a **FAIL** grade for the Laboratory component of the course.
- (c) Every student is required to have a proper hardcover scientific/engineering log-book for the laboratory sessions. It is to be submitted for grading at the end of each of these sessions. Any other form will not be accepted.
- (d) In the event of missed assessed term work (i.e. laboratory, mid-term examination) because of illness, it must be fully supported by a medical certificate dated no more than one day after the term work due date and it must be submitted to the instructor within 5 working days. Failure to do so will result in zero mark given to that component of the work.

### Recommended Text

P.P. Benham, R.J. Crawford and C.G. Armstrong, Mechanics of Engineering Materials, 2<sup>nd</sup> Edition, Longman Group Ltd. (1996)

**Department of Mechanical & Aerospace Engineering  
Carleton University**

**MAAE3202 Mechanics of Solids II**  
*(formerly 86.322\*)*

**PROBLEM SETS**

## MAAE3202 Mechanics of Solids II

**Problem Set 1: Complex Stresses and Strains**

1. At a point in a boiler rivet, the material of the rivet is undergoing a shear stress of 77 MPa while resisting movement between the boiler plates and a tensile stress of 62 MPa due to the extension of the rivet.

Find the magnitude of the tensile stresses acting at a point on the two planes making an angle of  $80^\circ$  to the axis of the rivet.

(86.5 MPa, 33.8 MPa)

2. At a point in a stressed material, the stresses on a certain plane are a direct compressive stress of 15.4 MPa and a shear stress of 62 MPa. The normal stress on the perpendicular plane is zero. Determine the magnitudes and directions of the principal stresses and the maximum shear stresses at the point.

(54.8 MPa at  $48.5^\circ$  ; -70.2 MPa at  $138.5^\circ$  ; and  $\pm 62.5$  MPa at  $3.5^\circ$  and  $93.5^\circ$ )

3. A thin cylindrical pressure vessel has a bore of 25 mm and a wall thickness of 2 mm. The cylinder is subjected to an internal pressure of 10 MPa and is known to carry an undetermined torsional load. In service, the cylinder fractures locally at an angle of  $35^\circ$  to the axis. Assuming the fracture to have taken place in the plane of maximum shear stress, determine the torsional load on the cylinder and the magnitudes of the principal stresses in the cylinder wall.

(5.68 MPa; 13.0 Nm; 63.5 MPa, 30.3 MPa)

4. An element of material is subjected to stresses  $\sigma_x = -120$  MPa,  $\sigma_y = -30$  MPa and  $\tau_{xy} = 90$  MPa. What is the maximum value of tensile stress in the element? The normal of the plane on which this occurs is inclined at an angle  $\phi$  to the x-axis. What is the value of  $\phi$ ?

If the stresses  $\sigma_x$  and  $\sigma_y$  are fixed in magnitude, determine the greatest permissible value of  $\tau_{xy}$  if tensile stresses are to be avoided anywhere in the element.

(25.6 MPa;  $58^\circ 17'$ ; 60 MPa)

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5. At a particular point on the surface of a component, the principal strain directions are known, but it is not convenient to attach electrical resistance strain gauges in these directions. However, it is possible to cement gauges at  $30^\circ$  and  $60^\circ$  anti-clockwise from the major principal strain direction, and the readings from these gauges are  $+0.0009$  and  $-0.0006$ , respectively. Construct the Mohr's strain circle and determine the principal stresses and maximum shear stress.  $E = 208 \text{ GPa}$ ,  $\nu = 0.29$ .

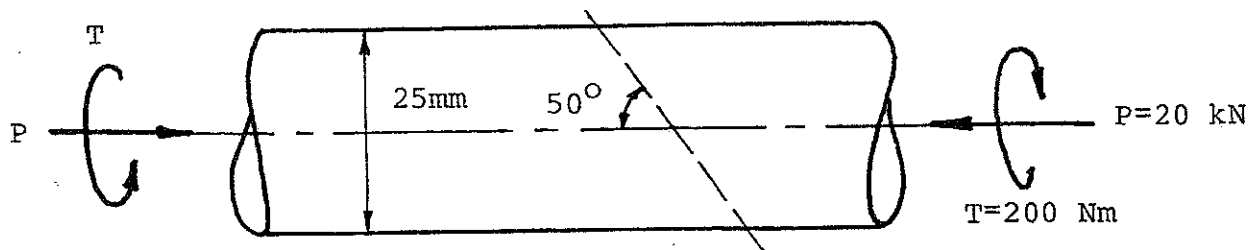
(287 MPa, -200 MPa, 243 MPa)

6. Three strain gauges A, B and C are fixed to a point on the surface of a test plate at  $120^\circ$  intervals and the strains recorded are  $\epsilon_A = +0.00108$ ,  $\epsilon_B = +0.00064$ ,  $\epsilon_C = +0.00090$ . Construct the Mohr's strain circle for this problem and determine the principal strains and the inclination of gauge A to the direction of the greater principal strain.

( $\epsilon_1 = +0.001135$ ,  $\epsilon_2 = +0.000615$ ;  $19^\circ$  c-wise)

7. A solid cylindrical shaft, 25 mm in diameter, is under a compressive force  $P = 20 \text{ kN}$  while transmitting power with a torque  $T = 200 \text{ Nm}$ .
- (a) Using a Mohr's stress circle, determine the principal stresses and the maximum shear stress at a point on the outer circumferential surface of the shaft. Show on a diagram the orientations of the physical planes with respect to the shaft axis where these stresses act.
- (b) What is the normal stress and the shear stress at the same point on the outer surface of the shaft but acting on a plane which is inclined at  $50^\circ$  to the axis, as shown in the figure.

[(a) 47.93 MPa,  $53.68^\circ$  c-wise; -88.67 MPa,  $36.32^\circ$  c-wise; 68.32 MPa.  
(b) 40.23 MPa; 31.39 MPa.]



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8. A thin-walled cylindrical tubing made of an aluminium alloy (Young's modulus, 69 GPa; Poisson's ratio, 0.33) has 50 mm mean diameter and 2.0 mm wall thickness, and may be treated as having closed ends. It contains a fluid under pressure that reaches 6 MPa in service. At the same time, the tubing carries an unknown torque,  $T$ , about its longitudinal axis and an end-thrust,  $F$ , of 8 kN. An important design criterion against failure of this component is that the maximum shear stress in the wall of the cylinder must not exceed 70 MPa.
- (a) Using the Mohr's stress circle, determine the largest permissible value of the torque  $T$ . What are the maximum and minimum direct stresses that exist in the wall of the cylinder under these service loads? State any assumptions made.
  - (b) If a strain gauge is to be mounted on the surface of the cylinder to verify the largest magnitude of the strain under the above loads, along what orientation with respect to the longitudinal axis should it be placed? What would be the expected reading?

For this problem, explain briefly and without doing any calculations, what effect, if any, would reducing the magnitude of the end thrust have on the maximum shear stress in the wall of the cylinder if all the other loads are fixed. You may find it useful to explain this with reference to the Mohr's circle.

[(a) 491 Nm; 113.5 MPa, -26.5 MPa. (b)  $31.6^\circ$ ; 0.00177]

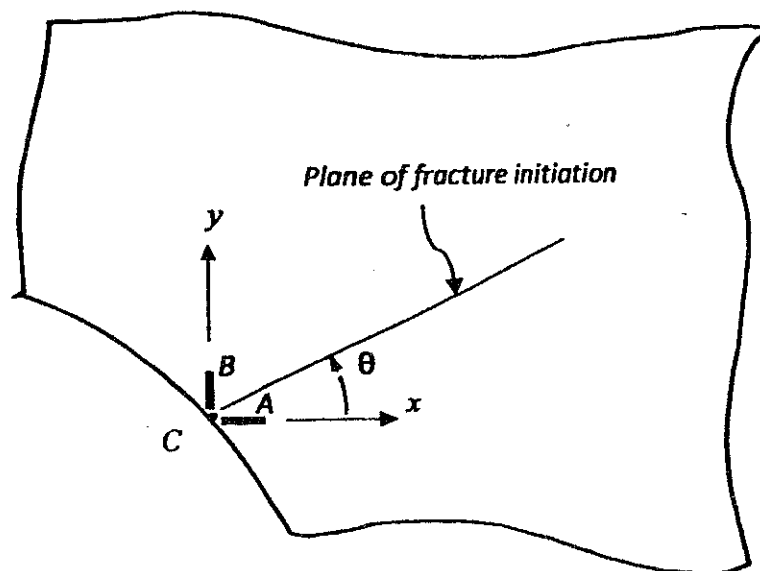
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9. An engineering component has a uniform thickness but is of relatively complex geometry in the  $x$ - $y$  plane. It is made of a glass ceramic which is linear elastic under the service conditions. The ceramic is a brittle material, with tensile rupture strength of 94 MPa but compressive rupture strength of 345 MPa. Its Young's modulus is 66.9 GPa and Poisson's ratio is 0.29

In service, the component is subjected to an in-plane external load  $P$  which gives rise to a complex stress state in the  $x$ - $y$  plane due to its geometry. From frequent overloads of  $P$ , it has been observed that fracture initiates locally at the point  $C$  and ruptures along the plane as indicated in the figure, with  $\theta = 20^\circ$ . To help establish the maximum value of  $P$  that can be applied to avoid crack initiation, two strain gauges,  $A$  and  $B$ , are bonded on the free surface at the critical point  $C$  along the  $x$ - and  $y$ -directions, respectively. When the load  $P = 3.2$  kN is applied, the following strains are recorded:  $\varepsilon_A = 1.586 \times 10^{-4}$  m/m and  $\varepsilon_B = 6.904 \times 10^{-4}$  m/m.

- Assuming that fracture initiates along the plane on which the maximum tensile stress acts, determine using the Mohr's circle, the principal stresses and the maximum shear stress at the point  $C$  when  $P = 3.2$  kN.
- What is then the largest permissible value of  $P$  to avoid rupture?
- Briefly, suggest an alternative way to determine the principal stresses using strain gauges with a different set-up from the one used above and without the need for the assumption in Part (a).

[(a) 58 MPa, 22 MPa, 18MPa; (b) 5.19 kN]



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**Problem Set 2: Torsion of Non-circular Thin-walled Sections**

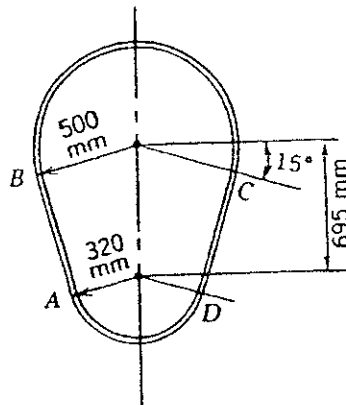
1. An extruded aluminum tubing used in structural applications is of rectangular cross-section with exterior side lengths 60 mm and 100 mm respectively. The wall thickness of the tubing is 4 mm. Determine the shear stress in each of the four walls when it is subjected to a torque of 3.5 kN-m.

As a result of defective fabrication, two perpendicular sides of the walls are 3 mm thick, while the other two are 5 mm thick. What will be the shear stresses in the walls when the tube is subject to the same torque?

(81.4 MPa; 108.5 MPa, 65.1 MPa)

2. A cross-section of an airplane fuselage made of aluminum alloy is shown in the figure. For an applied torque of 200 kN-m and an allowable shear stress of 50 MPa, determine the minimum thickness of the sheet required to resist the torque. The thickness may be assumed constant for the entire periphery.

(1.75 mm)



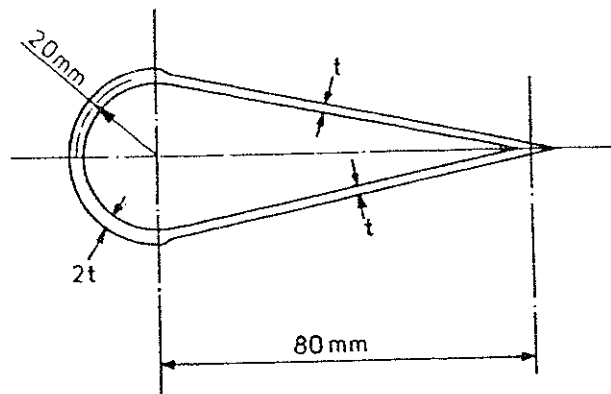
3. Compare the torsional strength and stiffness of thin-walled tubes of circular cross-section of mean radius  $R$  and thickness  $t$  with and without a longitudinal slot.

$((3R/t), (t^2/3R^2))$

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4. An aluminium alloy member having the cross-section shown in the figure below is 3 m in length. If the shear stress is not to exceed  $30 \text{ MN/m}^2$  and the applied torque is  $134 \text{ N-m}$ , determine the required thickness,  $t$ , of metal. What is the angle of twist?  $G = 28 \text{ GN/m}^2$ .

(1 mm, 0.142 rad.)



5. The figure below shows an idealised wing section in the region of an undercarriage bay of an aircraft. It is subjected to a torque of  $10 \text{ kN-m}$ .

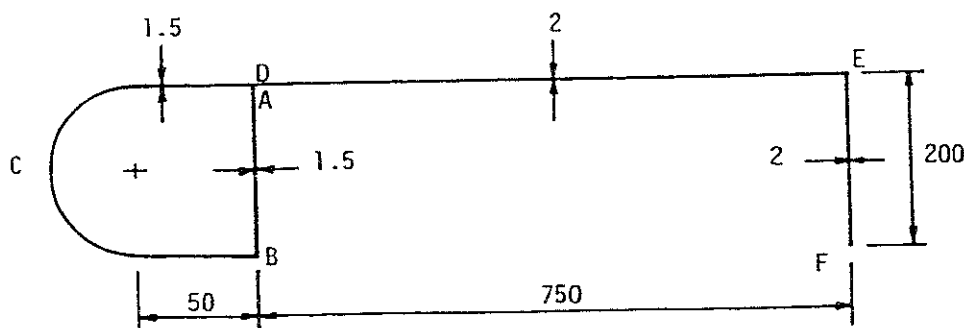
Determine

- the angle of twist per unit length in the wing, and
- the shear stresses in the walls of the section.

What will be the corresponding angle of twist per unit length if the end A is not bonded to the point D, that is, if A-B-C-D-E-F is an open section?

Assume that the length dimensions shown are measured between the mid-thickness planes. the shear modulus of the material  $G = 27 \text{ GPa}$ .

((a) 0.0573 rad/m; (b) 129.6 MPa, 3.094 MPa; 114.9 rad/m)



(All dimensions in mm.)

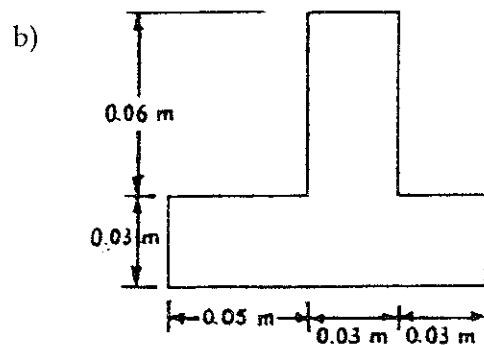
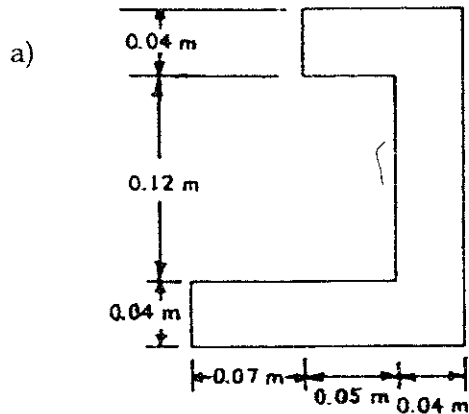
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## Problem Set 3: Unsymmetric Bending and Shear Centre

1. Determine for the sections shown in the figures below the principal centroidal axes and the principal second moments of area.

$$(a) I_v = 21.1 \times 10^{-6} \text{ m}^4, \quad I_w = 73.44 \times 10^{-6} \text{ m}^4, \quad \theta = -19.3^\circ$$

$$(b) I_v = 3.93 \times 10^{-6} \text{ m}^4, \quad I_w = 2.81 \times 10^{-6} \text{ m}^4, \quad \theta = 33.5^\circ$$



2. A bending couple  $M$  of magnitude 1.5 kN-m acting in a vertical plane is applied to a cantilever beam having the Z-shaped cross section as shown in the figure. Determine (a) the stress at point A, and (b) the angle which the neutral axis forms with the horizontal plane.

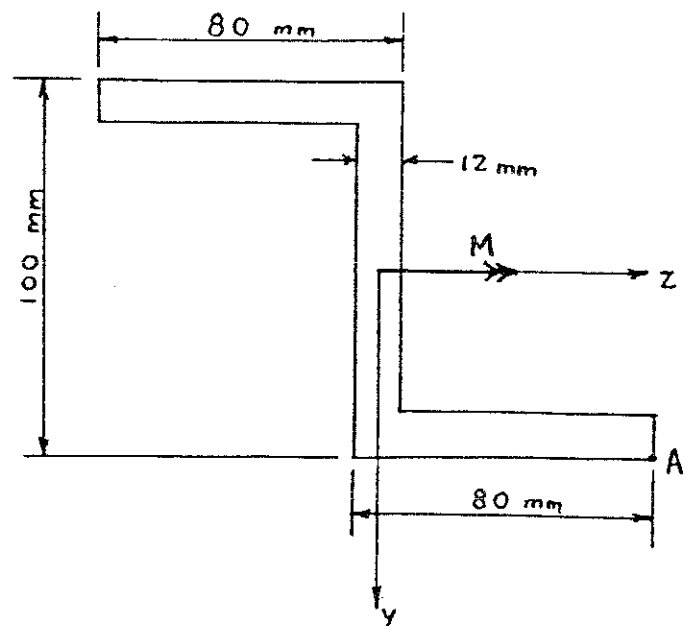
The second and product moments of area have been computed and are:

$$I_y = 3.25 \times 10^{-6} \text{ m}^4$$

$$I_z = 4.18 \times 10^{-6} \text{ m}^4$$

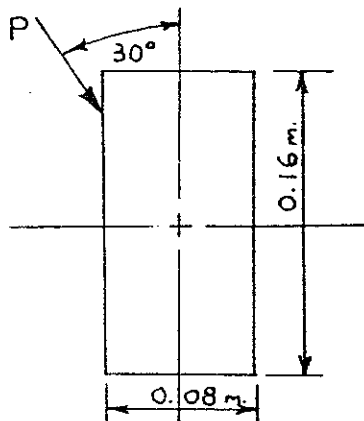
$$I_{yz} = 2.87 \times 10^{-6} \text{ m}^4$$

$$(+13.87 \text{ MPa}; 41.4^\circ)$$



## MAAE3202 Mechanics of Solids II

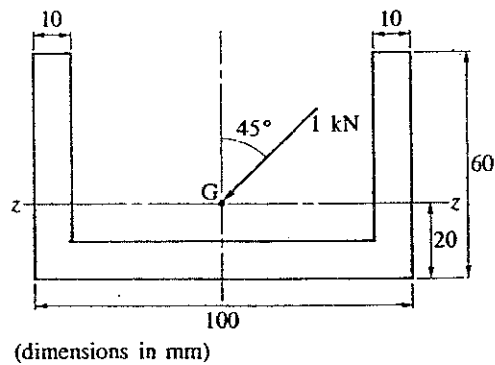
3. A cantilever beam 7 m long carries a concentrated load of 5 kN at its free end. The beam has a rectangular cross section with dimensions as shown in the figure. The plane of the load  $P$  is inclined at  $30^\circ$  to the vertical plane. Locate the neutral axis and compute maximum flexural stresses induced in the beam.



$$(\phi = -66.6^\circ ; +191.3 \text{ MPa} , -191.3 \text{ MPa} )$$

4. A channel-section beam 1 m long is built in at one end and subjected to a point load of 1 kN at the free end. If the direction of the load is as in the figure calculate the direction of the neutral axis and the maximum values of the tensile and compressive stresses on the section.

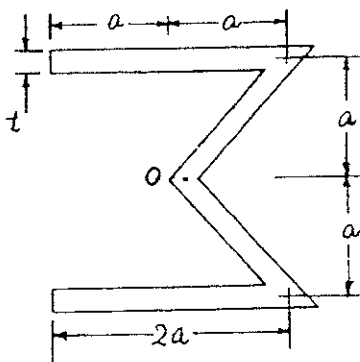
$$(54.7 \text{ MPa}; -33.5 \text{ MPa})$$



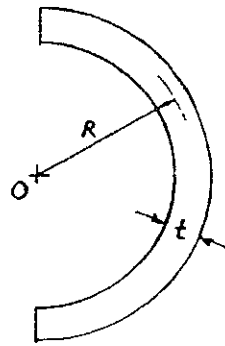
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5. Determine the location of the shear centre for the following symmetric cross sections. Assume the load to act perpendicular to the axis of symmetry and that the thickness  $t$  is small compared to the overall dimensions of the cross section. For problems (a) and (c), leave the solution in terms of the second moment of area about the axis of symmetry.

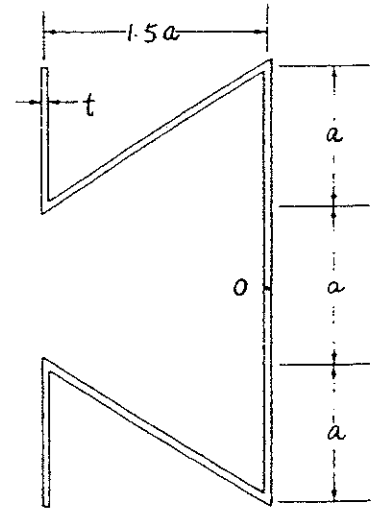
(Positions to the right of point O : (a)  $4ta^4/I$ ; (b)  $4R/\pi$ ; (c)  $6.126 ta^4/I$ )



(a)



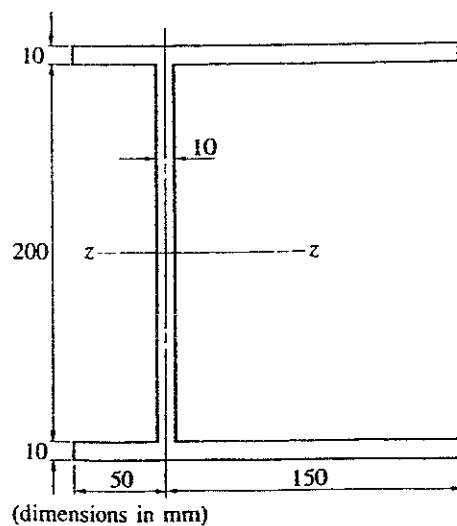
(b)



(c)

6. Determine the location of the shear centre for the beam cross-section shown in the figure. Also calculate maximum values of the horizontal and vertical shear stresses in a flange and the web respectively for a vertical load of 500 kN applied at the shear centre.

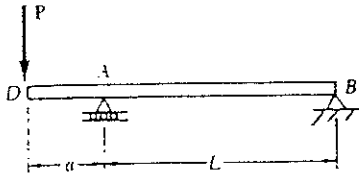
(43.4 mm; 155 MPa; 256 MPa)



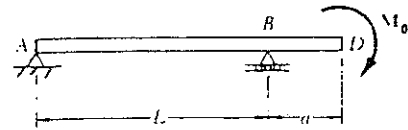
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**Problem Set 4: Energy Methods**

1. Using the direct energy method, determine the deflection and slope, respectively, at point D caused by the load P and the couple  $M_0$  as shown in the figures.

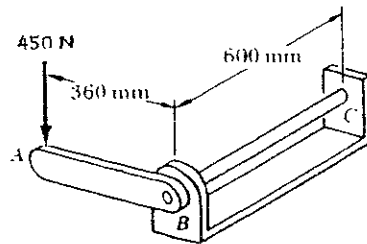


$$(Pa^2(a+L)/3EI; M_0(L+3a)/3EI)$$



2. The 18 mm diameter steel rod BC is attached to the lever AB and to the fixed support C. The uniform steel lever is 9 mm wide and 24 mm deep. Using the direct energy method, determine the deflection of point A. Take the Young's modulus of steel as 200 GPa and the shear modulus as 80 GPa.

$$(45.8 \text{ mm})$$

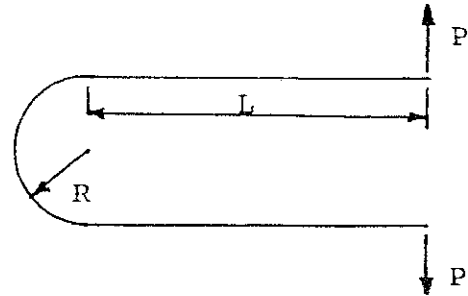


3. Solve Problems 1 - 2 using Castigliano's theorems.

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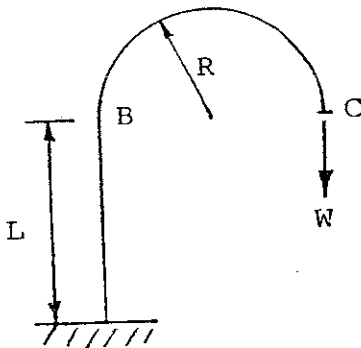
4. The bar of circular cross-section, diameter  $d$ , is bent as shown. Using Castigliano's second theorem, show that the relative vertical displacement between the two ends is:

$$\delta_p = \frac{32 P (4 L^3 + 6 \pi R L^2 + 24 R^2 L + 3 \pi R^3)}{3 \pi E d^4}$$



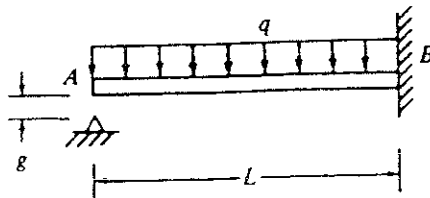
5. The structure shown is of constant flexural rigidity  $EI$  throughout. It carries a load  $W$  at point C. Determine expressions for the vertical deflection at C and horizontal deflection at B.

$$(\delta_{C_v} = 2WR^2[3\pi R/4 + 2L]/EI; \delta_{B_H} = WRL^2/EI)$$



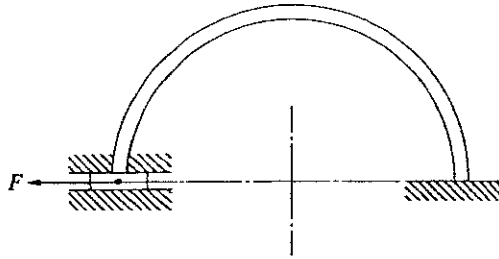
6. The uniform cantilever beam shown makes contact with the support at A only after gap  $g$  is closed by application of the uniformly distributed load of intensity  $q$  per unit length. What is the reaction at A in terms of  $q$ ,  $L$ ,  $g$  and  $EI$ ? What is the largest value of  $g$  for which the answer obtained above is valid?

$$(3(qL/8 - EIg/L^3); qL^4/8EI)$$



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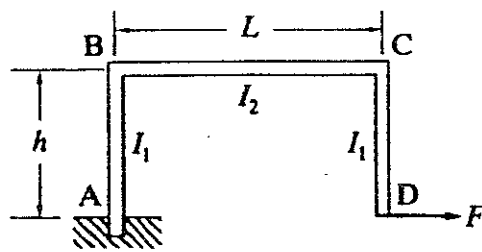
7. A curved beam is in the form of a semicircle as shown in the figure. The free end is pinned but is restrained to move in the horizontal direction only. If a horizontal force  $F$  is applied at the free end show that the vertical restraining force on the end of the beam is given by  $4F/3\pi$ .



8. The frame ABCD is fixed at A and a horizontal force  $F$  is applied at D. B and C are rigidly jointed and the lengths of the members are  $AB = CD = h$  and  $BC = L$ . The relevant second moments of area are  $I_1$  for AB and CD and  $I_2$  for BC. Show that if D moves under load at an angle of  $45^\circ$  to the horizontal then

$$\frac{I_1}{I_2} = \frac{b}{3L} \left\{ \frac{3L - 4b}{2b - L} \right\}$$

Ignore the effects of axial load and shear force on the frame.



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**Problem Set 5: Yield Criteria**

1. A test sample of a material is loaded in the form of a simple cantilever of 3 mm x 3 mm square cross-section and 50 mm length. Initial yielding of the cantilever occurs when an end load of 19.8 N is applied.

Determine the maximum load in pure torsion for a 12 mm diameter solid circular shaft of the same material if yielding is not to occur according to (a) the maximum shear stress and (b) the shear strain energy criteria of yielding.

Neglect shear stress and the clamping effect at the root of the cantilever.

(37.4 Nm; 43.1 Nm)

2. The pressure hull of a submarine is 3 metres in diameter and 50 metres long. The steel used in the construction has a yield point in tension of 280 MPa. Use (a) the shear strain energy criterion and (b) the maximum shear stress criterion, and considering the hull as a thin-walled cylindrical pressure vessel, find the minimum required wall thickness of the hull if yielding is not to occur at a depth of 300 metres. Use a factor of safety of two and take the density of sea water as 1040 Kg/m<sup>3</sup>.

Neglecting the deflection of the end plates and assuming that the vessel just has buoyancy on the surface, calculate the volume of water that must be pumped from the buoyancy tanks to maintain a depth of 300 metres.

((a) 28.4 mm; (b) 32.8 mm; (c) 0.525 m<sup>3</sup> using (a).)

3. A solid brass shaft is supported in bearings 3.0 m apart and transmits 40 kW at 1.0 rev/s. In addition, it carries a vertical load of 20 kN at a distance of 0.5 m from one bearing and 2.5 m from the other. If the brass has a 0.1% proof stress in tension of 210 MPa, find the required shaft radius, to give a factor of safety against yielding of 2.5, according to (a) the maximum principal stress, and (b) the maximum shear stress criteria of yielding.

If the shaft was designed according to criterion (a) whereas the material yielded according to criterion (b) what bending moment could be sustained for the given power transmission?

((a) 52.2 mm ; (b) 54.1 mm ; 6.9 kNm)

4. A bar of solid circular cross-section carries a bending moment of 12 kNm and a torque of 8 kNm. The tensile and compressive strengths of the bar material are 17 MPa and 200 MPa respectively. Using a safety factor of 3, what should be its minimum diameter?

(288 mm)

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## Problem Set 6: Analysis of Variation of Stress and Strain

1. Using the equations of equilibrium in Cartesian co-ordinates and assuming the longitudinal stress,  $\sigma_x$ , is given by the simple beam theory expression  $\sigma_x = (My/I)$ , show that in a rectangular beam of depth  $2d$  and of unit thickness, the stress through the depth of the beam, when used as a cantilever carrying a uniformly distributed load  $w$  per unit length, is given by:

$$\sigma_y = \frac{w}{2I} \left( -\frac{1}{3}y^3 + d^2y - \frac{2}{3}d^3 \right)$$

2. Check if each of the following fields is a valid solution of a two-dimensional elasticity problem. The  $a_i$ 's are constants.

(a)  $\sigma_x = -a_1x^2y$ ,  $\sigma_y = -a_1y^3/3$ ,  $\tau_{xy} = a_1xy^2$ .

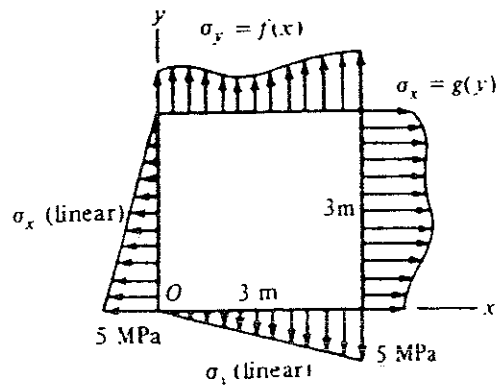
(b)  $\epsilon_x = a_1x^3$ ,  $\epsilon_y = a_1x^2y$ ,  $\gamma_{xy} = a_1xy^2$ .

(c)  $\sigma_x = a_1x + a_2y$ ,  $\sigma_y = a_3x + a_4y$ ,  $\tau_{xy} = -a_4x - a_1y$ .

3. The boundary of a 3 m x 3 m square plate is subjected to direct stresses  $\sigma_x$  and  $\sigma_y$  as shown. It is also subjected to a shear stress distribution on the boundary such that the shear stress  $\tau_{xy}$  at every point in the plate is given by

$$\tau_{xy} = 3x^2 + 7y^2 + 2x + 2.5 \quad (\text{in MPa})$$

- (a) Find the principal stresses at the origin.  
 (b) Find the functions  $f(x)$  and  $g(y)$  required for equilibrium.  
 (c) Does the stress field within the plate satisfy compatibility requirements?



**Problem Set 7: Axisymmetric Deformations - Thick-walled Pressurised Cylinders**

1. A steel pipe 0.125 m internal diameter, wall thickness 10 mm, is subjected to an internal pressure of 11.0 MPa. What are the maximum hoop, radial and shear stresses, if the axial stress is zero:
- (a) when treated as a thin-walled cylinder; and
- (b) when treated as a thick-walled cylinder?

(68.8 MPa, 0, 34.4 MPa; 74.6 MPa, -11.0 MPa, 42.8 MPa)

2. The cylinder of a hydraulic press has an internal diameter of 250 mm and the water pressure is 9.6 MPa. Find suitable thickness for the cylinder:
- (a) if the hoop stress is not to exceed 55 MPa; and
- (b) if the greatest shear stress is not to exceed 34 MPa.

Plot diagrams to show the distribution of hoop and radial stresses through the wall of the cylinder in case (a).

If the axial stress may be assumed to be negligible, calculate the change in inside and outside diameters of the cylinder in reaching maximum pressure from zero in case (a).

(24.1 mm, 22.6 mm, 0.07 mm, 0.066 mm)

3. A well-fitted steel sleeve is pressed onto a 25 mm diameter steel shaft and exerts a uniform radial pressure of 28 MPa. A tensile load of 40 kN is then applied to the shaft in the direction of its length, reducing the pressure between sleeve and shaft to 21 MPa. Find the reduction in diameter of the common surface of sleeve and shaft.

(0.0024 mm)

4. A compound cylinder is formed by shrinking a tube 200 mm external diameter and 150 mm internal diameter upon another tube 115 mm internal diameter. After shrinking, the radial pressure at the common surface is 14 MPa. Determine the hoop stresses at the inner and outer surfaces of each tube. What was the radial interference necessary to give these stresses for steel tubes?

Plot diagrams to show the variation of the hoop and radial stresses with radius for both tubes.

(-67.9 MPa, -54 MPa, +50 MPa, +36 MPa; 0.038 mm)

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5. The outer diameter of a compound steel tube is 0.45 m, the bore is 0.25 m diameter and the common diameter is nominally 0.33 m. The difference in diameters of the common surfaces before shrinking was 0.135 mm. Find the intensity of the radial pressure at the common surface.

If the greatest hoop stress in either tube is not to exceed 185 MPa, find the greatest permissible internal pressure that may be applied to the compound tube. Compare this with the pressure carrying capacity of a single cylinder of the same overall dimensions and having the same hoop stress limitation.

(12.1 MPa, 113.9 MPa, 97.7 MPa)

6. A compound cylinder is built with a stainless steel liner, internal radius 100 mm and 15 mm thick, shrink-fitted into another steel cylinder of external radius 150 mm. The radial interference at the common surface is 0.1 mm. The vessel is to be subjected to an internal pressure of 100 MPa.

The stainless steel used for the liner has a yield stress which is significantly higher than that of the steel used for the outer cylinder and there is no concern of yielding occurring. The steel used for the outer cylinder has a yield stress of 400 MPa.

- a) Will yielding occur in the outer cylinder? Use Tresca's criterion of yielding.
- b) Is the lack of concern for yielding in the liner justified?

**Problem Set 8: Axisymmetric Deformations – Thin Rotating Disc**

1. A thin steel disc has zero radial stress at the inner and outer surfaces and is rotated. If the inner diameter is 0.125 m and the outer diameter is 0.25 m, find the maximum permitted speed if the hoop stress is not to exceed 80 MPa.

(8275 rpm)

2. A solid steel turbine disc of 12.5 mm thickness and 0.5 m peripheral radius has 75 blades, each of mass 0.05 kg, attached to the rim. If their centre of mass is 0.05 m from the rim, what would be the stresses at the centre of the disc?

Assume that the centrifugal forces of the blades may be uniformly distributed around the disc periphery, and the disc rotates at 4500 rpm.

(191.4 MPa)

3. In a rotating disc of outside radius  $a$  and inside radius  $b$  with zero radial stresses at  $a$  and  $b$ , show that the maximum hoop stress is at  $b$  and that the maximum radial stress is at  $\sqrt{ab}$ . Show also that the maximum hoop stress always has a larger magnitude than the maximum radial stress.

4. A narrow steel tyre of inner and outer radii 0.25 m and 0.35 m, respectively, is shrunk on the outer surface of a solid steel disc of equal thickness and the compound wheel is then rotated. The design is such that the pressure which the tyre exerts on the periphery of the disc would reduce to zero if the speed were to reach 350 rev/s. What was the difference in diameter between the inside of the tyre and the outside of the disc before assembly?

(9.26 mm)