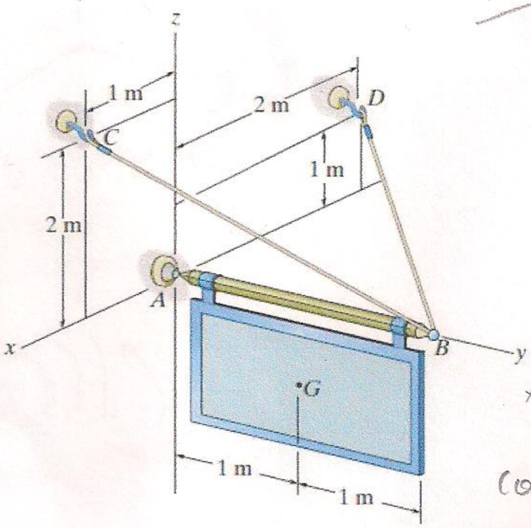
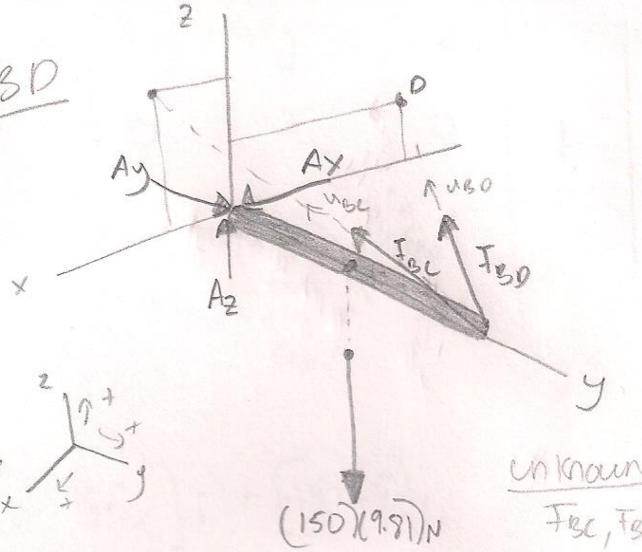
PLEASE NOTE, This exam should have a total of 9 pages, not including the first page. There are a total of 6 questions.

Q1. The sign has a mass of 150 kg with a center of mass at G. Determine the x,y,z components of reaction a the ball-and-socket joint A and the tension in the wires BC and BD.



FBD



unknowns

$F_{BC}, F_{BD}, A_y, A_x, A_z$

5 unk. 6 eqns ; can solve

Coordinates of
x y z
c (1, 0, 2)
d (-2, 0, 1)
a (0, 0, 0)
b (0, 2, 0)

Need u_{BD} and u_{BC}

D-B

$r_{BD} = (-2-0)\vec{i} + (0-2)\vec{j} + (1-0)\vec{k}$

$= -2\vec{i} + -2\vec{j} + 1\vec{k}$

$|r_{BD}| = 3$

$\boxed{u_{BD} = -\tfrac{2}{3}\vec{i} - \tfrac{2}{3}\vec{j} + \tfrac{1}{3}\vec{k}}$

C-B

$r_{BC} = (1-0)\vec{i} + (0-2)\vec{j} + (2-0)\vec{k}$

$|r_{BC}| = \sqrt{1\vec{i}^2 + -2\vec{j}^2 + 2\vec{k}^2} = 3$

$\boxed{u_{BC} = \tfrac{1}{3}\vec{i} + -\tfrac{2}{3}\vec{j} + \tfrac{2}{3}\vec{k}}$

$F_{BD} = F_{BD} \cdot u_{BD} = F_{BD} \cdot -\tfrac{2}{3}\,\vec{i} - \tfrac{2}{3}F_{BD}\vec{j} + \tfrac{1}{3}F_{BD}\vec{k}$

vector scalar

$T_{BC} = F_{BC} \cdot u_{BC} = F_{BC} \cdot \tfrac{1}{3}\vec{i} + -\tfrac{2}{3}F_{BC}\vec{j} + \tfrac{2}{3}F_{BC}\vec{k}$

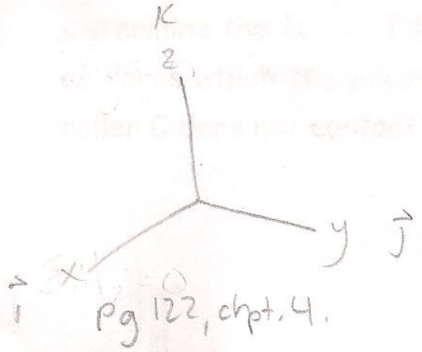
$\Sigma F_x = 0 = A_x - F_{BD}(\tfrac{2}{3}) + F_{BC}(\tfrac{1}{3}) = 0$

$\Sigma F_y = 0 \quad A_y - \tfrac{2}{3}F_{BD} - \tfrac{2}{3}F_{BC} = 0$

$\Sigma F_y = 0 \quad A_z + \tfrac{1}{3}F_{BD} + \tfrac{2}{3}F_{BC} - 1471.5 = 0$

$r \times F$

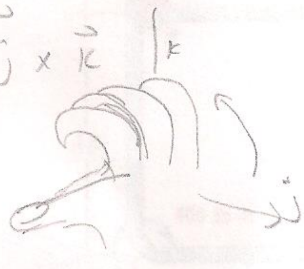
$$\sum M_A = 0 = (1\vec{j})[1471.5\vec{k}] + (2\vec{j})\Big[\big(\tfrac{2}{3}F_{BD} + F_{BC}\tfrac{1}{3}\big)\vec{i} + \big(-\tfrac{2}{3}F_{BC}\vec{j} + -\tfrac{2}{3}F_{BD}\vec{j}\big) + \big(\tfrac{1}{3}F_{BD} + \tfrac{2}{3}F_{BC}\big)\vec{k}\Big] = 0$$



pg 122, chpt. 4.

$$M = r \times f = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \left(\tfrac{2}{3}F_{BD} + \tfrac{1}{3}F_{BC}\right) & \left(-\tfrac{2}{3}F_{BC} - \tfrac{2}{3}F_{BD}\right) & \left(\tfrac{1}{3}F_{BD} + \tfrac{2}{3}F_{BC}\right) \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1471.5 \end{vmatrix}$$

$\vec{j} \times \vec{k}$



$j \times k = i$

$j \times i$

$$= \left[\tfrac{2}{3}F_{BD} + \tfrac{4}{3}F_{BC}\right]i + [-1471.5]i$$

$$+ [0 + 0]\vec{j}$$

$$+ \left[0 - \left[-\tfrac{4}{3}F_{BD} + \tfrac{2}{3}F_{BC}\right]\right]\vec{K}$$

$\sum M_x = 0 \quad \tfrac{2}{3}F_{BD} + \tfrac{4}{3}F_{BC} + 1471.5 = 0$

$\sum M_y = 0$

$\sum M_z = 0 = \tfrac{4}{3}F_{BD} - \tfrac{2}{3}F_{BC} = 0$

$\frac{4}{\cancel{3}}F_{BD} = \frac{2}{\cancel{3}}F_{BC}$

$F_{BD} = \frac{2}{4}F_{BC} = 0.5F_{BC}$

$\tfrac{2}{3}(0.5F_{BC}) + \tfrac{4}{3}F_{BC} = +1471.5$

$\tfrac{5}{3}F_{BC} = +1471.5$

$$\boxed{\begin{aligned} F_{BC} &= 882.9\,N \\ F_{BD} &= 441.45\,N \end{aligned}}$$

$A_y = 882.9\,N$

$A_z = 735.75\,N$

$A_x = 0\,N$

$\sum F_x \quad A_x = (\tfrac{2}{3})(441.45) - \tfrac{1}{3}(882.9) = 0$

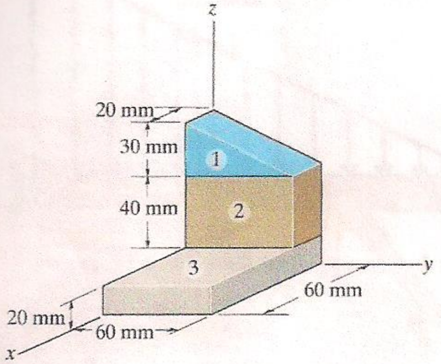
$A_y = (\tfrac{2}{3})(441.45) + (\tfrac{2}{3})(882.9) = 882.9\,N$

$A_z = 1471.5 - (\tfrac{1}{3} \cdot 441.45) - (\tfrac{2}{3} \cdot 882.9)$

$= 735.75\,N$

$1.5 \times \frac{0.001}{1\,m} = 0.0015\,m^3 \times 1000\,kg/m^3$

Q3. Locate the center of mass of the blockk. Materials 1, 2 and 3 have densities of 2.70 Mg/m^3, 7.5 Mg/m^3, 3.8 Mg/m^3 respectively.



① $\tilde{x} = 10mm$

$\tilde{y} = 20mm$

$\tilde{z} = 10m + 60mm = (30)(1/3) + 60$

$V = (60)(1/2)(30)(20) = 18000\ mm^3$

② $\tilde{x} = 10mm$

$\tilde{y} = 30mm$

$\tilde{z} = 20mm + 20mm = 40mm$

$V = (20)(40)(60) = 48000m^3$

③ $\hat{x} = 30mm$

$\bar{y} = 30mm$

$\hat{z} = 10mm$

$V = 60(60)(20) = 72000\ mm^3$

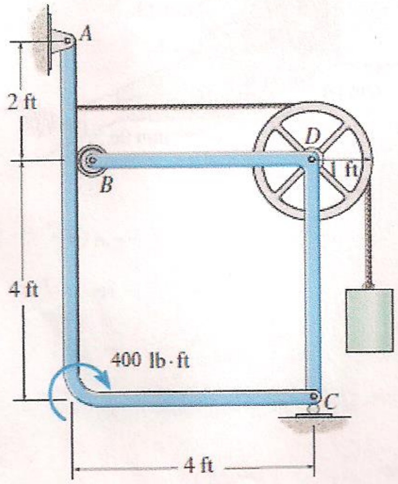
Segment	m(Kg) $kg/mm^3 \times mm^3$	$\tilde{x}$ mm	$\hat{y}$ mm	$\tilde{z}$ mm	$m\tilde{x}$	$m\hat{y}$	$m\tilde{z}$
1	(18000)(2.7)	10	20	70	486000	972000	3462000
2	360000 (48000)(7.5)	10	30	40	3600000	10800000	14400000
3	273600 (72000)(3.8)	30	30	10	8208000	8208000	2736000
total	682200 kg				12294000	19980,000	20538000

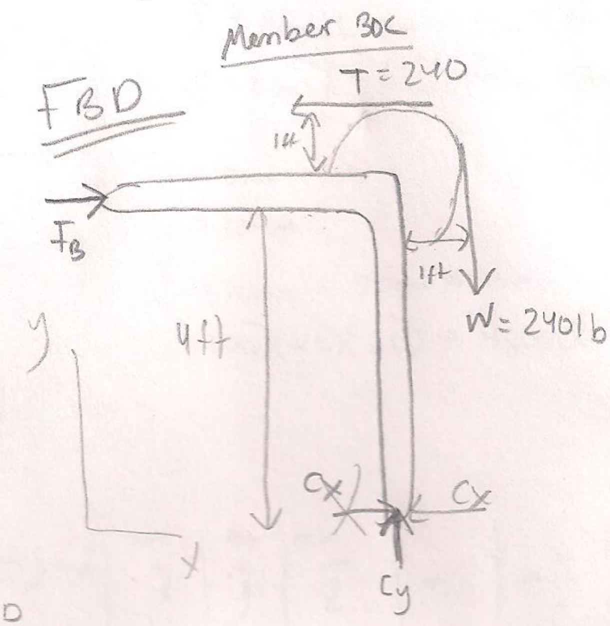
$\bar{x} = \dfrac{\Sigma m\bar{x}}{m} = \dfrac{12294000}{682200} = 18.02mm$ $\bar{x}$

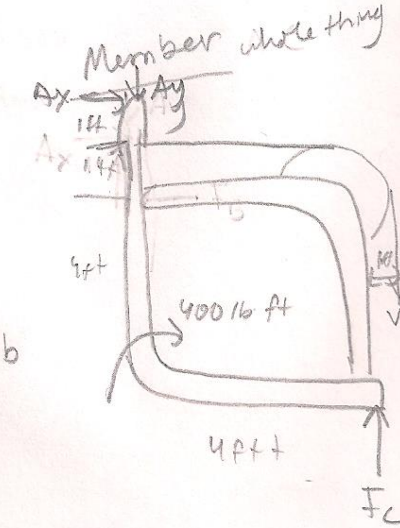
$\bar{y} = \dfrac{\Sigma m\tilde{y}}{m} = \dfrac{19980000}{682200} = 29.287mm$ $\bar{y}$

$\bar{z} = \dfrac{\Sigma m\tilde{z}}{m} = \dfrac{20538000}{682200} = 30.105mm$ $\bar{z}$

Q2. The two-member frame supports the 240-lb cylinder and 400 lb·ft couple moment. Determine the force of the roller at B on member AC and the horizontal and vertical component of force which the pin at C exerts on member CB and the pin at A exerts on member AC. The roller C does not contact member CB.







from entire Member FBD

$+\circlearrowleft \sum M_A = 0$

$F_C(4) - (240)(5) - 400 = 0$

$F_C = 400\,lb$

$\rightarrow \sum F_x = 0 \qquad A_x = 0$

$\uparrow \sum F_y = 0 \qquad F_C - A_y - W = 0$

$400 - 240 = A_y$

$A_y = 160\,lb$

$\rightarrow \sum F_x = 0$

$240 - 240 - C_x = 0$

$C_x = 0$

$+\uparrow \sum F_y = 0$

$C_y - 240 = 0$

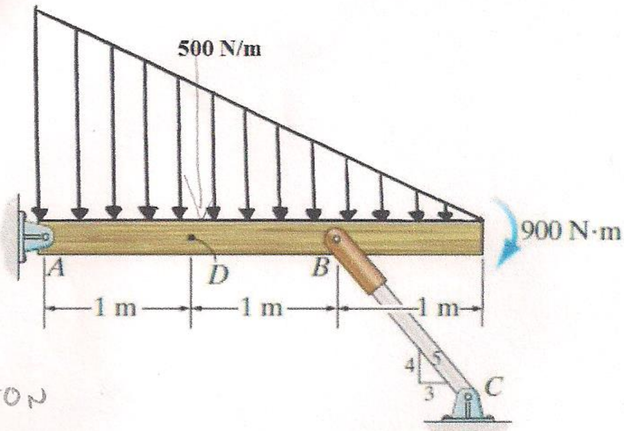
$C_y = 240$

for member BC

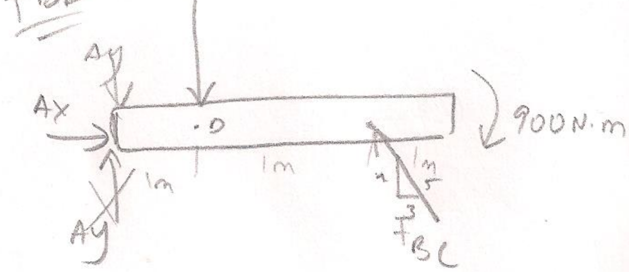
$+\circlearrowleft \sum M_C = 0 \qquad -(F_B)(4ft) + 240(5) - 240(1) = 0$

$\boxed{F_B = 240}$

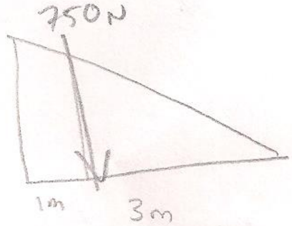
Q4. Determine the internal normal force, shear force and moment at point D in the beam.



FBD



3 unknowns, 3 eqns



$(500)(\tfrac{1}{2})(3) = 750\text{N}$

Solve first

$\circlearrowleft \sum M_A = 0$

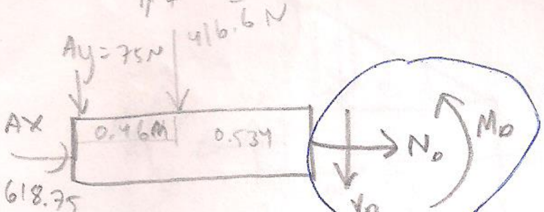
$+750(1\text{m}) - F_{BC}(4/5)(2) + 900 = 0$

$\boxed{F_{BC} = 1031.25\text{N}}$

$\sum F_x = 0 \quad A_x = (3/5)(1031.25) = 618.75$

$\sum F_y = 0 \quad -750 + A_y + (4/5)(1031.25) = 0$

$A_y = -75\text{N}$ assumption was wrong $75\text{N} \downarrow A_y$



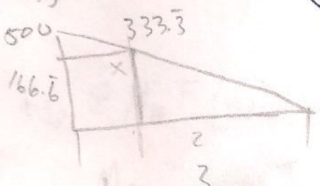
Postive directions, if one is negative, they are all negative (dont assume though, show your calculation) but make sure to draw them in positive direction, $\leftarrow N_D$ so if V_D is $\downarrow$ M_D has to be $\circlearrowright$ & $N_D \rightarrow$.

$\sum F_x = 0 \quad N_D + 618.75 = 0$

$N_D = -618.75\text{N}$

$\sum F_y = 0 \quad -75 - V_D - 416.6 = 0$

$V_D = -491.6\text{N} \quad V_D \uparrow$



$\frac{500}{3} = \frac{x}{2}$

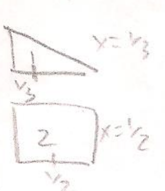
$x = 333.3$

force equivalent

$(333.3\text{N/m})(1\text{m}) + (166.6 \times \tfrac{1}{2} \times 1) = 416.6$

$\circlearrowleft \sum M_D = (75)(1) + (416.66 \times 0.534) + M_D = 0$

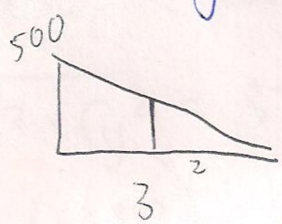
$M_D = -297.49\text{ N·m} \circlearrowleft$



$\bar{x} = \frac{(333.3 \times 0.5) + (166.6 \times 0.5 \times 1/3)}{416.6}$

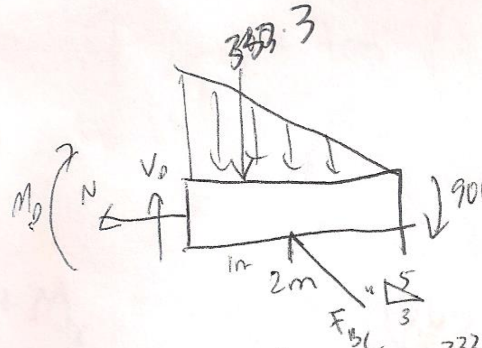
$\bar{x} = 0.466$

or you can do the right side of the beam to find



$$\frac{500}{3} = \frac{x}{2}$$

333.3



$(500)(\tfrac{1}{2})(2)$
$= 500\,N$

$\cancel{2/3} \times 2 = \tfrac{1}{3} \times 2$

$$+\circlearrowleft \Sigma M_D = (4/5)(1031.25)(1m) - (\cancel{500}\ 333.3)(0.66) - 900 + -M_D = 0$$

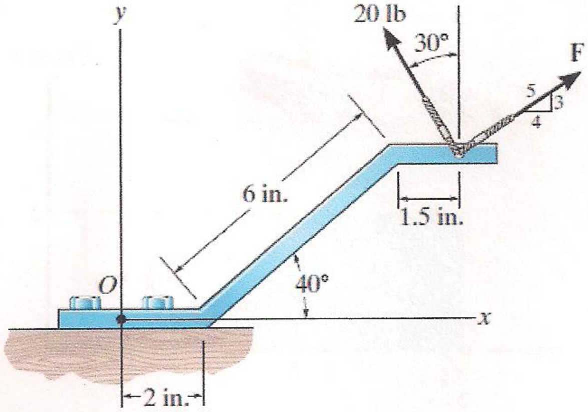
$M_D = \cancel{[illegible]} -294.97\ N{\cdot}m$

$\Sigma F_x = 0 \qquad N_D + (1031.25)(3/5) = 0$

$N_D = -618.75 \quad \rightarrow N_D$

$V_D = -491.6$

Q5. Replace the two forces by an equivalent resultant force and couple moment at point O. Set F= 30 lb.



c

3.856

40

x

4.596

$\frac{x}{6} = \cos 40$

$\xrightarrow{+} \Sigma F_{R_x} = \Sigma F_x = \left(\frac{4}{5}\right)(30) - 20\sin 30 = 14$

$\Sigma F_y = (3/5)(30) + 20\cos 30 = 35.32$

$F_R = \sqrt{F_x^2 + F_y^2} = 37.993 \text{ lb}$

$\theta = \tan^{-1}\left(\frac{39.993}{14}\right) = 69.77°$

y x 69.77° y x

x

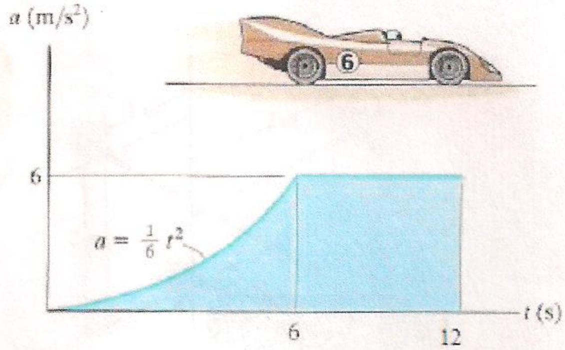
$\circlearrowleft + M_R = \Sigma M_O = (20\sin 30°)(6\sin 40°) + 20\cos 30°(3.5 + 6\cos 40)$

3.856

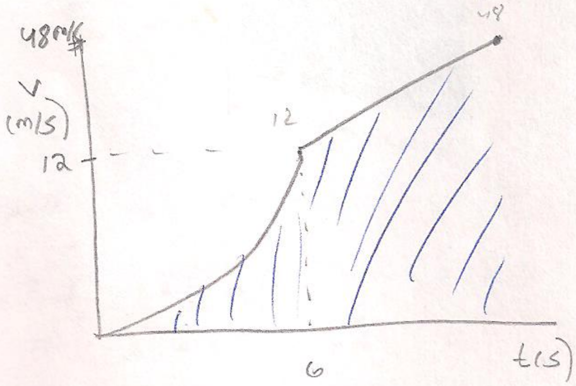
$- (4/5)(30)(6\sin 40) + (3/5)(30)(3.5 + 6\cos 40)$

$= 231.97 \text{ lb} \cdot \text{in} \circlearrowleft$

Q6. A race car starting from rest travels along a straight road and for 12 seconds has the accerlation shown. Construct the v-t graph that describes the motion and find the distance travelled in 12 seconds.



" area under curve or integrate — acceleration → Velocity → distence — take derivative



for v @ 6, for graph

$a = \frac{dv}{dt}$ $\quad \int_0^6 a\,dt = \int_{v_0}^{v} dv$

$\int_0^6 \frac{1}{6}t^2\,dt = \int_{v_0}^{v} dv$

$\frac{3}{3} \cdot \frac{1}{6}$

$\frac{1}{18}t^3 \Big|_0^6 = V - 0$

$V = \frac{6^3}{18} - 0$

$V = 12\,m/s$

distence is 198 m @ 12 seconds from when the car starts travelling. (from rest @ t=0)

for @ 12 seconds

$a\,dt = dv$

$\int_6^{12} 6\,dt = \int_{V_6}^{V_{12}} dv$

$6t \Big|_6^{12} = V_{12} - V_6$

$6(6) = V_{12} - 12\,m/s$

$V_{12} = 48\,m/s$

for total distence (area under curve ////) On v vs t curve $\Delta + \square + \Delta$

this area need to find $\quad a = \frac{d^2s}{dt}$

~~$\int a\,dt = \int v\,dv = \int\int$~~

$\iint a\,dt = \iint ds$

$\iint \frac{1}{6}t^2\,dt = \iint ds$

$\int \frac{1}{18}t^3\,dt = \int ds = \int_0^s ds$

$\frac{1}{72}t^4 \Big|_0^6 = s$

$s = 18 \;\Delta$

$s = 18 + (6 \times 12) + (0.5 \times (48-12) \times 6)$

$\boxed{= 198\,m}$