

CONCORDIA UNIVERSITY  
 Faculty of Engineering and Computer Science  
 ENGR 242/2 STATICS, Section X, Fall 2001

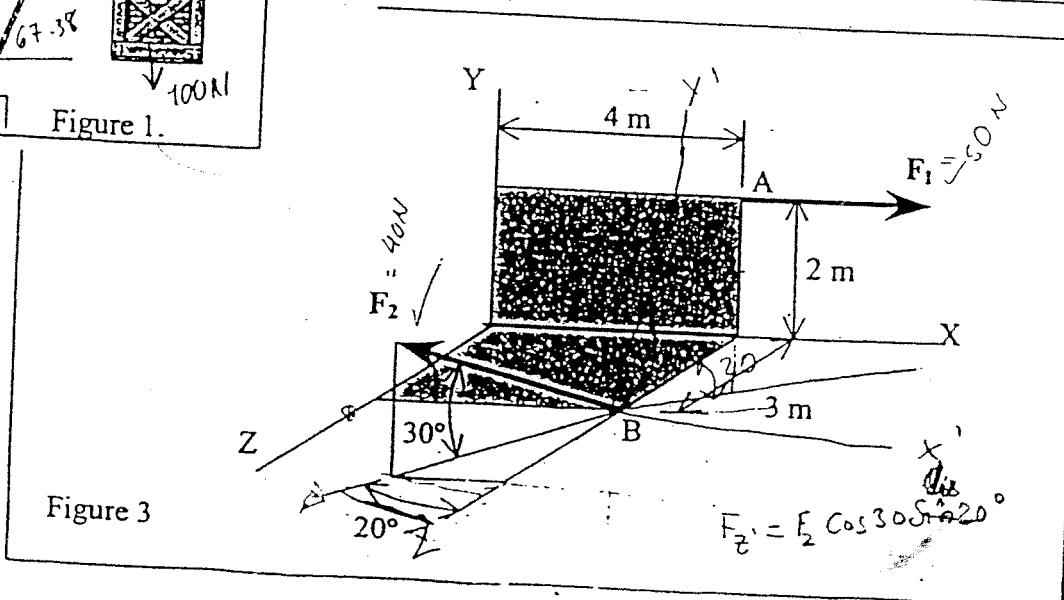
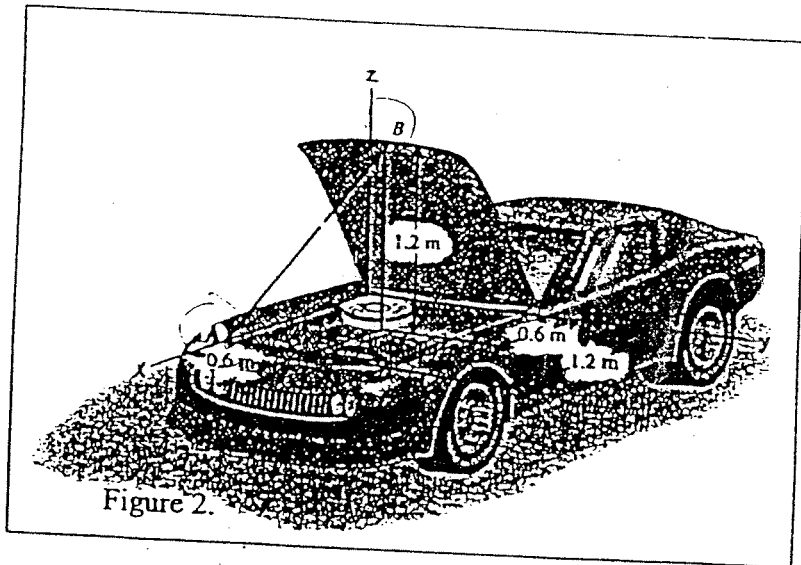
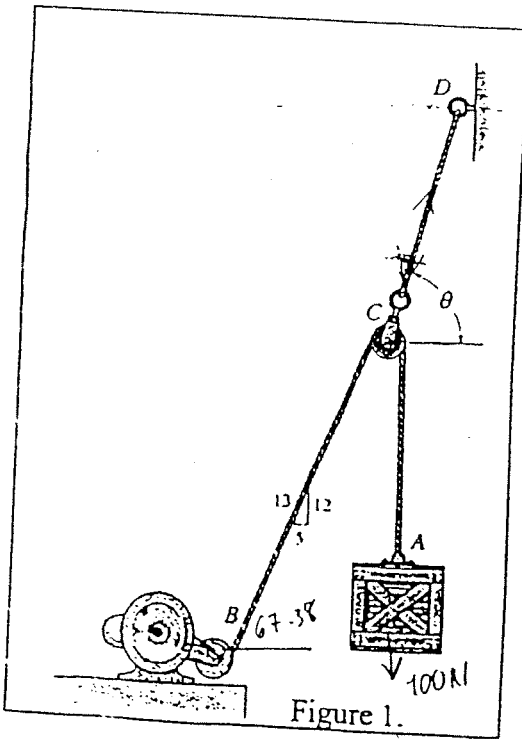
Test #1

Attempt all questions. Only non-programmable calculators are permitted.

Time: 60 minutes.

Marks: 10 points per problem (Total is 30).

- In Figure 1 below, if the crate has a weight of 100 N, determine the magnitude of the tension in cable CD and the angle  $\theta$ .
- In Figure 2 below, the hood of the automobile is supported by the strut AB, which exerts a force of  $F = 24$  N on the hood. Determine the moment of this force about the hinged axis y.
- Two forces  $F_1$  (50 N) and  $F_2$  (40 N) act as shown in Figure 3 below.
  - Replace the forces with an equivalent force-couple system at the origin.
  - Determine the magnitude and the direction cosines of the resultant force.

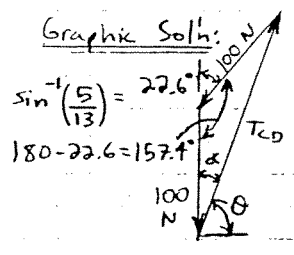
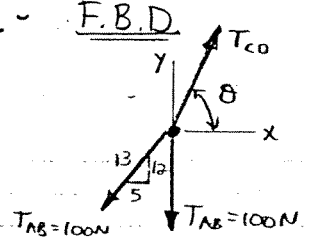


Handwritten calculations:  
 $4.5$   
 $10.7$   
 $6.504$

Solution Test #1

ENGR-242, Section X, October 2001

#1 - F.B.D



By the law of cosines:

$$T_{CD} = \sqrt{100^2 + 100^2 - 2 \times 100^2 \cos 157.4^\circ} = 196$$

By the law of sines:

$$\frac{\sin \alpha}{100} = \frac{\sin 157.4^\circ}{196}$$

$T_{CD} = 196$   
 $\theta = 78^\circ$

$$\alpha = 11.3^\circ \rightarrow \theta = 90^\circ - 11.3^\circ$$

In vector form:

#2 -

$$\vec{F} = F \vec{\lambda}_{B1A} = F \frac{\vec{r}_{B1A}}{r_{B1A}} = 24 \text{ [N]} \frac{(-0.6\vec{i} + 0.6\vec{j} + 1.2\vec{k})}{\sqrt{0.6^2 + 0.6^2 + 1.2^2}} \text{ [m]} = -9.8\vec{i} + 9.8\vec{j} + 19.6\vec{k}$$

Moment of F about y'

$$M_y = \vec{j} \cdot (\vec{r}_A \times \vec{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 1.2 & 0 & 0 \\ -9.8 & 9.8 & 19.6 \end{vmatrix} \Rightarrow \boxed{M_y = -23.52 \text{ N}\cdot\text{m}}$$

#3 - a) Vector Forces:  $\vec{F}_2 = 40 \text{ [N]} (-\cos 30^\circ \sin 20^\circ \vec{i} + \sin 30^\circ \vec{j} + \cos 30^\circ \sin 20^\circ \vec{k})$   
 $= -11.8\vec{i} + 20\vec{j} + 32.6\vec{k} \text{ [N]}$

$$\vec{F}_1 = 50\vec{i} \text{ [N]}$$

Resultant force at O:  $\vec{R} = \Sigma \vec{F} \rightarrow \boxed{\vec{R} = 38.2\vec{i} + 20\vec{j} + 32.6\vec{k} \text{ [N]}}$

Moments of Each Force about O:

$$\vec{M}_1 = \vec{F}_1 \times \vec{r}_1 = (2\vec{j}) \times (50\vec{i}) = -100\vec{k} \text{ [N}\cdot\text{m]}$$

$$\vec{M}_2 = \vec{F}_2 \times \vec{r}_2 = (4\vec{i} + 3\vec{k}) \times (-11.8\vec{i} + 20\vec{j} + 32.6\vec{k}) = -60\vec{i} - 165.8\vec{j} + 19.6\vec{k}$$

Resultant moment at O:  $\vec{M}_O = \Sigma \vec{M} \rightarrow \boxed{\vec{M}_O = -60\vec{i} - 165.8\vec{j} - 20\vec{k} \text{ [N}\cdot\text{m]}}$

b)  $R = \sqrt{38.2^2 + 20^2 + 32.6^2} \Rightarrow \boxed{R = 54.1 \text{ N}}$

$$\cos \theta_x = R_x / R = 38.2 / 54.1 \Rightarrow \theta_x = 45.0^\circ$$

$$\cos \theta_y = R_y / R = 20 / 54.1 \Rightarrow \theta_y = 68.3^\circ$$

$$\cos \theta_z = R_z / R = 32.6 / 54.1 \Rightarrow \theta_z = 52.9^\circ$$

**Test #1**

Attempt all questions. Only non-programmable calculators are permitted.

Time: 60 minutes.

Marks: 10 points per problem (Total is 30).

1. In Figure 1 below, three cables are jointed at the junction ring C. Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.
2. In Figure 2 below, the rectangular plate is supported by hinges along its side BC and by the cable AE. If the cable tension is 300 N, determine the moment of the force about the axis of the hinges. Note that E is the midpoint of the horizontal upper edge of the structural support.
3. In Figure 3 below, a motor with a weight of 32 N is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

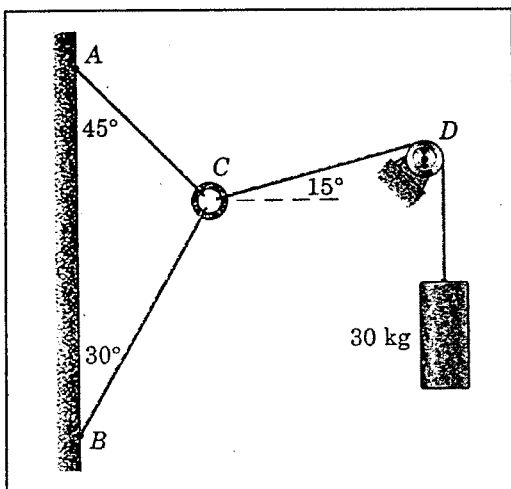


Figure 1

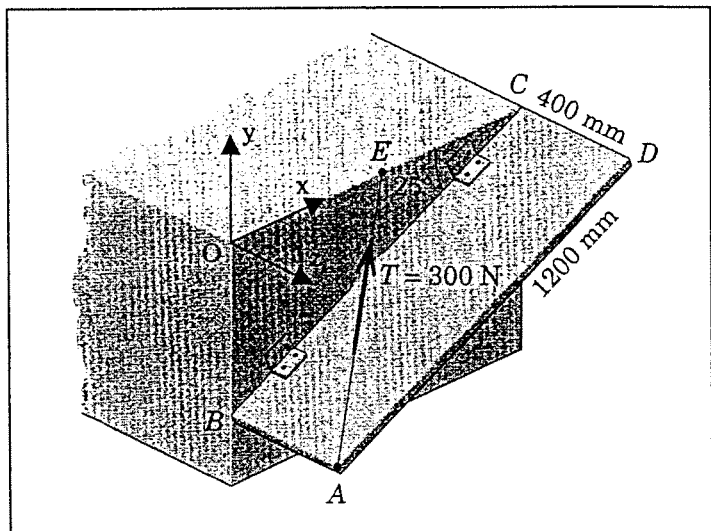


Figure 2

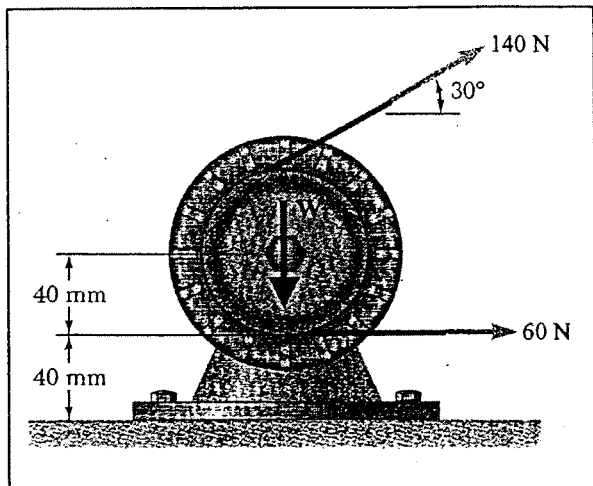


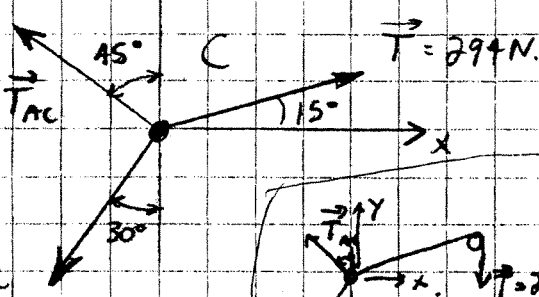
Figure 3

= evaluation system

#1.

FBD

xy



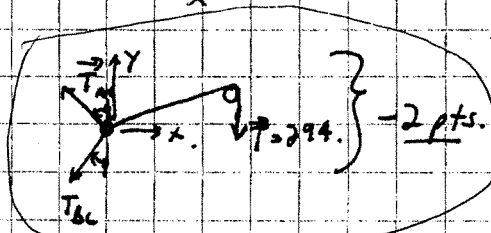
$$T = 30 \text{ kg} * 9.81 \text{ m/s}^2$$

$$T = 294 \text{ N}$$

1 pt to find T

4 pts FBD.

- ↳ 1 pt. All 3 forces shown
- ↳ 1 pt. angles
- ↳ 1 pt. coordinate axes.
- ↳ 1 pt. identify forces.



-2 pts.

1st approach:

1.5 pt Equation

$$\sum F_x = -T_{AC} \sin 45^\circ - T_{BC} \sin 30^\circ + 294 \cos 15^\circ = 0$$

$$T_{AC} = 0.7071 T_{BC} + 401.6$$

1.5 pt Equation

$$\sum F_y = T_{AC} \cos 45^\circ - T_{BC} \cos 30^\circ + 294 \sin 15^\circ = 0$$

$$0.7071 (0.7071 T_{BC} + 401.6) - 0.8660 T_{BC} + 76.09 = 0$$

$$-0.5 T_{BC} + 284.0 - 0.866 T_{BC} + 76.09 = 0$$

$$1.366 T_{BC} = 360$$

$$T_{BC} = 264 \text{ N}$$

$$T_{AC} = 215 \text{ N}$$

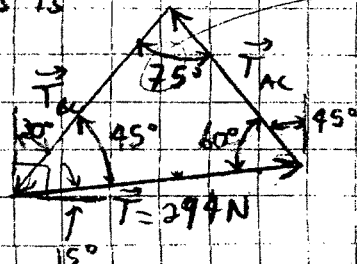
1 pt For solving

2 equations, 2 unknowns.

1 pt. Solution and units.

2nd approach: Graphically

The sum of the three vectors is zero.



$$= 180^\circ - 45^\circ - 60^\circ$$

By Sine Law:

$$\frac{294}{\sin 75^\circ} = \frac{T_{AC}}{\sin 45^\circ} = \frac{T_{BC}}{\sin 60^\circ}$$

2 pt: Sine law Equation

$$T_{AC} = 215 \text{ N}$$

$$T_{BC} = 264 \text{ N}$$

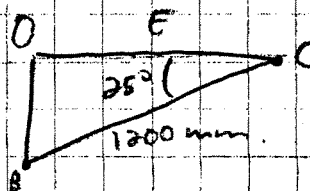
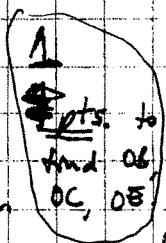
1 pt: Solution and units.

2 pts. Force triangle and angles

1.5

#2.

Force T as vector:



pt. A = (0, -507, 400) mm

pt. E = (545, 0, 0)

pt. B = (0, -507, 0)

pt. C = (1089, 0, 0)

OB = 1200 \* sin 25° = 507 mm

OC = 1200 \* cos 25° = 1089 mm

OE = OC / 2 = 545 mm

Let: for equation  $\vec{T} = T \lambda_T = T \frac{\vec{r}_{E1A}}{r_{E1A}} = 300 [N] \frac{(545\vec{i} + 507\vec{j} - 400\vec{k})}{\sqrt{545^2 + 507^2 + 400^2}}$  [mm]

Let: position vector  $\vec{r}_{E1A}$

845 mm

Let: Force vector:  $\vec{T} = 193\vec{i} + 180\vec{j} - 142\vec{k}$  [N]

Unit vector along BC

Let: for equation  
Let: for unit vector  $\lambda_{BC} = \frac{\vec{r}_{C1B}}{r_{C1B}} = \frac{(1089\vec{i} + 507\vec{j} + 0\vec{k})}{\sqrt{1089^2 + 507^2}} = 0.9066\vec{i} + 0.4221\vec{j}$

1200

Position vector from B to A

Let: position vector:  $\vec{r}_{A1B} = (0\vec{i} + 0\vec{j} + 400\vec{k})$  mm

or  $\vec{r}_{E1B}$  or  $\vec{r}_{A1C}$  or  $\vec{r}_{E1C}$

The mixed triple product

Let: equation  $M = \lambda_{BC} \cdot (\vec{r}_{A1B} \times \vec{T}) =$

0.9066	0.4221	0	0.9066	0.4221
0	0	400	0	0
193	180	-142	193	180

Alternate approach: compute  $M_0 = \vec{r}_{A1B} \times \vec{T}$  then  $M = M_0 \cdot \lambda_{BC}$  -ve.

Let's find the determinant:  $= 32586 - 62275 = -32689$  N·mm

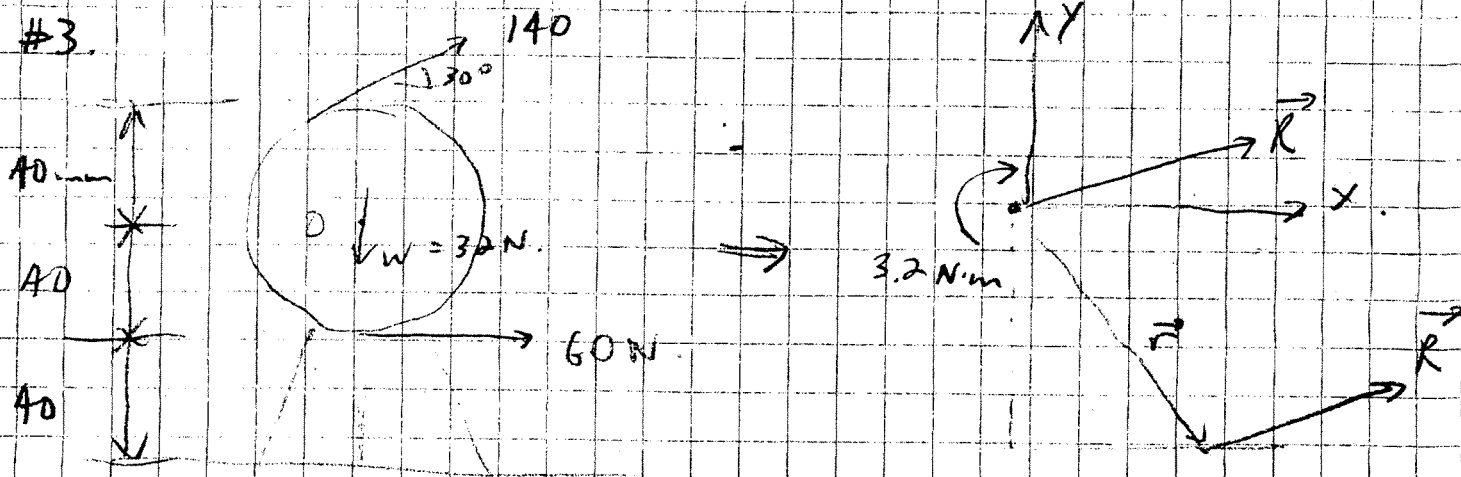
true

Let: Solution and Units

$M = -32.7$  N·m

↳ No point if M Vector from

#3.



Equivalent force-couple system at O.

(2 pts)  $\vec{R} = \sum \vec{F} = -32\vec{j} + 60\vec{i} + (140 \cos 30^\circ \vec{i} + 140 \sin 30^\circ \vec{j})$

(1 pt)  $|\vec{R} = 181\vec{i} + 38\vec{j} \text{ N.}|$

(2 pts)  $\vec{M}_R = \sum (\vec{r} \times \vec{F}) = 0.04 * (60 - 140) \vec{k} = -3.2 \vec{k} \text{ N}\cdot\text{m}$

Move resultant on floor to cancel moment.

(1 pt for  $\vec{r}$ )  $\vec{r} = (x\vec{i} - 0.08\vec{j}) \text{ m}$

(Let  $\rightarrow$  equation)

$$\vec{M}_R = \vec{r} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & -0.08 & 0 \\ 181 & 38 & 0 \end{vmatrix} = 38x\vec{k} + 14.48\vec{k} = -3.2\vec{k}$$

(2 pts) Finding determinant and equal to previous moment found  
 $+38x = -17.68$

$x = -0.465 \text{ m}$

(1 pt. answer and units)  $|x = -465 \text{ mm}|$

Resultant will be 465 mm to the left of pt O on floor.

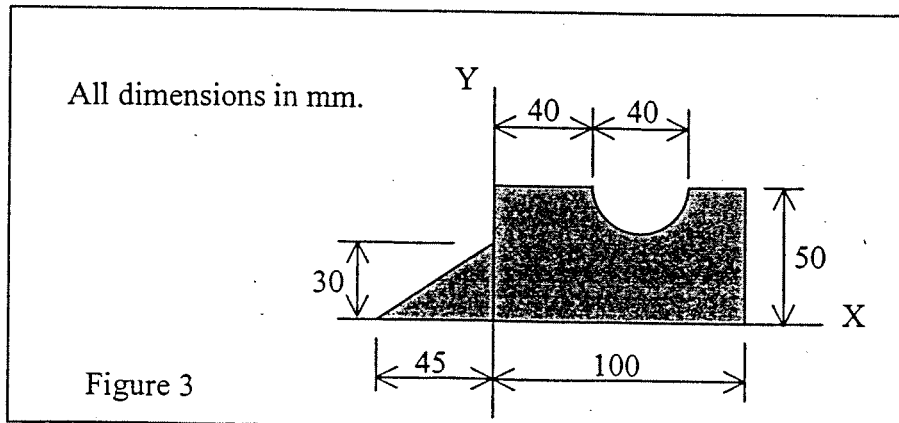
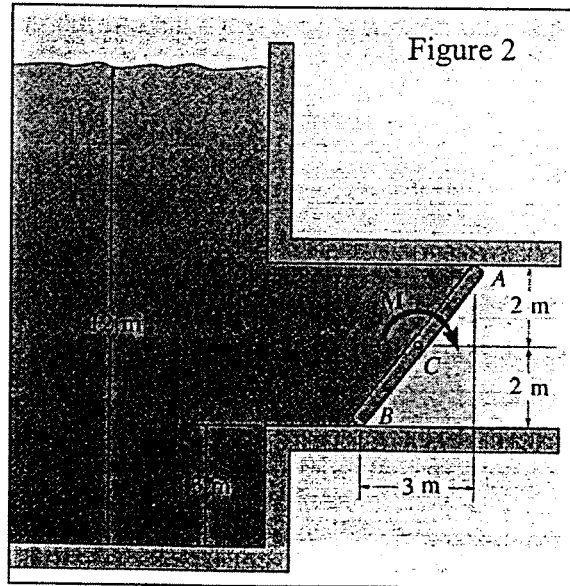
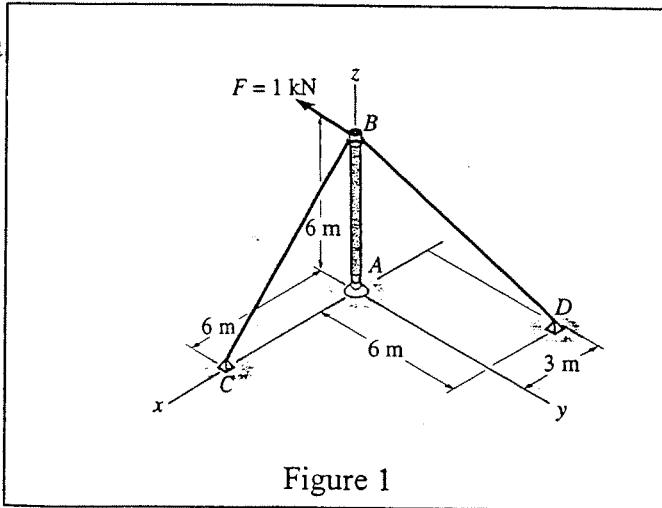
**Test #2**

Attempt all questions. Only non-programmable calculators are permitted.

Time: 60 minutes.

Marks: 10 points per problem (Total is 30).

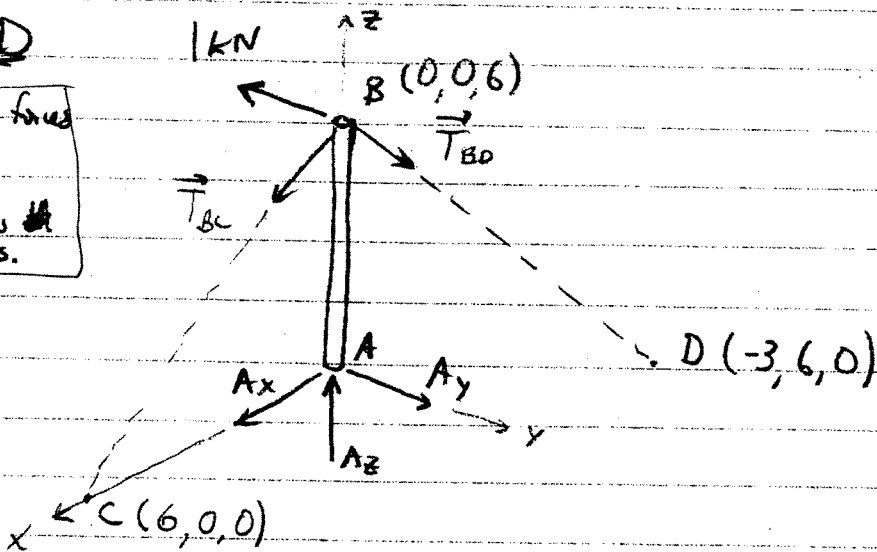
1. Determine the tension in cables BC and BD and the reactions at the ball-and-socket joint A for the mast shown in Figure 1 below.
2. The 3-m-wide rectangular gate is pinned at its center C. Determine the torque M that must be applied to its central shaft in order to open the gate. Specific weight of water:  $\gamma_w = 9810 \text{ N/m}^3$ .
3. For the plane area shown in Figure 3 below, determine
  - (a) the first moments of area with respect to the X and Y axes,
  - (b) the location of the centroid, and
  - (c) the volume of the body of revolution obtained by rotating the area about the X axis.



#1.

FBD

Reaction forces  
Axes.  
Dimensions &  
or coords.



Define the tension forces as vectors.

①  $\vec{T}_{BD} = T_{BD} \frac{\vec{r}_{D|B}}{r_{D|B}} = T_{BD} \frac{(-3\vec{i} + 6\vec{j} - 6\vec{k})}{\sqrt{3^2 + 6^2 + 6^2}} = T_{BD} \left( \frac{-1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k} \right)$

①  $\vec{T}_{BC} = T_{BC} \frac{\vec{r}_{C|B}}{r_{C|B}} = T_{BC} \frac{(6\vec{i} + 0 - 6\vec{k})}{\sqrt{6^2 + 6^2}} = T_{BC} (0.707\vec{i} - 0.707\vec{k})$

Find reactions:

①  $\sum M @ A_x = 6 * 1 - 6 * (T_{BD})_y = 0 \quad \boxed{T_{BD} = 1.5 \text{ kN.}}$

①  $\sum M @ A_y = 6 * (T_{BC})_x - 6 * (T_{BD})_x = 0$   
 $\rightarrow (T_{BC})_x = (T_{BD})_x$   
 $0.707 T_{BC} = 0.333 * 1.5 \rightarrow \boxed{T_{BC} = 0.707 \text{ kN}}$

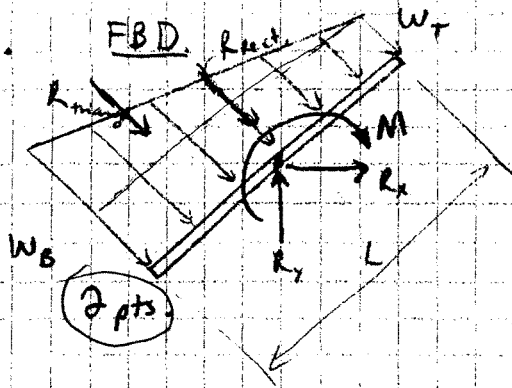
①  $\sum F_x = A_x + 0.707 T_{BC} - \frac{1.5}{3} = 0$

①  $\sum F_y = A_y - 1 + \frac{2}{3} T_{BD} = 0$

①  $\sum F_z = A_z - 0.707 T_{BC} - \frac{2}{3} T_{BD} = 0$

$A_x = 0 \text{ kN}$   
 $A_y = 0 \text{ kN}$   
 $A_z = 1.5 \text{ kN}$

#2.



(1 pt) When M is about to open the gate, the walls of the pipe do not exert any reactions.

$$W_T = \gamma h_T b = 9810 \left[ \frac{\text{N}}{\text{m}^3} \right] \times (12-7) \text{ [m]} \times 3 \text{ [m]}$$

$$W_T = 147.5 \text{ kN/m} \quad (1 \text{ pt.})$$

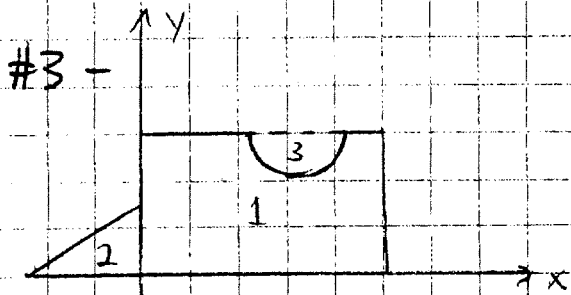
$$W_B = 9810 \times (12-3) \times 3 = 264.9 \text{ kN/m} \quad (1 \text{ pt.})$$

Length of gate:  $L = \sqrt{3^2 + 4^2} = 5 \text{ m} \quad (1 \text{ pt.})$

$$\sum M_{DC} = -M + \underbrace{(W_B - W_T)}_{\text{Resultant}} \times \frac{L}{2} \times \underbrace{\left( \frac{L}{2} - \frac{L}{3} \right)}_{\text{Distance}} = 0$$

$$M = (264.9 - 147.5) \left[ \frac{\text{kN}}{\text{m}} \right] \times \frac{5}{2} \text{ [m]} \times \underbrace{\left( \frac{5}{2} - \frac{5}{3} \right)}_{2.0833 \dots} \text{ [m]}$$

$$\boxed{M = 245 \text{ kN}\cdot\text{m}} \quad (1 \text{ pt.})$$



Shapes	Area (mm <sup>2</sup> )	$\bar{X}$ (mm)	$\bar{Y}$ (mm)	$\bar{X}A$ (mm <sup>3</sup> )	$\bar{Y}A$ (mm <sup>3</sup> )
+ [1]	$100 \times 50 = 5000$	$\frac{100}{2} = 50$	$\frac{50}{2} = 25$	250000	125000
+ [2]	$30 \times 15 / 2 = 675$	$-\frac{45}{3} = -15$	$\frac{30}{3} = 10$	-10125	6750
- [3]	$\pi \times \frac{20^2}{2} = -628$	$40 + 20 = 60$	$50 - \frac{4 \times 20}{3\pi} = 41.5$	-37680	-26062
$\Sigma A = 5047$					

check sign equation computation

- ① Correct naming of column sums

1<sup>st</sup> Moment of Area about Y axis:  $Q_y = \Sigma \bar{X}A = 202195 \text{ mm}^3$   
 1<sup>st</sup> Moment of Area about X axis:  $Q_x = \Sigma \bar{Y}A = 105688 \text{ mm}^3$

- ① Location of centroid: eq. + answer
- ① eq. + answer

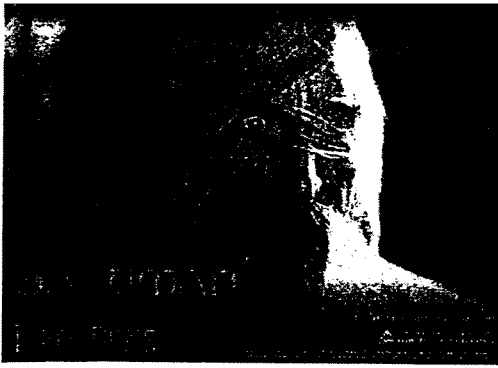
$$\bar{X} = \frac{\Sigma \bar{X}A}{\Sigma A} = \frac{202195 \text{ mm}^3}{5047 \text{ mm}^2} \Rightarrow \bar{X} = 40.1 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{Y}A}{\Sigma A} = \frac{105688 \text{ mm}^3}{5047 \text{ mm}^2} \Rightarrow \bar{Y} = 20.9 \text{ mm}$$

By the 2<sup>nd</sup> theorem of Pappus-Guldinus:

① equation  $V_{B.O.R.} = A \times 2\pi \bar{y} = 5047 \text{ [mm}^2\text{]} \times 2\pi \times 20.9 \text{ [mm]}$

① answer  $V_{B.O.R.} = 664000 \text{ mm}^3$



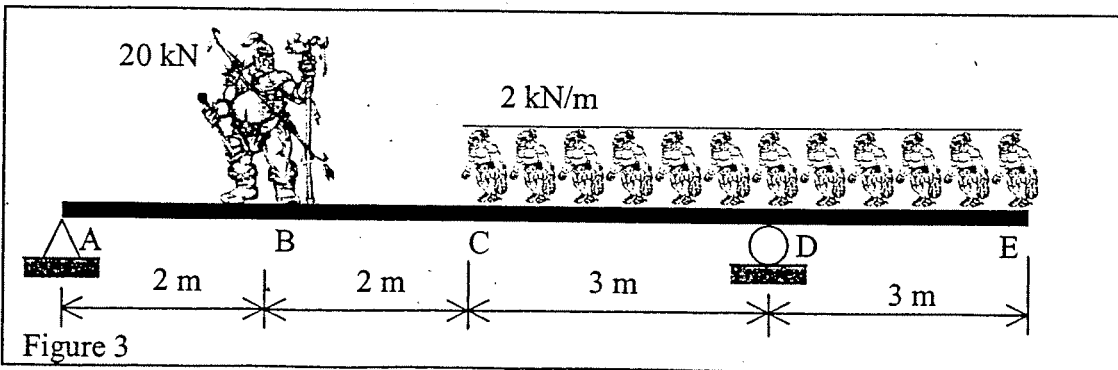
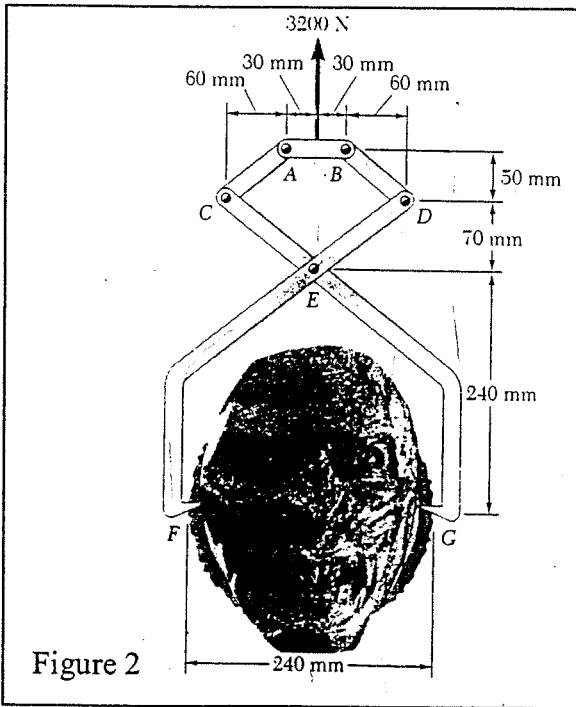
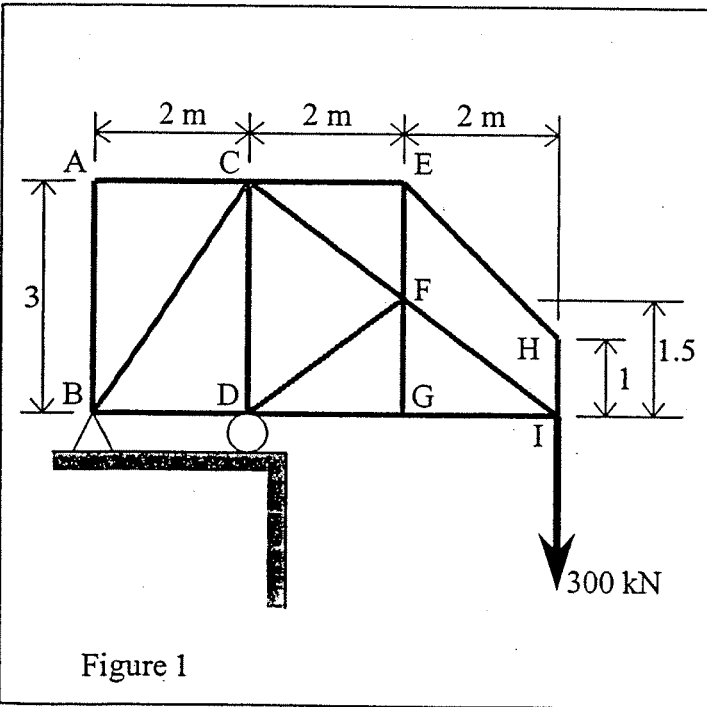
For/16

### Test #3

Only non-programmable calculators are permitted.  
Marks: 10 points per problem (Total is 30).

Saruman, the wizard turned evil, is preparing an army in his lair in Isengard. Your help is needed to analyze some of the machines and structures to prevent Darkness from laying its grip on Middle-Earth.

1. A crane used to hoist material in Isengard is a truss loaded as shown in Figure 1 below. Determine the force in each member and state whether each member is in tension or compression.
2. The pair of tongs shown in Figure 2 are used to tame Trolls when they misbehave. Determine the forces exerted at E and F on tong DEF.
3. A beam is used to bridge a deep chasm as shown in Figure 3 below. The Troll at B on the left represent a point load of 20 kN while the army of goblins on the right between C and E represent a distributed load of 2 kN/m. Draw the shear and bending-moment diagrams for the beam.



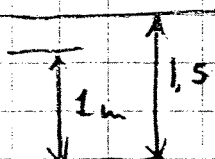
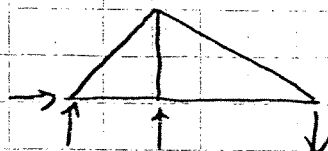
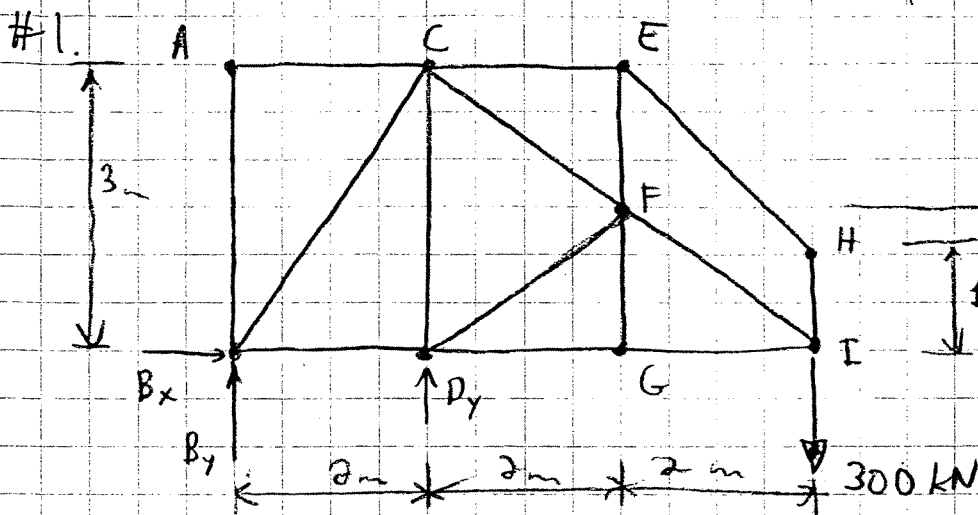
10pts

Solution

Test #3

ENGR-242, 2002

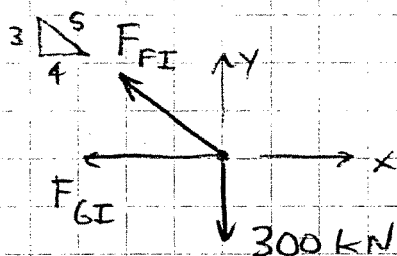
F02/26



Zero force members:  $F_{AB} = F_{AC} = F_{CE} = F_{EF} = F_{EH} = F_{HI} = F_{FG} = F_{FO} = 0$   
 (0.5) (0.5) (0.5) (0.5) (0.5) (0.5) (0.5) (0.5)

FBD Node I:

1. FBD



$$\sum F_y = \frac{3}{5} F_{FI} - 300 = 0$$

(1)

$F_{FI} = F_{CF} = 500 \text{ kN}$   
 FI, CF in tension

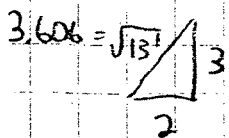
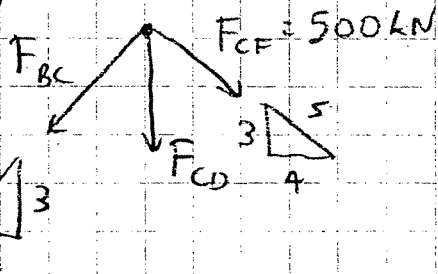
$$\sum F_x = -\frac{4}{5} F_{FI} - F_{GI} = 0$$

(1)

$F_{GI} = F_{GO} = F_{CO} = -400 \text{ kN}$   
 BO, GI, CO in Comp.

FBD Node C

1. FBD



$$\sum F_x = \frac{4}{5} * 500 - \frac{2}{3.606} * F_{BC} = 0$$

(1)

$F_{BC} = 721 \text{ kN}$   
 BC in tension

$$\sum F_y = -F_{CD} - \frac{3}{3.606} * 721 - \frac{3}{5} * 500 = 0$$

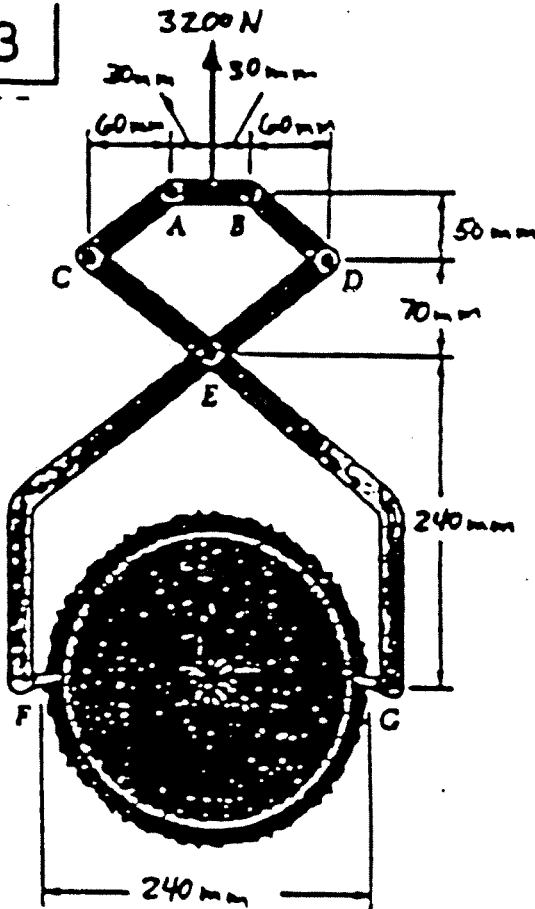
(1)

$F_{CD} = 900 \text{ kN}$   
 CD in Comp.

#2

6.143

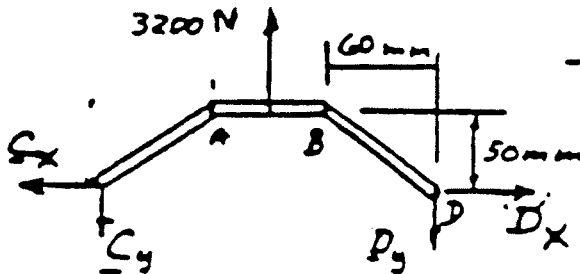
F02/3b



GIVEN: WEIGHT OF LOG IS 3200N

FIND: FORCES EXERTED AT E AND F ON TONG DEF.

2 pt.  
1st FBD



FREE BODY:

MEMBERS CA, AB, BD

BY SYMMETRY

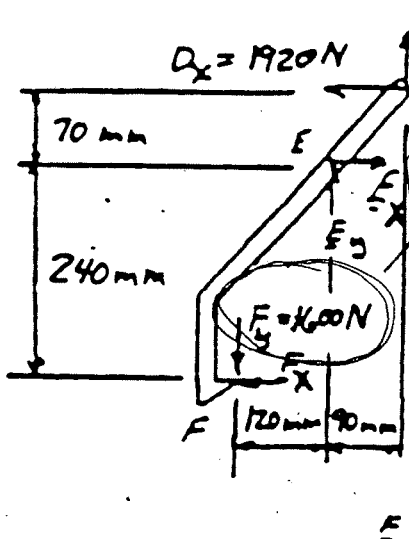
$$D_y = \frac{1}{2}(3200N) = 1600N \quad (1 \text{ pt})$$

SINCE BD IS A TWO-FORCE MEMBER

$$\frac{D_y}{50 \text{ mm}} = \frac{D_x}{60 \text{ mm}} ; \frac{1600N}{50 \text{ mm}} = \frac{D_x}{60 \text{ mm}} ; D_x = 1920N \quad (1 \text{ pt})$$

to find  $D_y$   
to find  $D_x$

2 pt.  
2nd FBD



FREE BODY: TONG DEF

$$\uparrow \Sigma F_y = 0: 1600N + E_y - 1600N = 0$$

$$E_y = 0 \quad (1 \text{ pt})$$

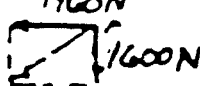
$$\uparrow \Sigma M_E = 0$$

$$(1600N)(210 \text{ mm}) + (1920N)(30 \text{ mm}) - E_x(240 \text{ mm}) = 0$$

$$E_x = +3880N \quad E = 3880N \rightarrow (1 \text{ pt})$$

$$\pm \Sigma F_x = 0: 3880N - 1920N - F_x = 0$$

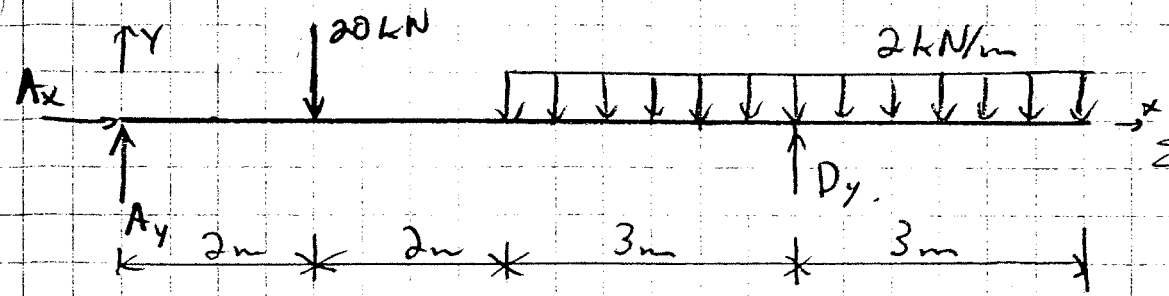
$$F_x = 1960N \quad (1 \text{ pt})$$



$$F = 2530N @ 30.2^\circ$$

(No need)

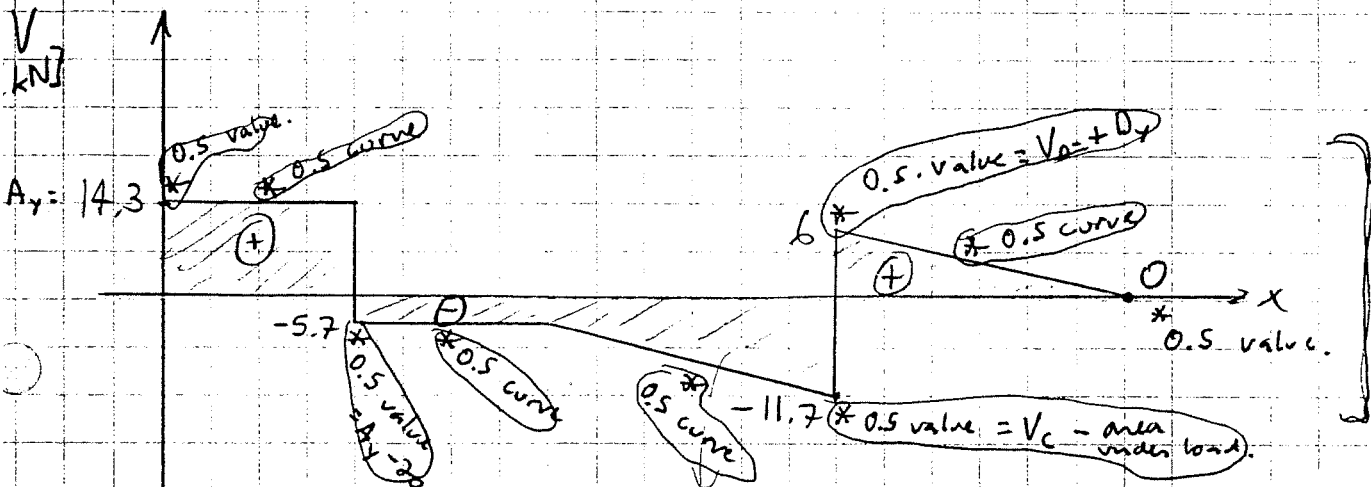
#3. FBD



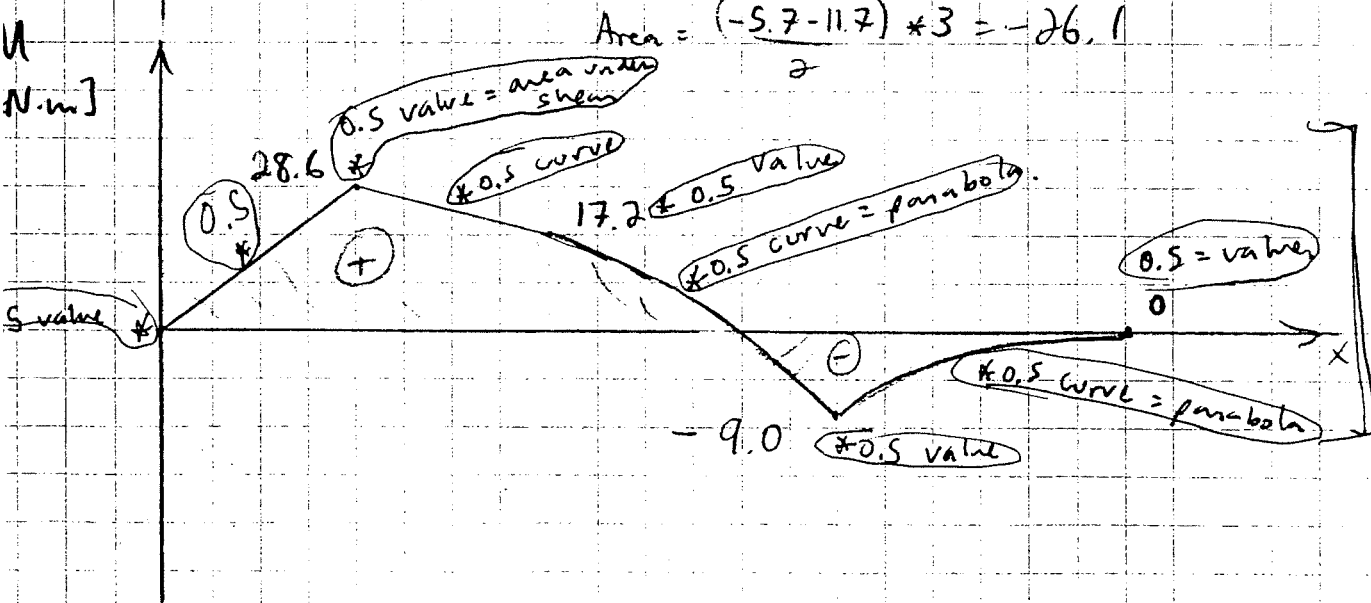
$\sum F_x = A_x = 0$

$\sum M_A = -2 \times 20 + 7 D_y - 2 \times 6 \times 7 = 0$   
 $\sum F_y = A_y + D_y - 20 - 2 \times 6 = 0$

$D_y = 17.7 \text{ kN}$   
 $A_y = 14.3 \text{ kN}$



Total 4.5 pts.



Total 4.5 pts.

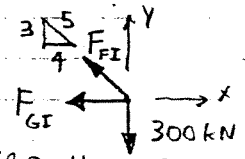
Area =  $\frac{(-5.7 - 11.7) \times 3}{2} = -26.1$

702/5<sup>6</sup>

Solution Test #3 ENGR-242 Fall 2002

#1. Zero force members:  
 $F_{AB} = F_{AC} = F_{CE} = F_{EF} = 0$   
 $F_{EH} = F_{HI} = F_{FG} = F_{FD} = 0$

FBD Node I



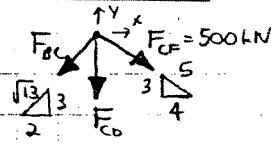
$$\sum F_y = \frac{3}{5} F_{FI} - 300 = 0$$

$F_{FI} = F_{CF} = 500 \text{ kN}$   
 FI, CF in tension

$$\sum F_x = -\frac{4}{5} F_{FI} - F_{GI} = 0$$

$F_{GI} = F_{GD} = F_{BD} = -400 \text{ kN}$   
 GI, GD, BD in comp.

FBD Node C



$$\sum F_x = \frac{4}{5} \cdot 500 - \frac{2}{\sqrt{13}} F_{BC} = 0$$

$F_{BC} = 721 \text{ kN}$   
 BC in tension

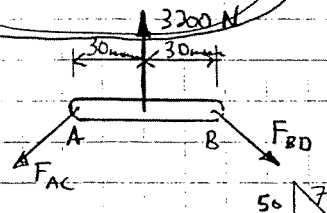
$$\sum F_y = -F_{CD} - \frac{3}{\sqrt{13}} \cdot 721 - \frac{3}{5} \cdot 500 = 0$$

$F_{CD} = -900 \text{ kN}$   
 CD in comp.

10 pts.  
 Alternative Solution

#2. Members BD and AC are two-force members.

2 pt. FBD of AB.



$$\sum M_A = 30 \times 3200 - \frac{50}{78.1} F_{BD} \times 60 = 0$$

$$\rightarrow F_{BD} = 2500 \text{ N (2 pt.)}$$

$$\sum M_E = -240 F_x + 70 \times 60 F_{BD} + 240 \times 90 + \frac{50}{78.1} F_{BD} \times 120 + F_y = 0$$

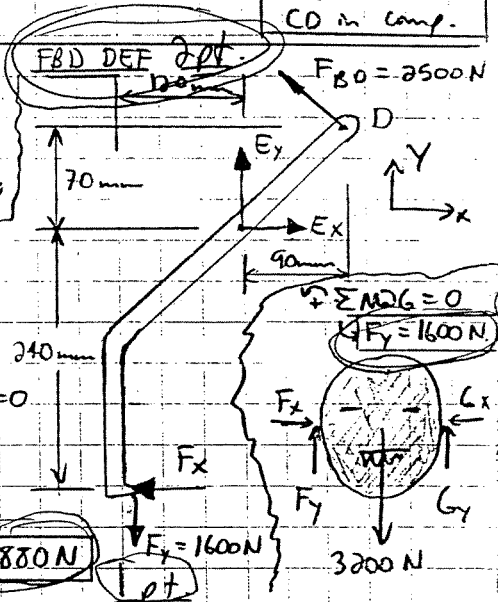
$$\rightarrow F_x = -1960 \text{ N (1 pt.)}$$

$$\sum F_x = -\frac{60}{78.1} F_{BD} + E_x - F_x = 0$$

$$\rightarrow E_x = 3880 \text{ N (1 pt.)}$$

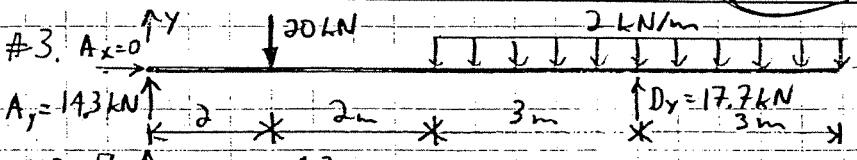
$$\sum F_y = \frac{50}{78.1} F_{BD} + E_y - F_y = 0$$

$$\rightarrow E_y = 0 \text{ (1 pt.)}$$



FBD DEF 2 pt.

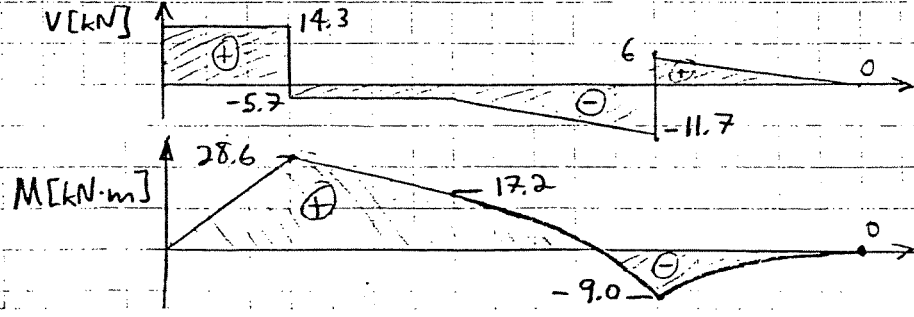
$$\sum M_G = 0 \rightarrow F_y = 1600 \text{ N (1 pt.)}$$



$$\sum M_A = 0 \rightarrow D_y = 17.7 \text{ kN}$$

$$\sum F_y = 0 \rightarrow A_y = 14.3 \text{ kN}$$

$$\sum F_x = 0 \rightarrow A_x = 0$$



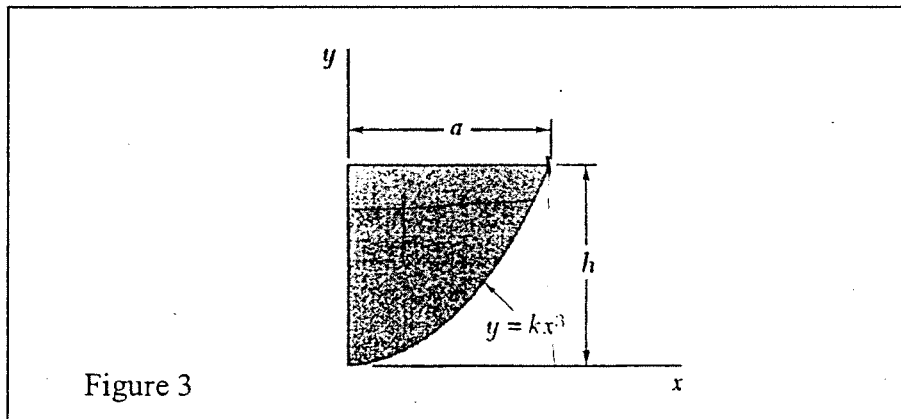
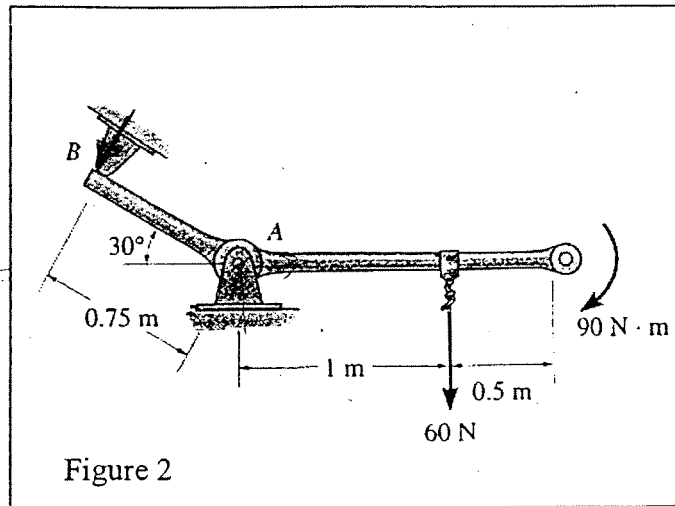
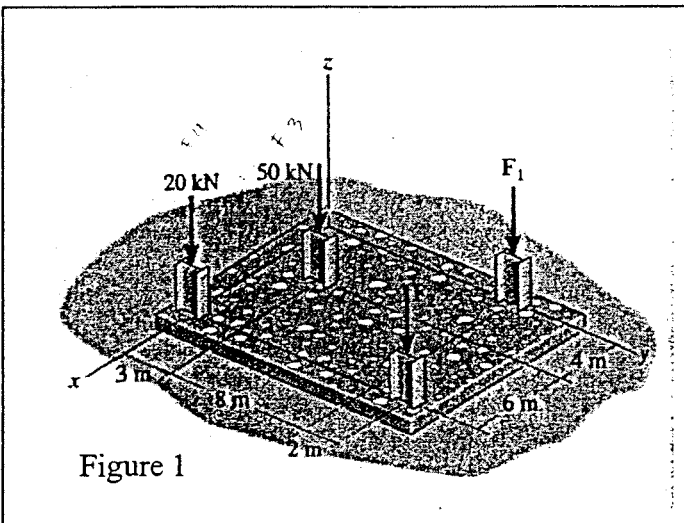
**Test #2**

Attempt all questions. Only non-programmable calculators are permitted.

Time: 60 minutes.

Marks: 10 points per problem (Total is 30).

- In Figure 1 below, the concrete slab is subjected to four parallel forces. Determine the magnitude and direction of the equivalent resultant force and specify its location  $(x, y)$  on the slab. Take  $F_1 = 30 \text{ kN}$  and  $F_2 = 40 \text{ kN}$ .
- The link shown in Figure 2 below is pin connected at A and rests against a smooth support at B. Find the reaction components at A and B.
- For the area shown in Figure 3 below, determine by direct integration
  - the first moments of area with respect to the X and Y axes, and
  - the location of the centroid. (Express your answer in terms of  $a$  and  $h$ .)



Solution Test #2

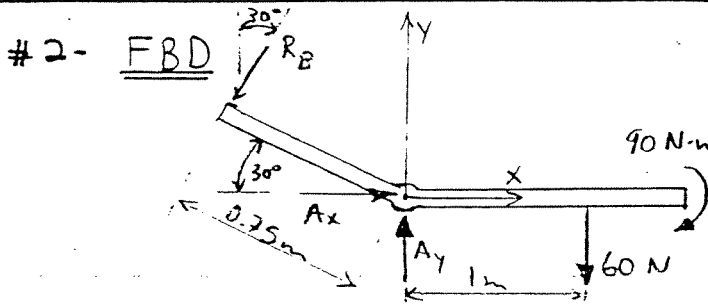
ENGR-242, Section X, Oct. 2001

#1- Equivalent force-couple system at origin:

$$R = \sum F = 20 + 50 + 30 + 40 \rightarrow R = 140 \text{ N.}$$

$$M_x^R = \sum y_i F_i = -0 \cdot 20 - 3 \cdot 50 - 11 \cdot 30 - 13 \cdot 40 = -1000 \text{ N}\cdot\text{m} = yR \rightarrow y = \frac{-1000}{-140} = 7.14 \text{ m}$$

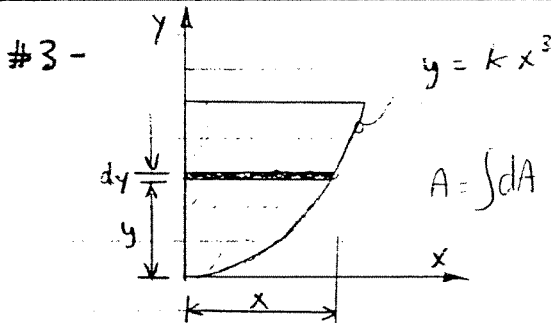
$$M_y^R = \sum x_i F_i = -10 \cdot 20 - 4 \cdot 50 - 0 \cdot 30 - 10 \cdot 40 = -800 \text{ N}\cdot\text{m} = xR \rightarrow x = \frac{-800}{-140} = 5.71 \text{ m}$$



$$\sum M_{\text{at } A} = 0.75 R_B - 1 \cdot 60 - 90 = 0 \rightarrow R_B = 200 \text{ N}$$

$$\sum F_x = A_x - \sin 30^\circ R_B = 0 \rightarrow A_x = 100 \text{ N}$$

$$\sum F_y = A_y - \cos 30^\circ R_B - 60 = 0 \rightarrow A_y = 233 \text{ N}$$



$$y = kx^3$$

$$\text{at } x=a, y=h \rightarrow k = \frac{h}{a^3}$$

$$\therefore y = \frac{h}{a^3} x^3 \quad \text{or} \quad x = \frac{a}{\sqrt[3]{h}} \sqrt[3]{y}$$

$$dA = x dy = \frac{a}{\sqrt[3]{h}} \sqrt[3]{y} dy$$

$$A = \int dA = \frac{a}{\sqrt[3]{h}} \int_0^h y^{1/3} dy = \frac{a}{\sqrt[3]{h}} y^{4/3} \cdot \frac{3}{4} \Big|_0^h = \frac{3ah}{4}$$

1<sup>st</sup> Moment of area about x:

$$Q_x = \int y_{el} dA = \frac{a}{\sqrt[3]{h}} \int_0^h y^{4/3} dy = \frac{a}{\sqrt[3]{h}} y^{7/3} \cdot \frac{3}{7} \Big|_0^h = \frac{3}{7} ah^2$$

1<sup>st</sup> Moment of area about y:

$$Q_y = \int x_{el} dA = \int_0^h \frac{x}{2} x dy = \frac{a^2}{2h^{2/3}} \int_0^h y^{2/3} dy = \frac{a^2}{2h^{2/3}} y^{5/3} \cdot \frac{3}{5} \Big|_0^h = \frac{3a^2h}{10}$$

$$\bar{x} = \frac{Q_y}{A} = \frac{3a^2h}{10} \cdot \frac{4}{3ah} \rightarrow \bar{x} = \frac{2}{5} a$$

$$\bar{y} = \frac{Q_x}{A} = \frac{3ah^2}{7} \cdot \frac{4}{3ah} \rightarrow \bar{y} = \frac{4}{7} h$$

$$\bar{x} = \frac{2}{5} a$$

$$\bar{y} = \frac{4}{7} h$$



56

COURSE	NUMBER	SECTION	
Statics	ENGR. C242/2	01, T, V, X, Y, XX	
EXAMINATION	DATE	TIME	OF PAGES
Final	December 15, 1984	9:30 - 12:30	3
INSTRUCTOR		DIVISION	
Prof. Douglass, Goldman, Stathopoulos, Troitsky & Zielinski		Day and Evening	

MATERIALS ALLOWED:

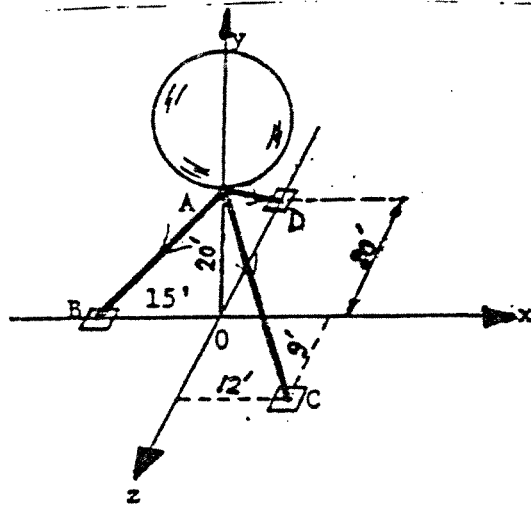
Any calculating device

SPECIAL INSTRUCTIONS:

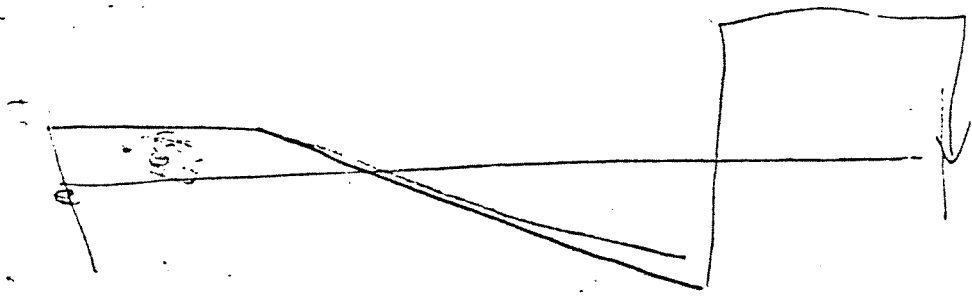
Do any five questions. Only the first five problems presented will be graded.

All questions carry equal weight.

1. A balloon is held by three cables as shown. If the tension of cable AC is 2500 lb., what is the total lift force provided by the balloon?

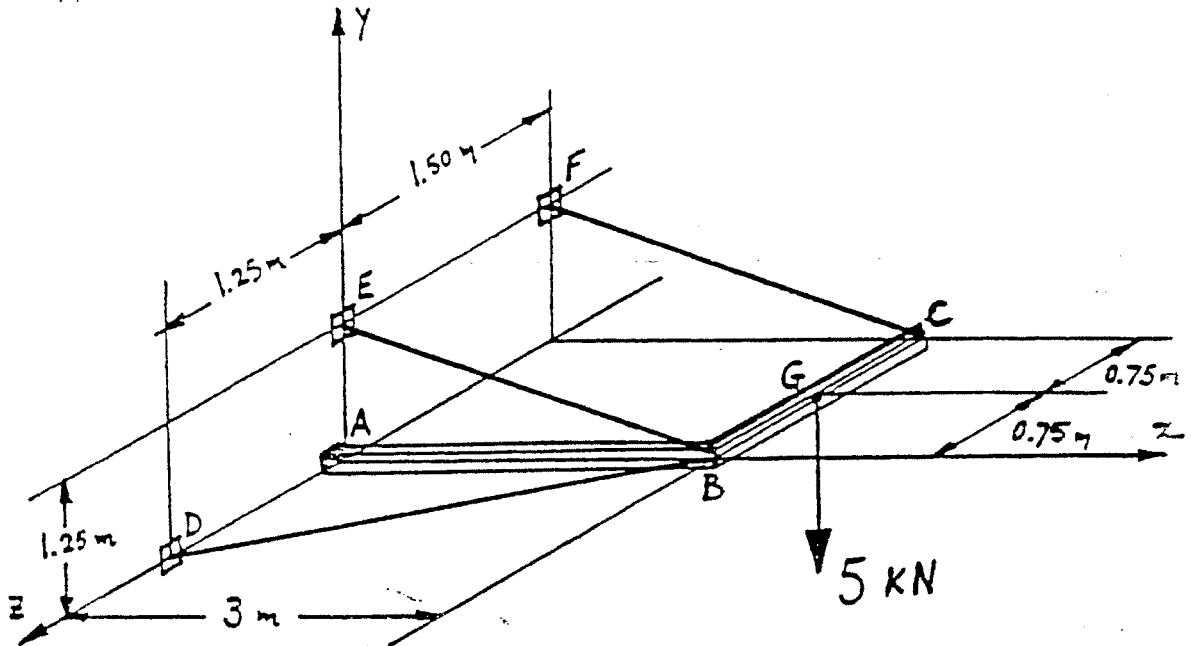


$T_B = 5367.5$



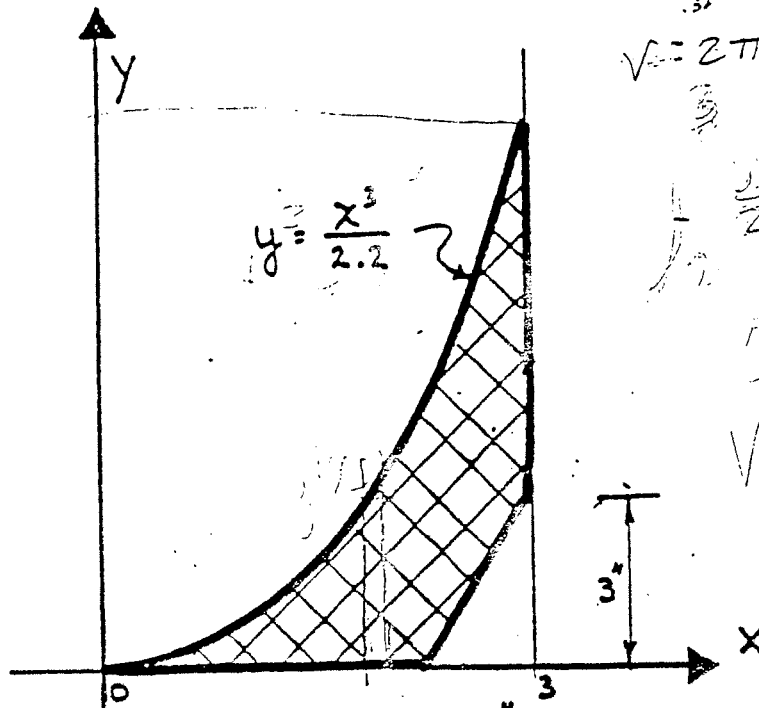
2. The rigid L-shaped member ABC is supported by a ball and socket at A and by the three cables BD, BE and CF as shown. Determine the tension in each cable and the reaction components at A caused by the 5-KN force applied at G.

$\frac{DB}{OB}$



- 3) For the cross-hatched area shown:

- a) Locate the centroid with respect to the OX axis.  
 b) What is the volume generated if this area is rotated 100 degrees about the OX axis?



Handwritten calculations for the volume of the solid of revolution:

$$V = 2\pi \int_0^3 y \cdot A \cdot dL$$

$$V = 2\pi \int_0^3 \frac{x^2}{2.2} \cdot \frac{100}{360} \cdot \sqrt{1 + \frac{4x}{2.2}} \cdot dx$$

$$V = \frac{100}{360} \cdot 2\pi \cdot \frac{1}{2.2} \int_0^3 x^2 \sqrt{1 + \frac{4x}{2.2}} \cdot dx$$

$$V = \frac{100}{360} \cdot 2\pi \cdot \frac{1}{2.2} \cdot \frac{1}{3} \cdot \frac{100}{360}$$



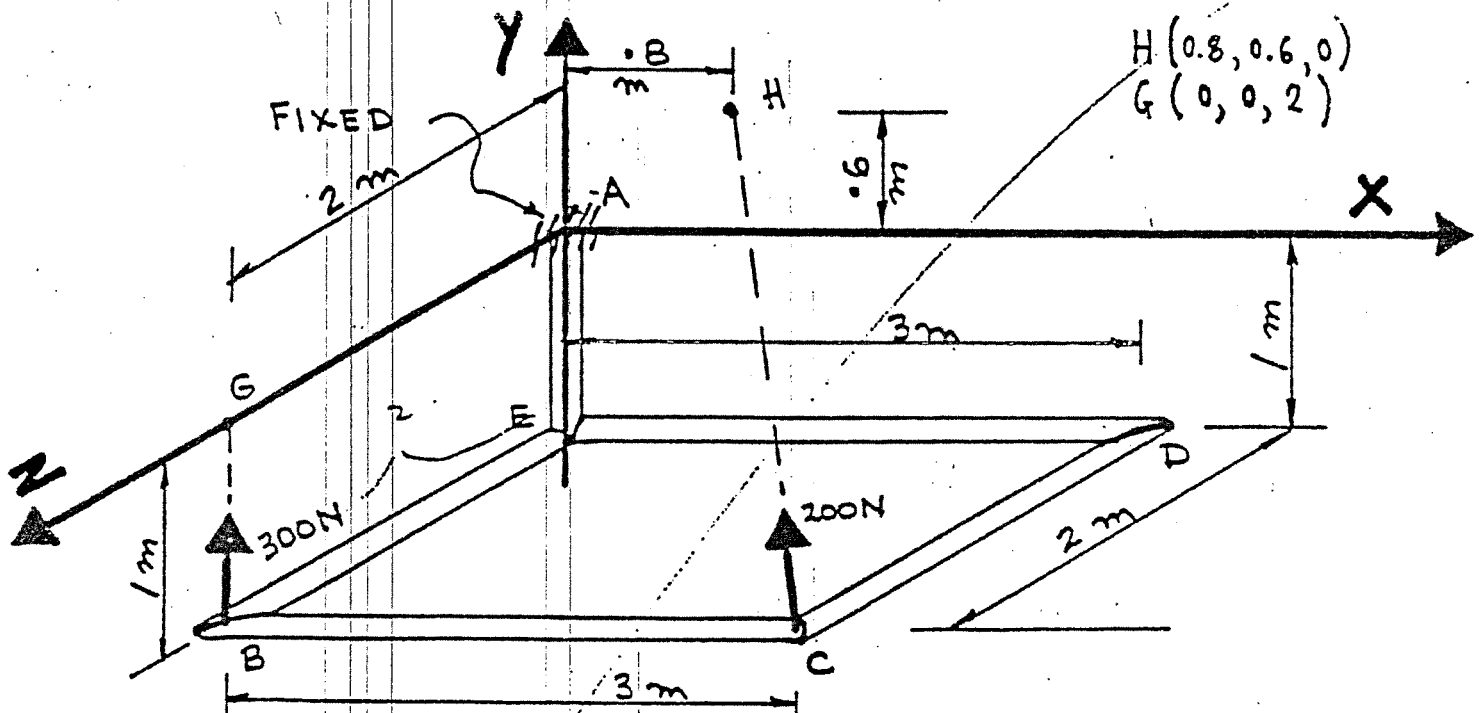
COURSE Statics	NUMBER ENGR. C242/2	SECTION 01, V, X, Y, AA
EXAMINATION Final	DATE December 15, 1983	TIME 19:00 - 22:00
INSTRUCTOR Prof. Goldman, Stathopoulos, Troitsky & Zielinski	DIVISION Day and Evening	

OF PAGES  
4

MATERIALS ALLOWED:  
Any calculating device

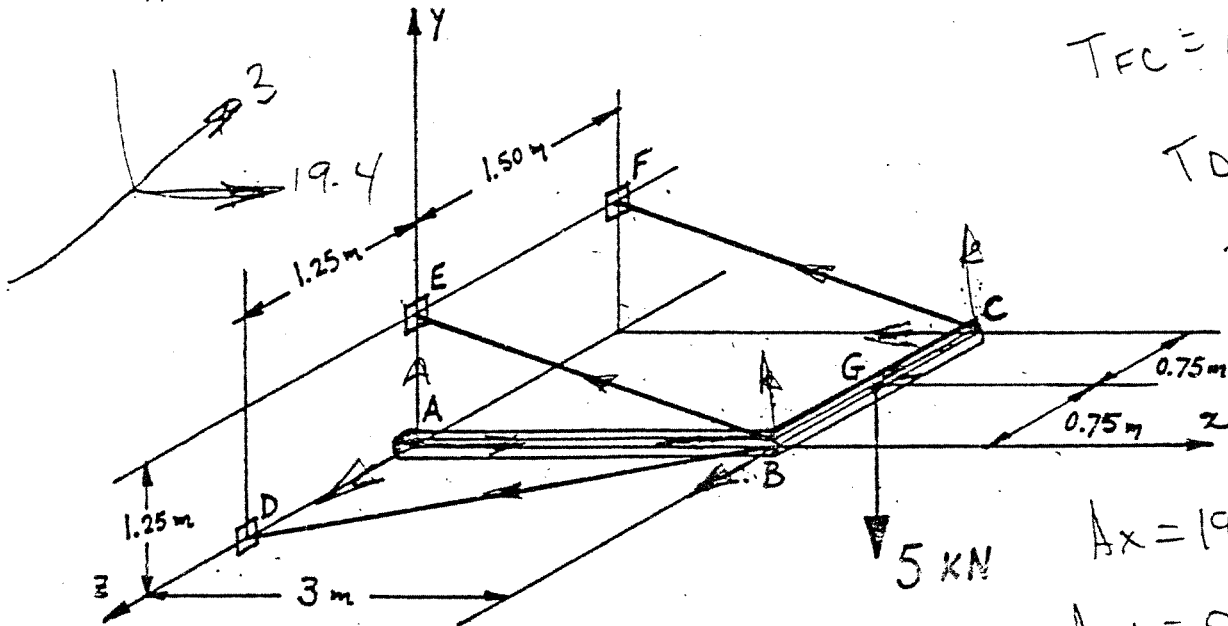
SPECIAL INSTRUCTIONS:  
Do any five problems. Only the first five problems presented will be graded.  
All questions carry equal weight.

- The steel frame AEBCDE is supported at a fixed connection at A. For the loading shown, determine the components of the reaction in the X, Y, Z direction at A and the moments about each axis to keep the frame in equilibrium.



$\frac{1}{2} \text{ kN}$   
 $\frac{1}{3} \text{ kN}$

2. The rigid L-shaped member ABC is supported by a ball and socket at A and by the three cables BD, BE and CF as shown. Determine the tension in each cable and the reaction components at A caused by the 5-KN force applied at G.

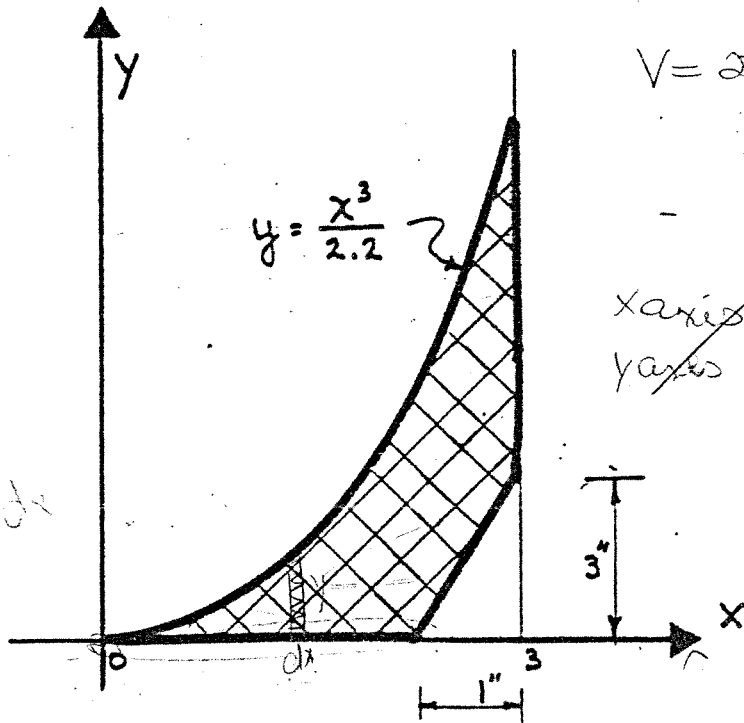


$T_{FC} = 6.5 \text{ kN}$   
 $T_{DB} = 7.8 \text{ kN}$   
 $T_{EB} = 6.5 \text{ kN}$

$A_x = 19.2 \text{ kN}$   
 $A_y = 0$   
 $A_z = -3 \text{ kN}$

- 3) For the cross-hatched area shown:

- Locate the centroid with respect to the OX axis.
- What is the volume generated if this area is rotated 100 degrees about the OX axis?



$V = 2\pi \bar{y} A \cdot \frac{100}{360}$

$y = \frac{x^3}{2.2}$

~~x-axis~~  $\rightarrow \bar{y}$   
~~y-axis~~  $\rightarrow \bar{x}$

$\int y dx$

CONCORDIA UNIVERSITY  
Faculty of Engineering & Computer Science

ENGR 242/2 STATICS, Section V, Fall 1996

Test # 1

Attempt all questions; only calculators permitted.

Time: 75 minutes

Marks

- 35 (1) Struts  $AB$  and  $AC$  in Fig. 1 can transmit only axial tensile or compression forces. Determine the forces in struts  $AB$  and  $AC$  and the tension in cable  $AD$  when force  $F = 1250$  kN.

use coordinates

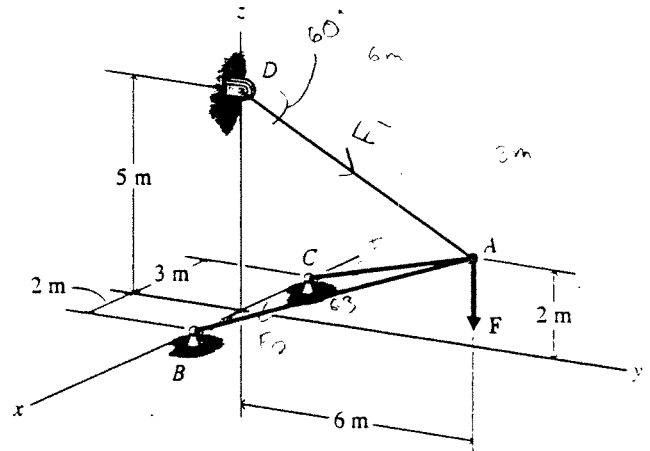


Fig. 1

- 35 (2) The tension in cable  $AB$  is 2 kN in Fig. 2. (a) What are the magnitude and direction of the moment about point  $D$  due to the force exerted by the cable? (b) What is the moment about the shaft  $CD$  due to the same force? Draw a sketch to indicate the sense of the moment about the shaft.
- 30 (3) A bracket is subjected to the force-couple system shown in Fig. 3. Determine (a) the magnitude and direction of the single equivalent force; (b) the perpendicular distance from support  $O$  to the line of action of the equivalent force.

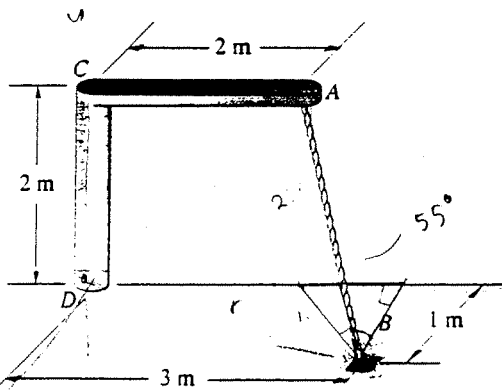


Fig. 2

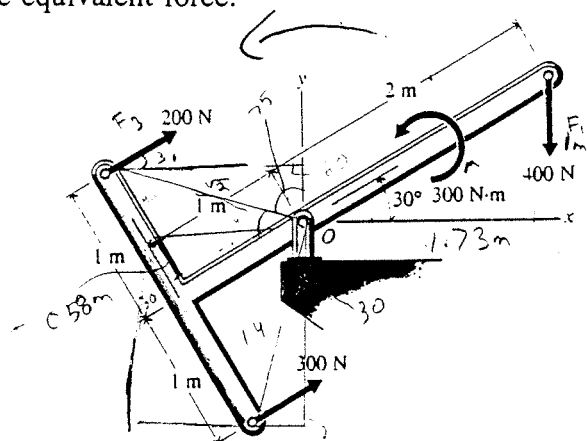
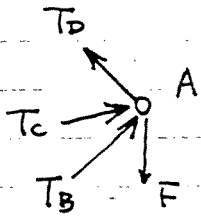


Fig. 3



$$\vec{AD} = 0\vec{i} - 6\vec{j} + 3\vec{k}$$

$$AD = \sqrt{45}$$

$$\vec{BA} = -2\vec{i} + 6\vec{j} + 2\vec{k}$$

$$BA = \sqrt{44}$$

$$\vec{CA} = 3\vec{i} + 6\vec{j} + 2\vec{k}$$

$$CA = 7$$

$$\sum \vec{F} = 0$$

$$x: -\frac{2}{\sqrt{44}} T_B + \frac{3}{7} T_C = 0$$

$$y: -\frac{6}{\sqrt{45}} T_D + \frac{6}{\sqrt{44}} T_B + \frac{6}{7} T_C = 0$$

$$z: \frac{3}{\sqrt{45}} T_D + \frac{2}{\sqrt{44}} T_B + \frac{2}{7} T_C = 1250$$

$$\therefore T_D = 1677 \text{ kN}, T_B = 995 \text{ kN}, T_C = 700 \text{ kN} \quad \Delta$$

1a)  $\vec{r} = \vec{DB} = 3\vec{i} + \vec{k}$

$$\vec{F} = 2 \frac{\vec{AB}}{AB} = 2 \frac{(\vec{i} - 2\vec{j} + \vec{k})}{\sqrt{1^2 + (-2)^2 + 1^2}}$$

$$= \frac{2}{\sqrt{6}} (\vec{i} - 2\vec{j} + \vec{k})$$

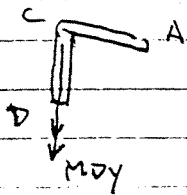
$$\vec{M}_D = \vec{r} \times \vec{F} = \frac{2}{\sqrt{6}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \frac{4}{\sqrt{6}} (\vec{i} - \vec{j} - 3\vec{k}) \quad (\text{kN}\cdot\text{m})$$

Magnitude:  $M_D = \frac{4}{\sqrt{6}} \sqrt{1^2 + (-1)^2 + (-3)^2} = \frac{4}{\sqrt{6}} \sqrt{11} = 5.42 \text{ (kN}\cdot\text{m)} \quad \Delta$

Direction:  $\theta_x = \cos^{-1}(\frac{1}{\sqrt{11}}) = 72^\circ$ ;  $\theta_y = \cos^{-1}(\frac{-1}{\sqrt{11}}) = 108^\circ$ ;  $\theta_z = \cos^{-1}(\frac{-3}{\sqrt{11}}) = 155^\circ \quad \Delta$

(b) about CD:  $M_{Dy} = -\frac{4}{\sqrt{6}} = -1.63 \text{ (kN}\cdot\text{m)} \quad \Delta$



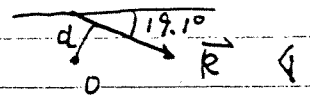
Sense:  $\downarrow$

3. (a)  $R_x = (200 + 300) \cos 30^\circ = 433 \text{ N}$

$$R_y = (200 + 300) \sin 30^\circ - 400 = -150 \text{ N}$$

$$\therefore R = \sqrt{R_x^2 + R_y^2} = 458.3 \text{ (N)} \quad \Delta$$

$$\theta = \tan^{-1}(\frac{R_y}{R_x}) = -19.1^\circ$$



(b)  $\vec{M}_D = -300(1) + 200(1) + 400(2 \cos 30^\circ) - 300$

$$= 292.8 \text{ (N}\cdot\text{m)}$$

$$= d R$$

$$\therefore d = 292.8 / 458.3 = 0.64 \text{ (m)} \quad \Delta$$

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Faculty of Engineering & Computer Science

ENGR 242/2 STATICS, Section T, Fall 1995

Test # 2

Attempt all questions; only calculators permitted.

Time: 75 minutes

Marks

35 (1) As shown in Fig. 1, the boom  $AC$  is supported at  $A$  by a ball-and-socket joint and by two cables  $BDC$  and  $CE$ . Cable  $BDC$  is continuous and passes over a frictionless pulley at  $D$ . Calculate the  $x, y, z$  components of reaction at  $A$  if a crate, having a weight of 80 lb, is suspended from the boom.

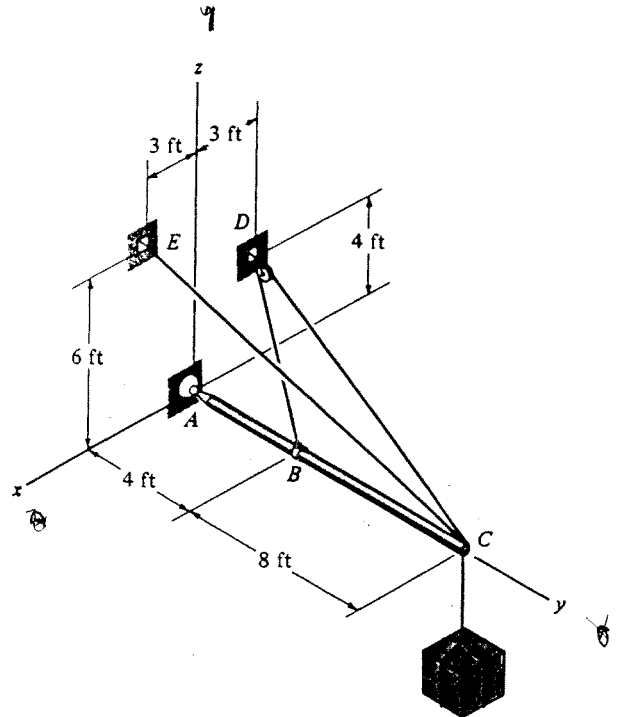


Fig. 1

30 (2) The plate in Fig. 2 is of uniform thickness and is made of homogeneous material whose mass per unit area of the plate is  $2 \text{ kg/m}^2$ . The vertical reactions at  $A$  and  $B$  are 6 N and 10 N, respectively. What is the  $x$  coordinate of the centroid of the hole?

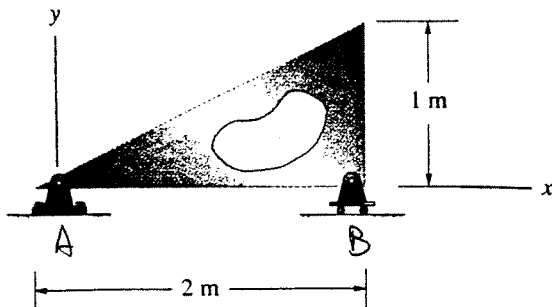


Fig. 2

35 (3) Determine the force in members  $GH$  and  $DE$  of the symmetric truss under the given loads in Fig. 3. State whether the two members are in tension or compression.

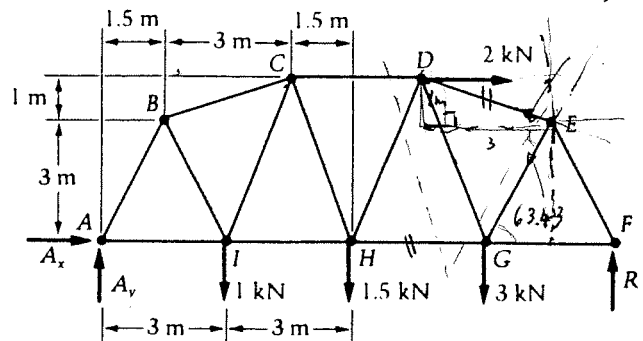


Fig. 3

$R \times F$

0.5976

**Test # 2**

Attempt all questions; only calculators permitted.

Time: 90 minutes

- (1) The door shown in Fig. 1 has a mass of 25 kg and is supported in a horizontal position by two hinges and a bar. The hinges have been properly aligned; therefore, they exert only force reactions on the door. Assume that the hinge at *B* resists any force along the axis of the hinge pins. Determine the reactions at supports *A*, *B* and *D*. (35 marks)
  
- (2) A square viewing window is located in a large water tank as shown in Fig. 2. Determine the resultant force **R** exerted by the water pressure on the window and locate its line of action with respect to point *A* at the top of the window if  $l = 2\frac{1}{2}$  m,  $d = 3$  m and  $h = 5$  m. Use  $1000 \text{ kg/m}^3$  for the density of water. (25 marks)

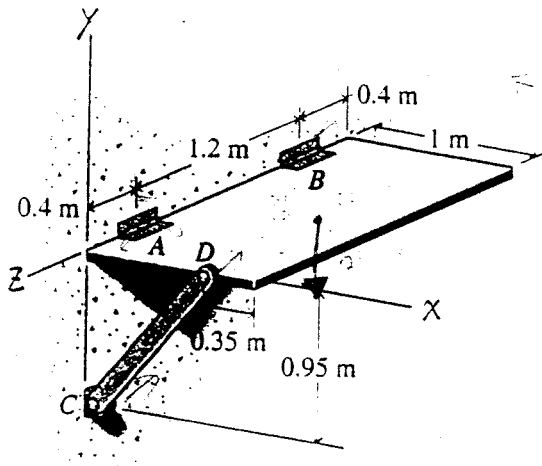


Fig. 1

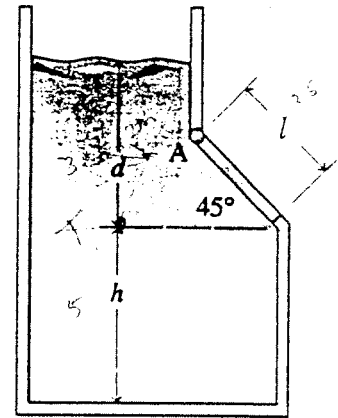


Fig. 2

- (3) Determine the forces in members *BC*, *BD*, *CD* and *CE* of the truss shown in Fig. 3. (25 marks)
  
- (4) Two prismatic homogeneous bars *AB* and *BC* of length  $l$  and  $2l$ , respectively, are rigidly joined at *B* at an angle of  $120^\circ$  to each other, as shown in Fig. 4. In the suspended position shown, *AB* makes an angle  $\alpha$  with the vertical string *OA*. Determine the value of  $\alpha$ . (Hint: the resultant of the weight of the bars should be along *OA*, 15 marks)

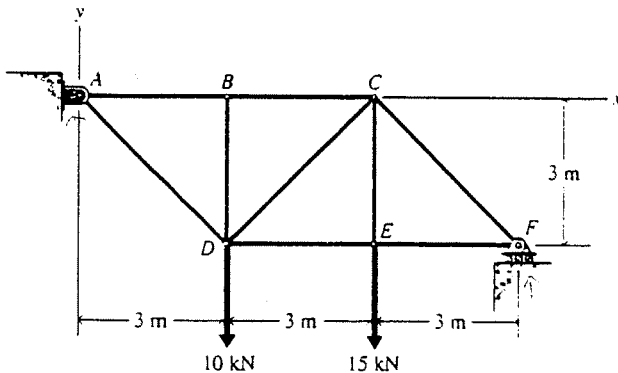


Fig. 3

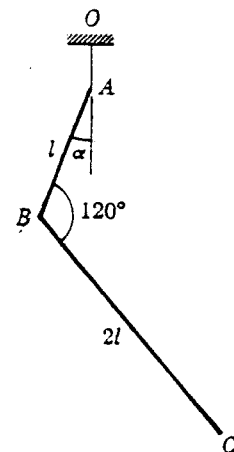


Fig. 4

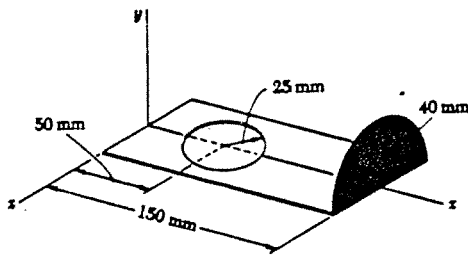
CONCORDIA UNIVERSITY  
DEPARTMENT OF CIVIL ENGINEERING  
TEST # 3 ENGR 242/2 - STATICS  
PROFESSOR M.S. TROITSKY

TIME: 11:45 - 13:00

DATE: Nov., 27, 1987

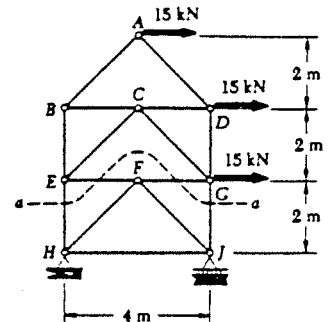
INSTRUCTIONS :      CLOSED BOOK EXAMINATION.  
                             SOLVE ANY THREE PROBLEMS.      TOTAL MARKS: 30

1. Locate the center of gravity of the sheetmetal form shown below.
- ✓ 2. Determine the force in member GJ of the truss shown. (Hint: Use section a-a.)
3. Draw the shear and bending-moment diagrams for the beam and loading shown.
- ✓ 4. A block of mass  $m = 20$  kg rests on a rough plane as shown. Knowing that  $\alpha = 25^\circ$  and  $\mu = 0.20$ , determine the magnitude and direction of the smallest force  $P$  required (a) to start the block up the planer, (b) to prevent the block from moving down the plane.

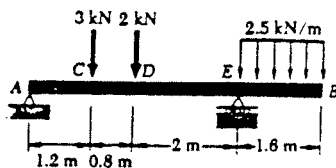


P# 1

$\frac{4r}{3\pi}$

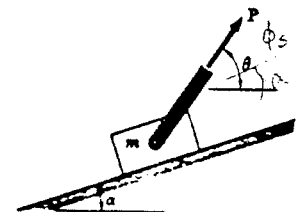


P# 2



P# 3

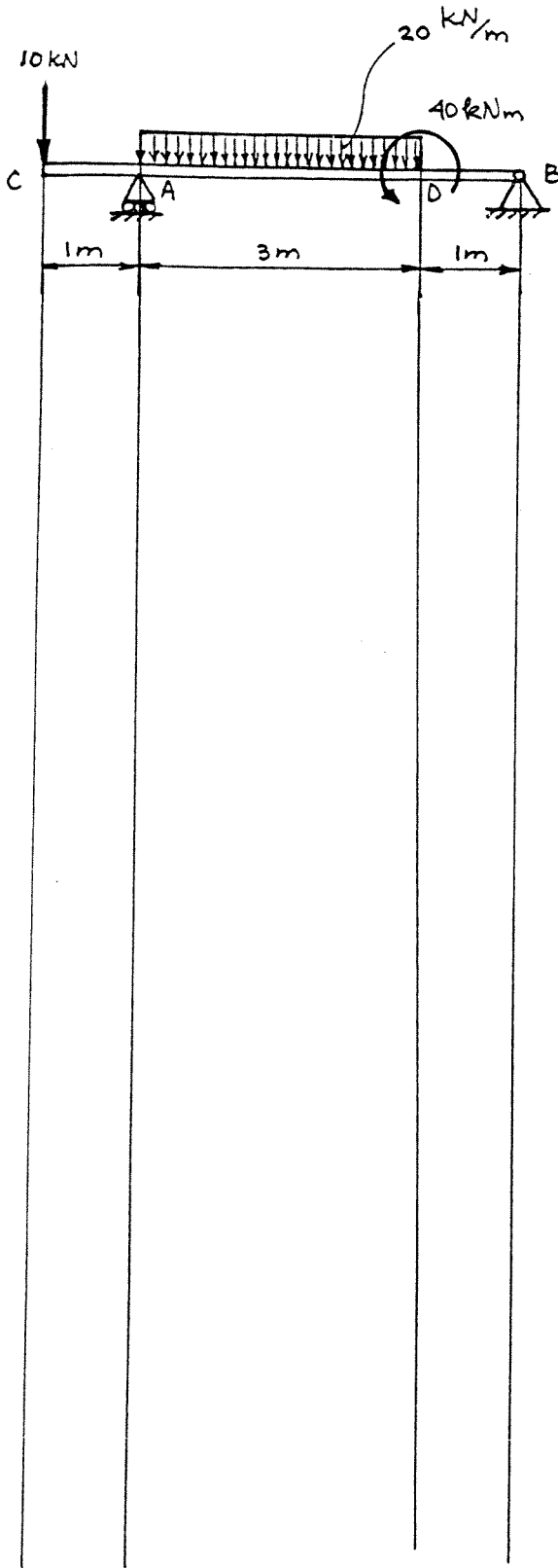
189



P# 4

$\theta = \phi_s + \alpha$

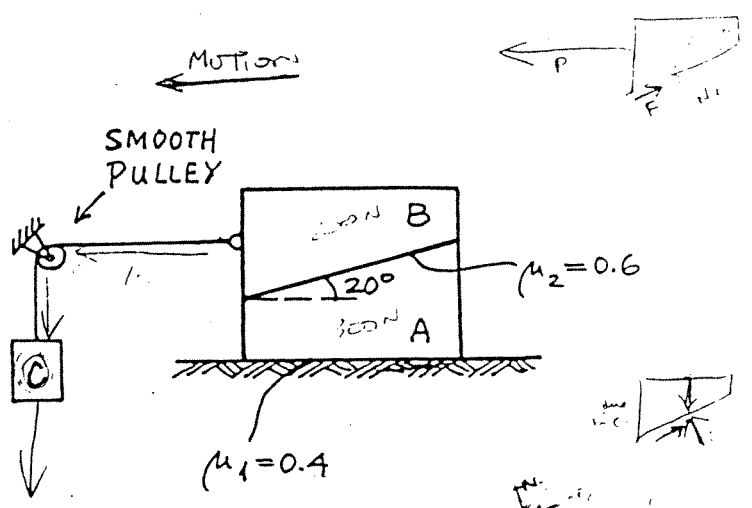
2. Draw the shear force and bending moment diagrams for the beam loaded as shown; indicate peak values. Write an equation for the shear force and the bending moment for section AD. Also, locate the points of zero moment on the beam. Answer this question on this sheet.



COURSE	NUMBER	SECTION	
Statics	ENGR 242/2	T, V, X, XX, Z	
EXAMINATION	DATE	TIME	# OF PAGES
Final	December 7, 1995	14:00 - 17:00	Five (5)
STRUCTOR			
Prof. Athienitis, Goldman, Haseganu, Stathopoulos (Coordinator), Wu			
MATERIALS ALLOWED:			
Standard calculator only			
SPECIAL INSTRUCTIONS:			
Do questions 1 to 5 and either one of 6 or 7. All questions carry equal weight.			

### QUESTION 1

Blocks A and B weigh 300 N and 200 N, respectively. Using the coefficients of static friction indicated in the diagram, determine the greatest weight of block C without causing motion.

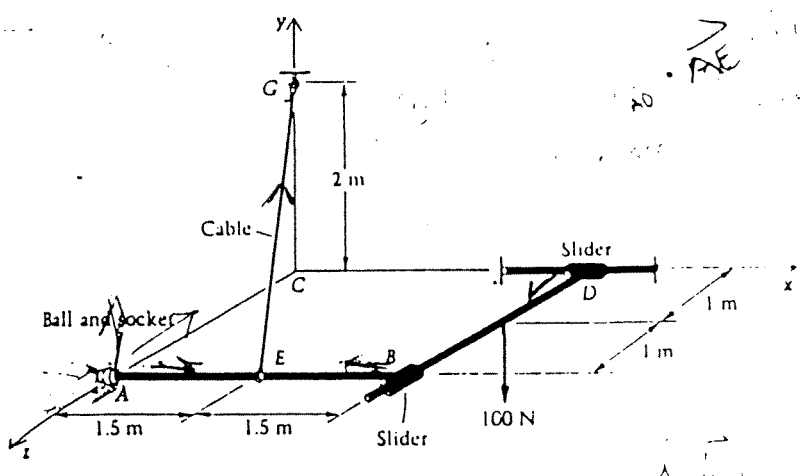


Equilibrium  
 $F = P$   
 $F < F_m$

$$\sum F_x = 0 \quad -C + F \cos 20^\circ - \mu_2 N_2 - (200 \sin 20^\circ)$$

### QUESTION 2

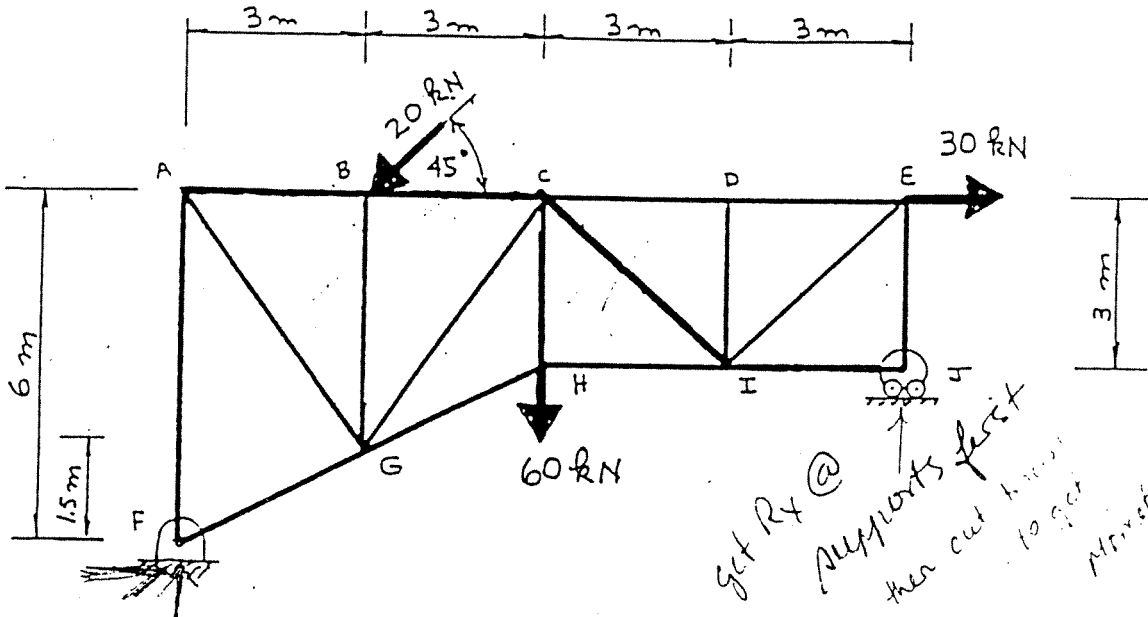
The structure shown is composed of two light bars and a cable. There is no resistance to rotation about and sliding along the axes of the two slides at B and D. Find the tension in the cable when the structure is subjected to the 100-N load.



*Handwritten signature*

QUESTION 3

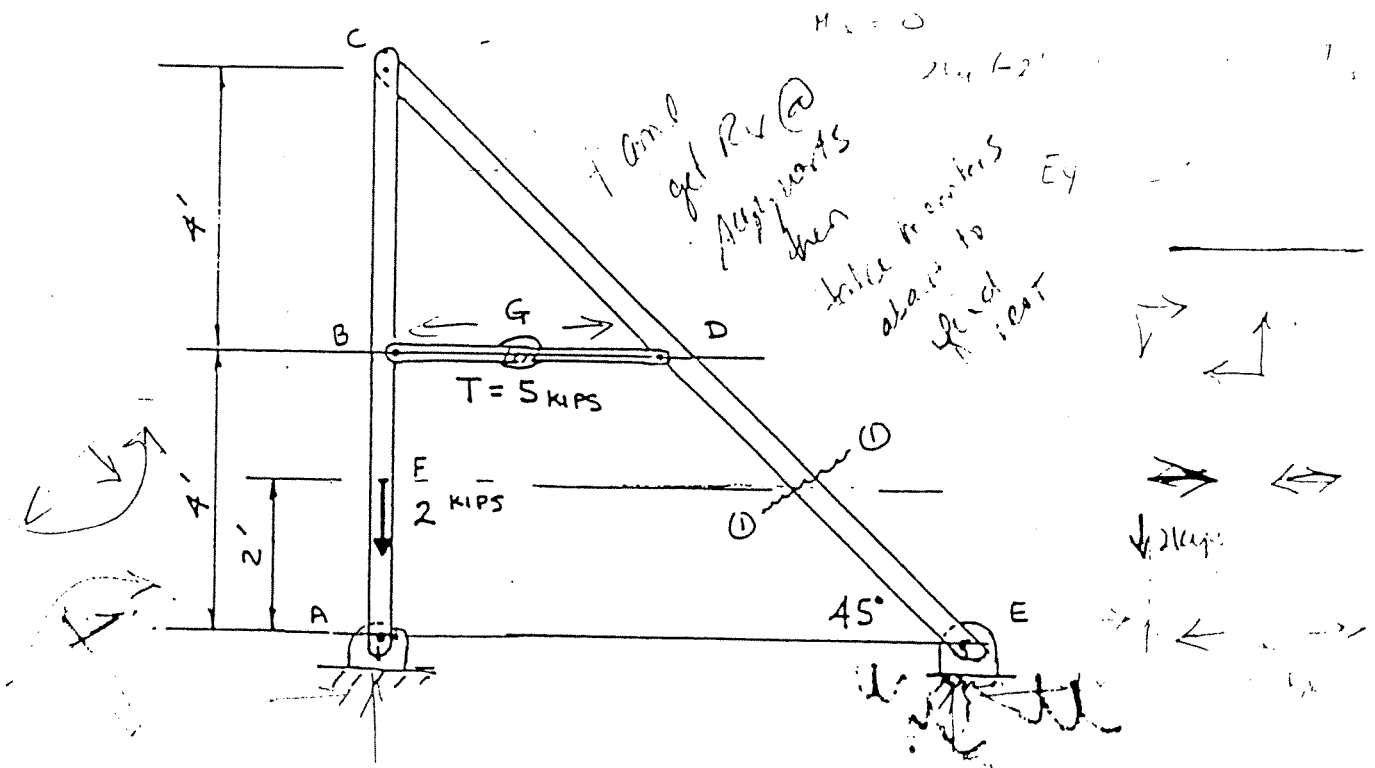
For the truss shown, calculate the reactions at  $F$  and  $J$  and obtain the forces in the members  $BC$ ,  $BG$ ,  $CI$  and  $IJ$ . Show these in a clear sketch of the truss indicating tension or compression.



QUESTION 4

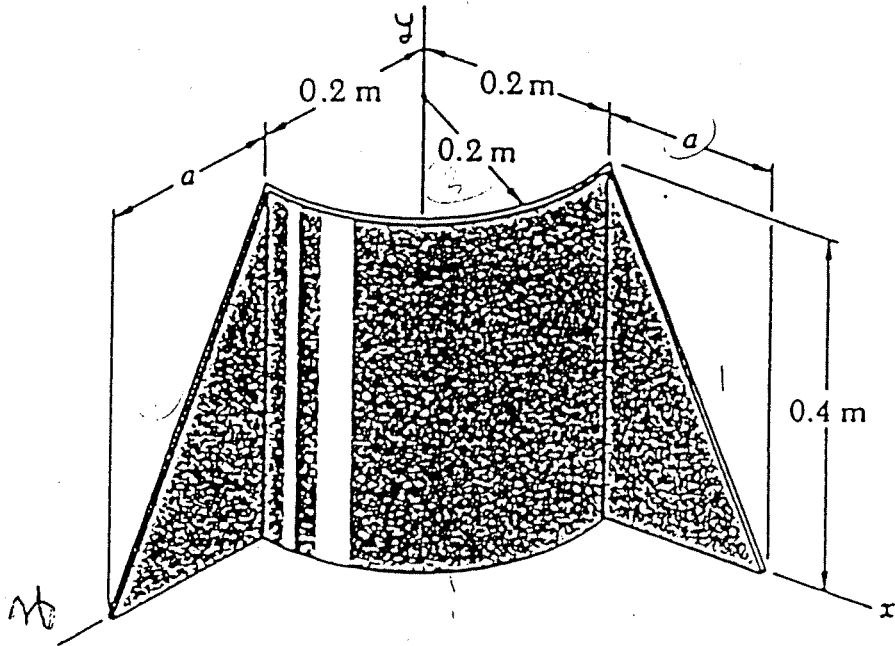
After the frame is set up, a 2-kip load is placed at point  $F$  and a turn buckle  $G$  is tightened so that the tension in member  $BD$  is 5 kips.

- Find the reactions at  $A$  and  $E$  and the forces in the pin at  $C$ .
- At section 1-1, what is the shear, bending moment and axial force in the member?



QUESTION 6

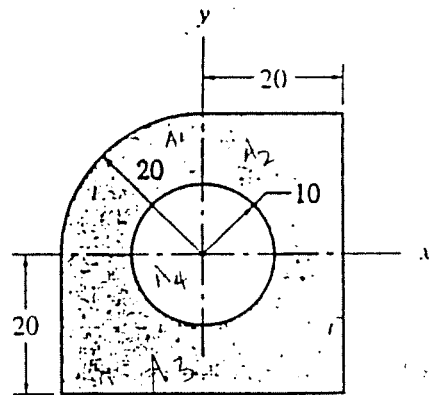
Determine the distance  $a$  so that the center of gravity of the sheet metal form is located 0.2 m from the  $y$ -axis. The thickness of the sheet is negligible.



QUESTION 7

For the plane region with the circular cutout shown, find the moments of inertia with respect to

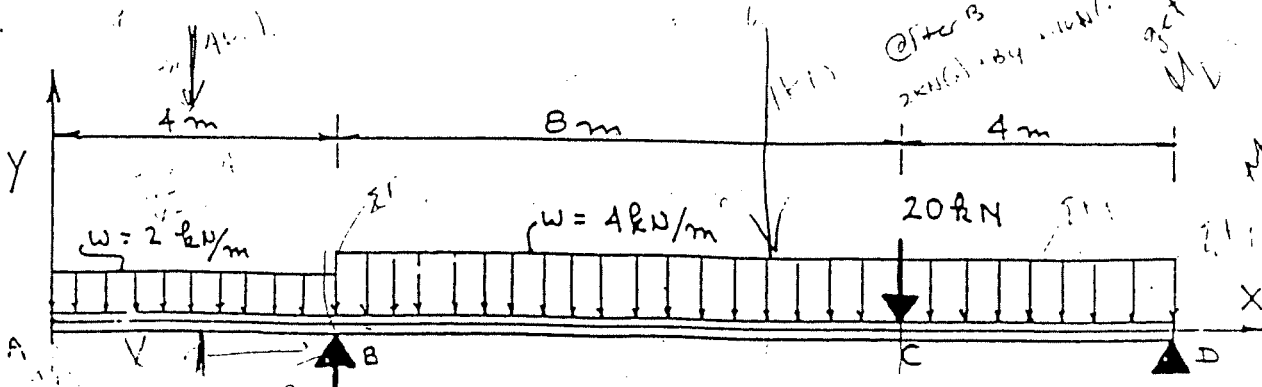
- (a) the horizontal  $x$ -axis, i.e.  $I_x$ ; and
- (b) the horizontal centroidal axis, i.e.  $\bar{I}_x$



QUESTION 5

For the beam and loading shown,

- a) sketch the bending moment and shearing force diagrams and indicate peak values;
- b) write an equation for the bending moment valid in section  $BC$  of the beam, and locate the point of zero moment in this section.



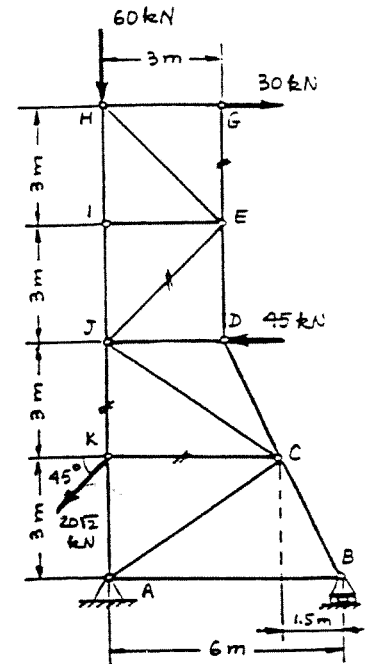
41

# CONCORDIA UNIVERSITY

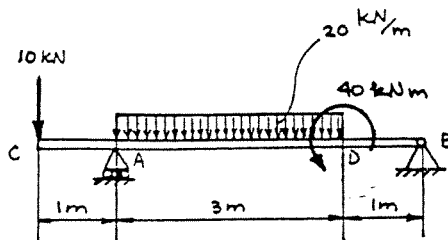
## DEPARTMENT OF MECHANICAL ENGINEERING

COURSE:	NUMBER:	SECTION:
STATICS	ENGR 242/2	T
EXAMINATION:	DATE & TIME:	# OF PAGES:
MIDTERM 2	November 25, 1999, 8:30 – 9:45 pm	3
INSTRUCTOR: Dr. Eliza M. Haseganu		
MATERIAL ALLOWED:	<input checked="" type="checkbox"/> NO <input type="checkbox"/> YES	
CALCULATORS ALLOWED:	<input type="checkbox"/> NO <input checked="" type="checkbox"/> YES	(non-programmable)
SPECIAL INSTRUCTIONS:	Answer questions 1 and 2 for 35 marks each, and question 3 OR 4, for 30 marks. Question 2 should be answered on the separate sheet provided.	

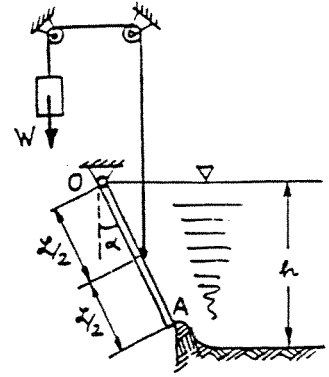
1. Determine the forces in members GE, EJ, JK, and KC of the truss shown, and indicate whether these members are in tension or compression. Use the method of joints combined with the method of sections. Indicate with justification the zero force members.



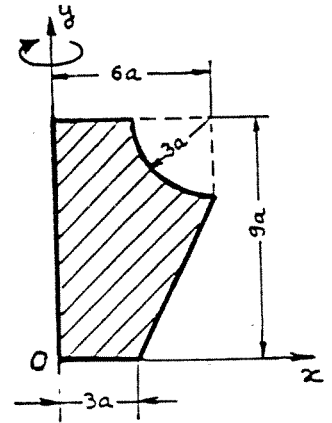
2. Draw the shear force and bending moment diagrams for the beam loaded as shown; indicate peak values. Write an equation for the shear force and the bending moment for section AD. Also, locate the points of zero moment on the beam. Answer this question on the separate sheet provided.



3. The gate  $OA$  is located at the end of a 2m wide channel and is supported by hinges along its top edge. The gate is held in its closed position by a cable and a weight  $W$ , as shown. Determine the minimum magnitude of the weight  $W$  necessary to keep the gate closed when the water level reaches  $O$ . Consider the density of water  $\rho = 1000 \text{ kg/m}^3$ ,  $L = 3 \text{ m}$ ,  $\alpha = 30^\circ$  and  $h = 3 \text{ m}$ . Neglect the weight of the gate.



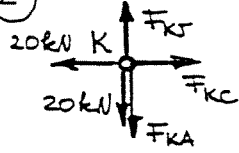
4. For the figure shown determine:
- (1) the location of the centroid of the area;
  - (2) the volume generated by a complete rotation of the shaded area about the  $y$  axis.
  - (3) If  $a = 0.1 \text{ m}$ , and the mass density of the material is  $\rho = 100 \text{ kg/m}^3$ , determine the mass and the weight of the body of revolution obtained at (2).



1.

• METHOD OF JOINTS:

② FBD JOINT K:

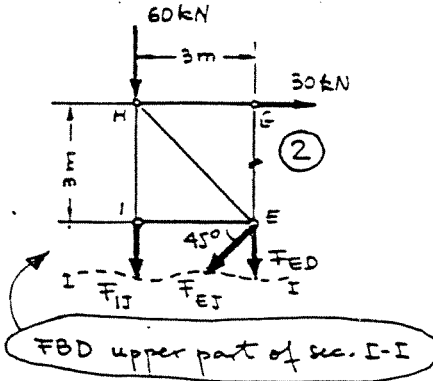


$\oplus \sum F_x = 0; F_{KC} - 20 = 0$

$F_{KC} = 20 \text{ kN}$

⑥ TENSION

• METHOD OF SECTIONS:

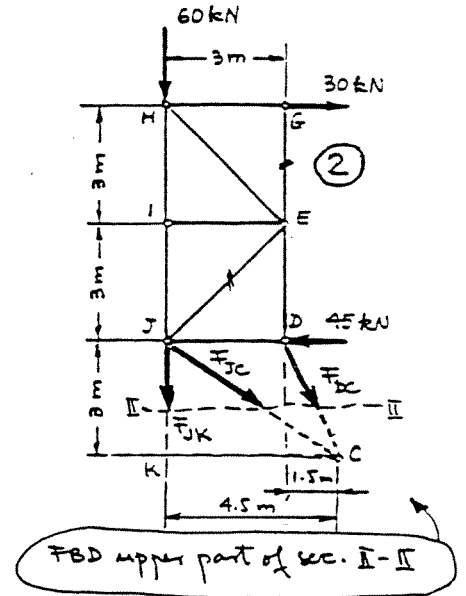


$\oplus \sum F_x = 0;$

$-F_{EJ} \frac{\sqrt{2}}{2} + 30 = 0$

$\rightarrow F_{EJ} = 42.43 \text{ kN}$  ⑥

TENSION



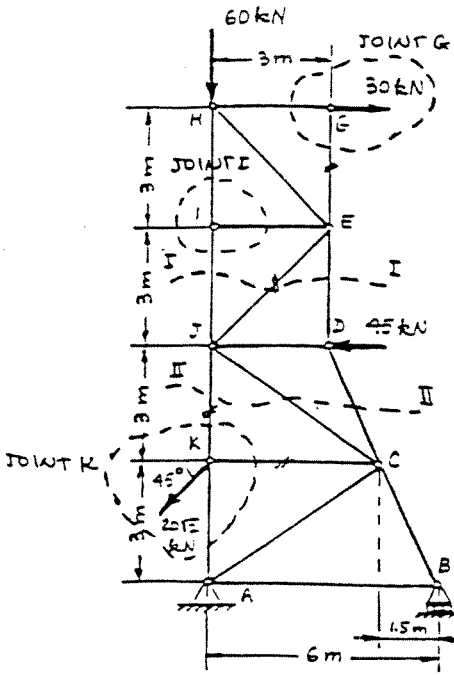
$\oplus \sum M_C = 0;$

$F_{JK} (4.5) + 60(4.5) - 30(9) + 45(3) = 0$

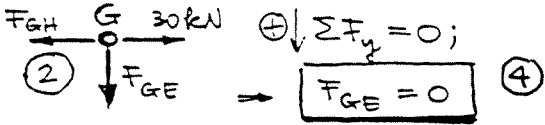
$\rightarrow F_{JK} = -30 \text{ kN}$

$F_{JK} = 30 \text{ kN}$

COMPRESSION



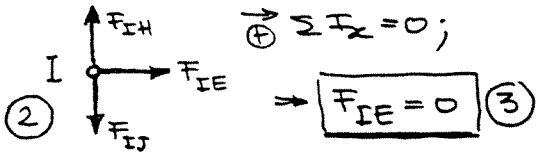
FBD JOINT G:



$\oplus \sum F_y = 0;$

$F_{GE} = 0$  ④

FBD JOINT I:

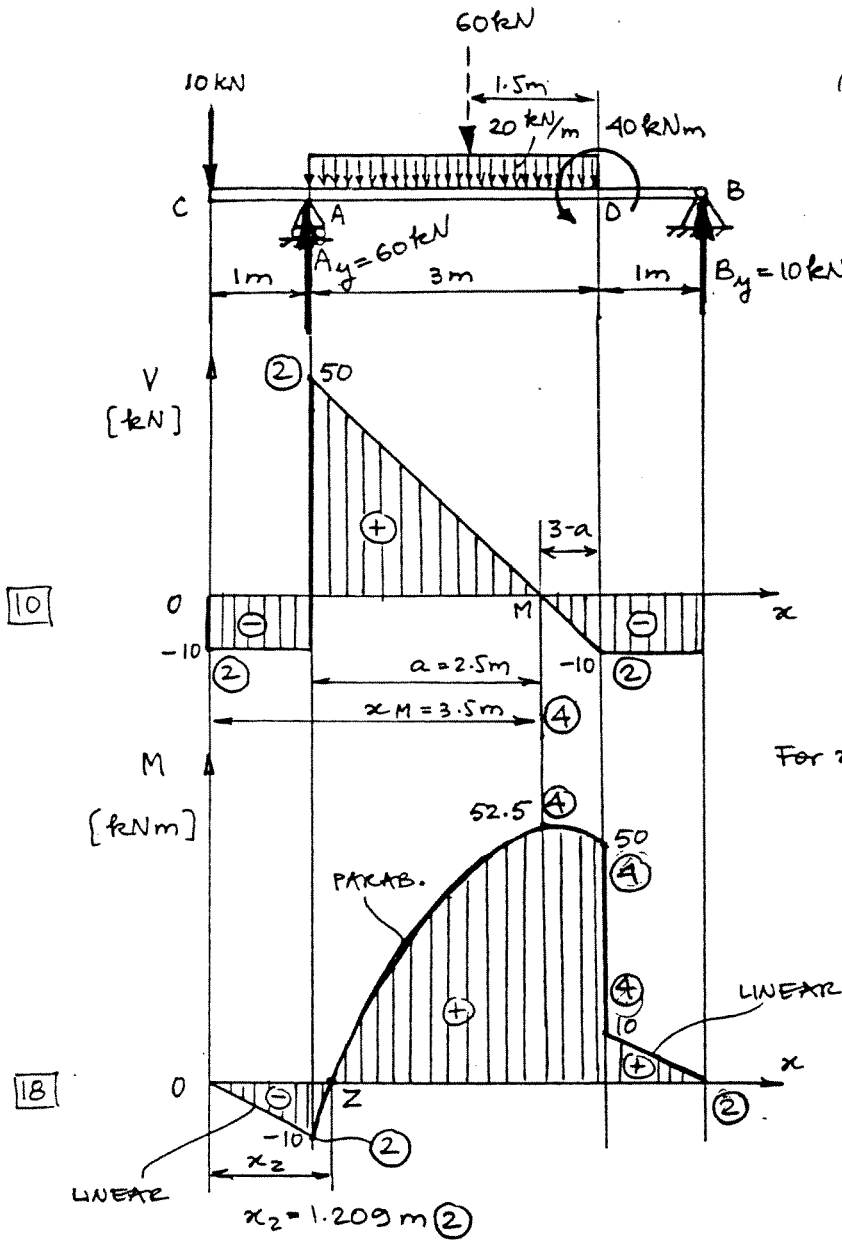


$\oplus \sum F_x = 0;$

$F_{IE} = 0$  ③

$\Sigma 35$

2. Draw the shear force and bending moment diagrams for the beam loaded as shown; indicate peak values. Write an equation for the shear force and the bending moment for section AD. Also, locate the points of zero moment on the beam. Answer this question on this sheet.



• REACTION FORCES

$$\sum M_B = 0; 4A_y - 10(5) - 60(2.5) - 40 = 0$$

$$\Rightarrow A_y = 60 \text{ kN} \quad (2)$$

$$\sum M_A = 0; 4B_y + 40 - 60(1.5) + 10(1) = 0$$

$$\Rightarrow B_y = 10 \text{ kN} \quad (2)$$

Check:  $\sum F_y = 0; 60 + 10 - 10 - 60 = 0$

• SHEAR FORCE

$$V_{C\ominus} = 0; V_{C\oplus} = -10 \text{ kN}$$

$$V_{A\ominus} = -10 \text{ kN}; V_{A\oplus} = -10 + 60 = 50 \text{ kN}$$

$$V_D = -10 + 60 - 60 = -10 \text{ kN} = V_{B\ominus}$$

$$V_{B\oplus} = 0$$

For  $x_M$ :  $V_M = 60 - 10 - 20(x_M - 1) = 0$

$$\Rightarrow x_M = 3.5 \text{ m}$$

or: similarity  $\frac{a}{50} = \frac{3-a}{10} \Rightarrow a = 2.5$

$$\Rightarrow x_M = a + 1 = 3.5 \text{ m}$$

• BENDING MOMENT

$$M_C = 0; M_A = -10 \text{ kNm}$$

$$M_M = 60(2.5) - 10(3.5) - \frac{20(2.5)^2}{2} = 52.5 \text{ kNm}$$

$$M_{D\ominus} = 60(3) - 10(4) - 60(1.5) = 50 \text{ kNm}$$

$$M_{D\oplus} = 50 - 40 = 10 \text{ kNm}$$

$$M_B = 0$$

• INTERVAL AD

$$(1) V = -10 + 60 - 20x = -20x + 50 \text{ kN}$$

$$(2) M = -10x + 60(x-1) - 20\frac{(x-1)^2}{2} \text{ kNm}$$

• POINTS OF ZERO MOMENT

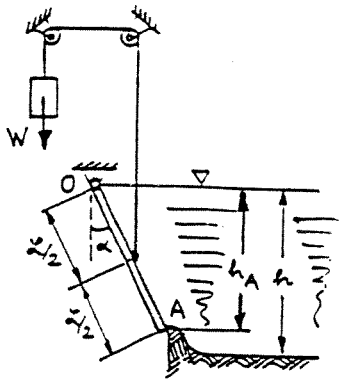
$$M_z = -10x_z + 60(x_z - 1) - 20\frac{(x_z - 1)^2}{2} = 0$$

$$x_z^2 - 7x_z + 7 = 0 \Rightarrow x_z = \begin{cases} 5.79 \text{ m} \\ 1.209 \text{ m} \end{cases}$$

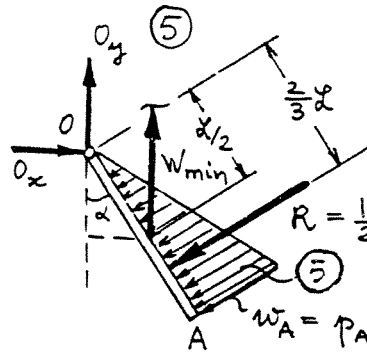
$$V = \left[ \oplus \uparrow \sum F_{iy} \text{ LEFT of } (1) \right]$$

$$M = \left[ \oplus \curvearrowright \sum M_{(1)} \text{ LEFT of } (1) \right]$$

$\Sigma 35$



FBD: Since  $W_{min}$  is required, reaction force at support A is zero



$$R = \frac{1}{2} w_A L = \frac{1}{2} 50.9743 (3) = \underline{76.46 \text{ kN}} \quad (5)$$

$$w_A = \rho_A b = \rho g h_A b = 1000 (9.81) \frac{3\sqrt{3}}{2} 2 = 50.9743 \frac{\text{kN}}{\text{m}} \quad (5)$$

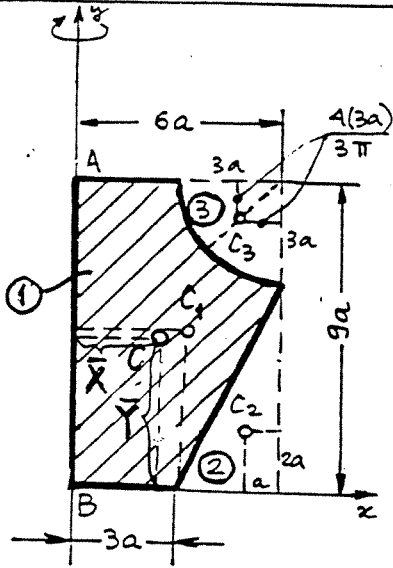
$$h_A = L \cos \alpha = 2 \cos 30^\circ = 2 \frac{\sqrt{3}}{2}$$

$$\oplus \Sigma M_O = 0 \quad W_{min} \frac{L}{2} \sin \alpha - R \frac{2}{3} L = 0$$

$$\underline{W_{min} = \frac{8R}{3} = \frac{8(76.46)}{3} = 203.89 \text{ kN}} \quad (10)$$

Σ 30

4.



PART	$\bar{x}$	$\bar{y}$	A	$\bar{x}A$	$\bar{y}A$
①	3a	4.5a	54a <sup>2</sup>	162a <sup>3</sup>	243a <sup>3</sup>
② (-)	5a	2a	-9a <sup>2</sup>	-45a <sup>3</sup>	-18a <sup>3</sup>
③ (-)	6a - $\frac{4(3a)}{3\pi}$	9a - $\frac{4(3a)}{3\pi}$	$-\frac{9\pi a^2}{4}$	-33.41a <sup>3</sup>	-54.62a <sup>3</sup>
Σ	-	-	37.93a <sup>2</sup>	83.59a <sup>3</sup>	170.38a <sup>3</sup>

$$\bar{X} \Sigma A = \Sigma \bar{x} A \quad \bar{X} 37.93a^2 = 83.59a^3 \rightarrow \underline{\bar{X} = 2.2a} \quad (5)$$

$$\bar{Y} \Sigma A = \Sigma \bar{y} A \quad \bar{Y} 37.93a^2 = 170.38a^3 \rightarrow \underline{\bar{Y} = 4.49a} \quad (5)$$

$$V = 2\pi \bar{X} A = 2\pi \Sigma \bar{x} A = 2\pi (83.59a^3) = \underline{525.2a^3} \quad (5)$$

$$m = \rho V = 100 (525.2) (0.1)^3 = \underline{52.52 \text{ kg}} \quad (5)$$

$$W = mg = \underline{515.22 \text{ N}} \quad (1)$$

Σ 30

① + ② + ③ / ④

TOTAL: 100

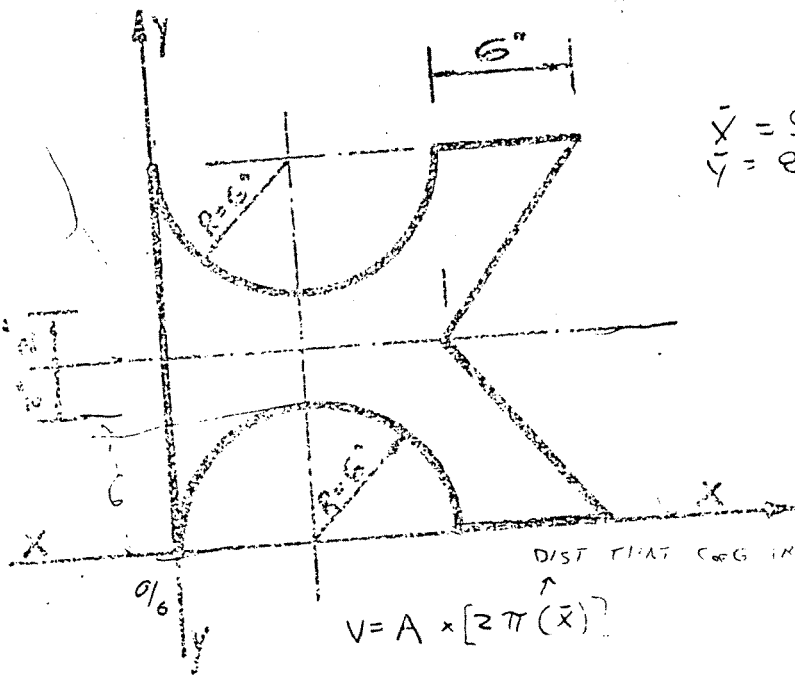
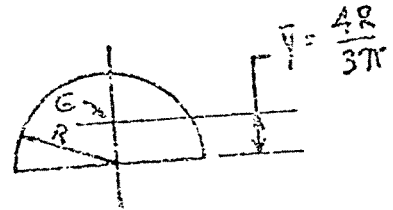
CONCORDIA UNIVERSITY  
ENGINEERING 0242

Class Test 2  
Time: 1 1/2 Hours

Statics  
Section Y

Prof. C. Goldman

1. a) Find the centroid of the area shown in the figure.  
b) What is the volume generated if this area is rotated 300° about the Y axis.



$$\bar{X} = 9.8''$$

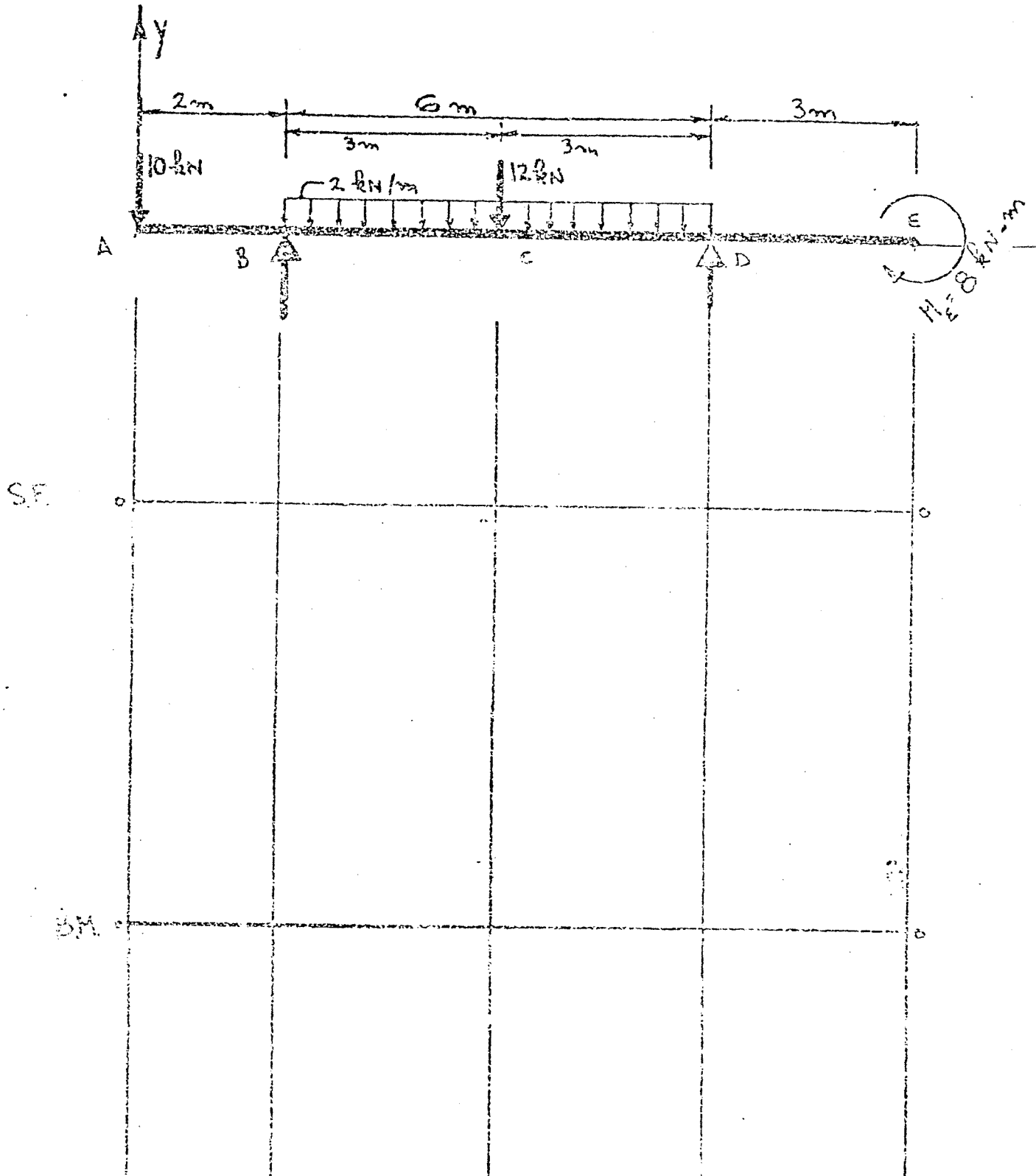
$$\bar{Y} = 8''$$

DIST THAT C.G. TRAVELS CIRCUMFERENCE

$$V = A \times [2\pi(\bar{x})]$$

$$V_{@300^\circ} = A \times 2\pi\bar{x} \times \frac{300}{360}$$

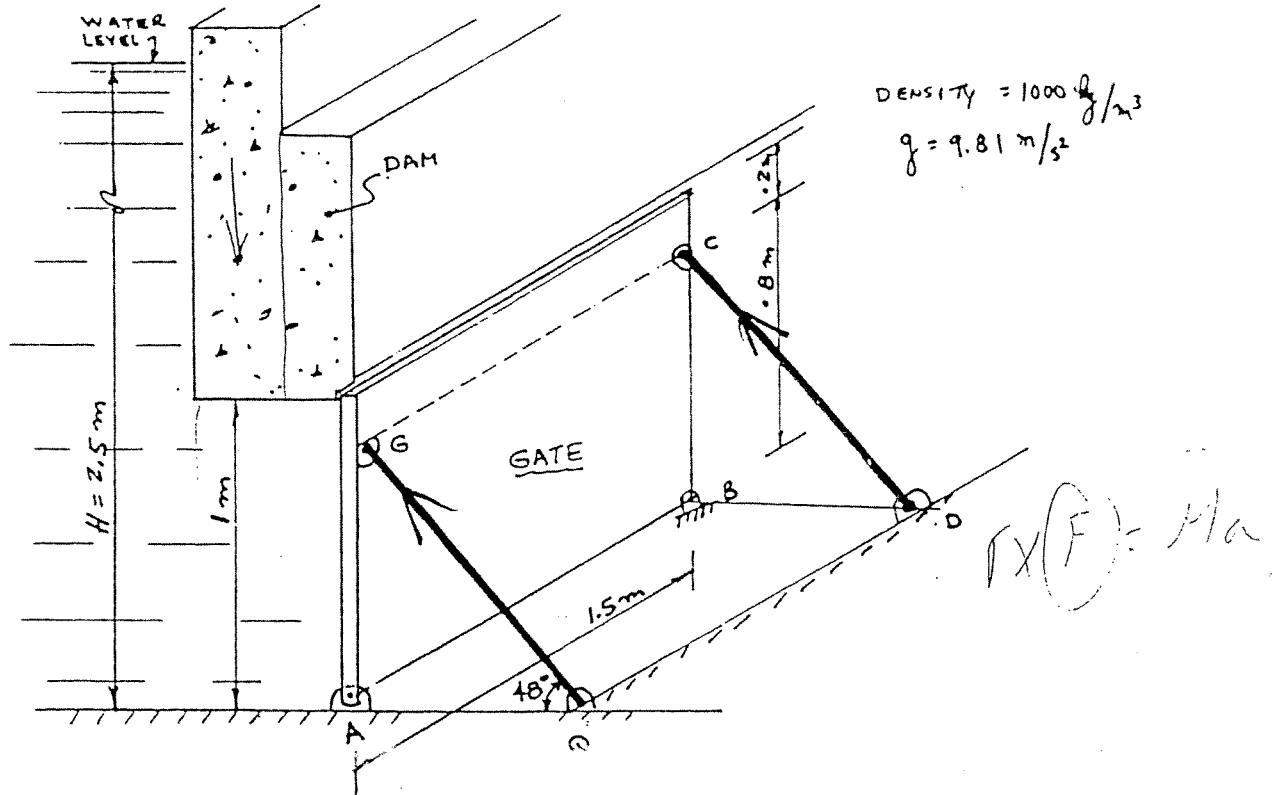
2. a) Sketch a bending moment, and shearing force diagram for the beam shown in the figure. SHOW PEAK VALUES  
 b) Locate the point of inflection. i.e. zero moment, BETWEEN  
 c) Write an equation for the shear, and moment, valid between points B and C of the beam.





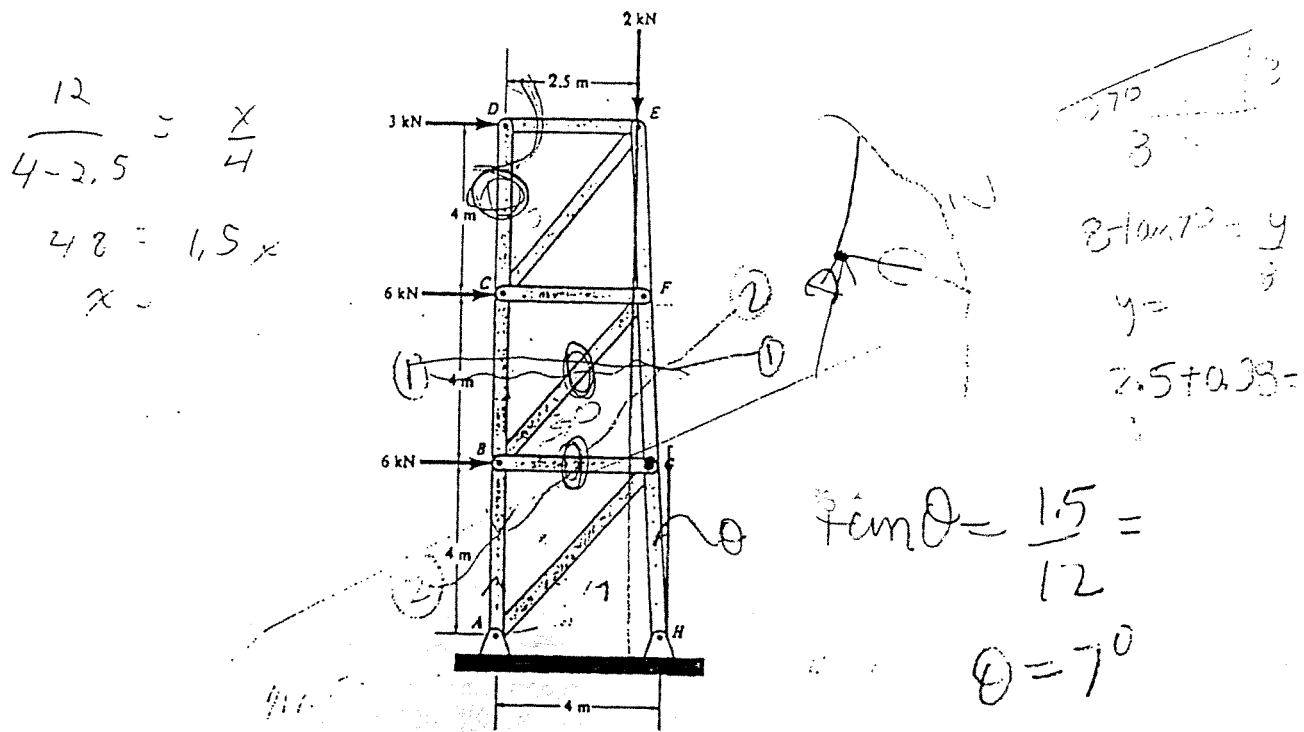
### Problem 2

Determine the forces in members CD and GQ holding the rectangular gate in a dam, as shown. Note that members CD and GQ are parallel and carry only axial loading.



### Problem 3

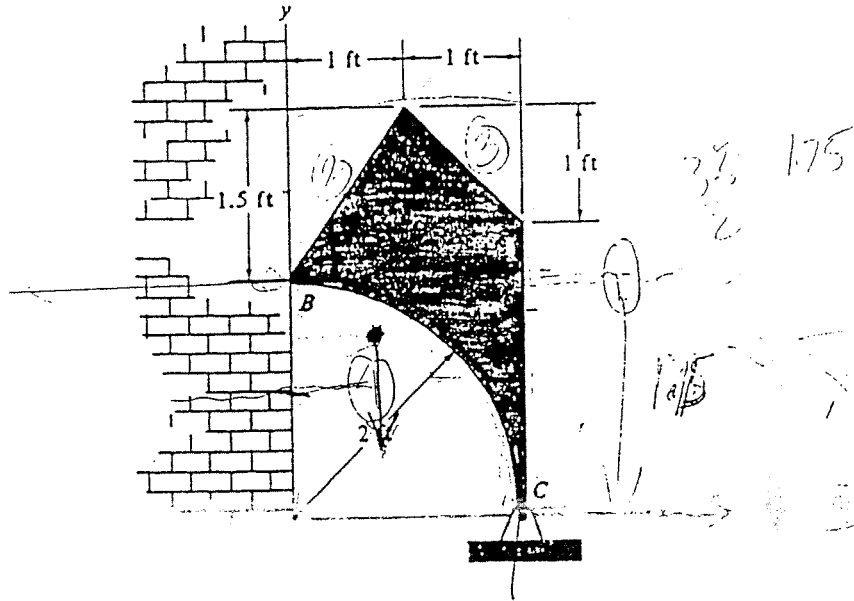
Determine the force in members DC, BG and BF of the tower truss shown. Indicate if the members are in tension or compression. Note that the left side ABCD stands vertical.



**Problem 6**

The 3-ft thick arched structure is supported by a pin at C and bears against the smooth wall at B. If the specific weight of the material of the structure is  $90 \text{ lb/ft}^3$  determine:

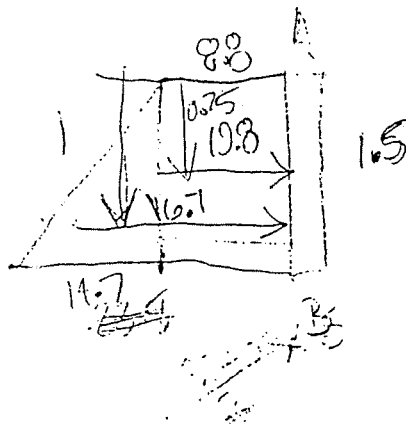
- the reactions at supports B and C; and
- the moment of inertia of the cross-section of the structure (shaded area) with respect to x axis.



$$P_B = 2.4 \times 1000 \times 9.81 = 23.5 \text{ kN/m}^2$$

$$P_H = 0.9 \times 1000 \times 9.81 = 8.8 \text{ kN/m}^2$$

$$T_{\text{total}} = 36.5$$



$$I_x = \frac{1}{12} b h^3 = \frac{1}{12} (1.5) (1.5)^3 = 0.28125$$

$$I_{x'} = I_x + A d^2 = 0.28125 + (1.5 \times 1.5) (1.67)^2 = 3.55$$

$$I_x = 3.55$$

$$I_y = 0.86$$

CONCORDIA UNIVERSITY  
Faculty of Engineering & Computer Science

ENGR 242/2 STATICS, Section T, Fall 1995

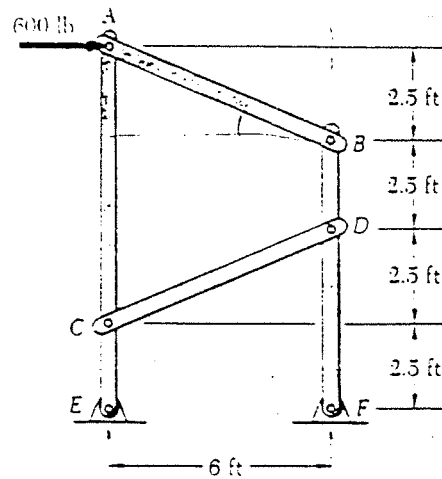
Test # 3

Attempt all questions; only calculators permitted.

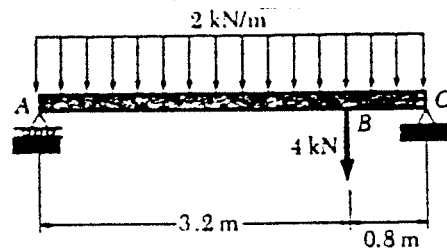
Time: 75 minutes

Marks

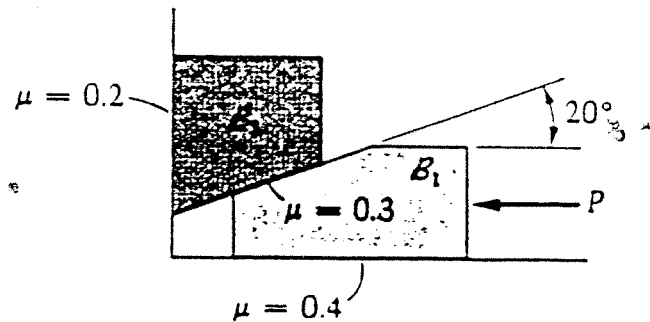
- 35 (1) A 600-lb horizontal force is applied to member  $ACE$  at  $A$  of the frame shown. Determine the forces acting on the two vertical members of the frame.



- 35 (2) For the beam and loading shown, (a) draw the shear and bending-moment diagrams; and (b) write equations for the shearing force and bending moment between  $A$  and  $B$ .



- 30 (3) For the given friction coefficients, find the force  $P$  for which the 200-lb block  $B_2$  will be on the verge of sliding upward. The wedge  $B_1$  weighs 50 lb.



CONCORDIA UNIVERSITY  
Faculty of Engineering & Computer Science

ENGR 242/2 STATICS, Section V, Fall 1996

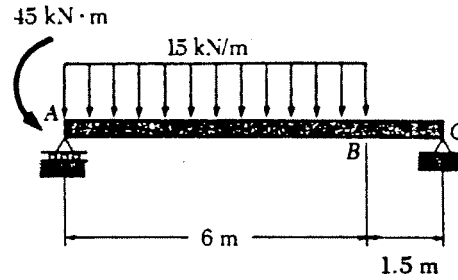
Test # 3

Attempt all questions; only calculators permitted.

Time: 90 minutes

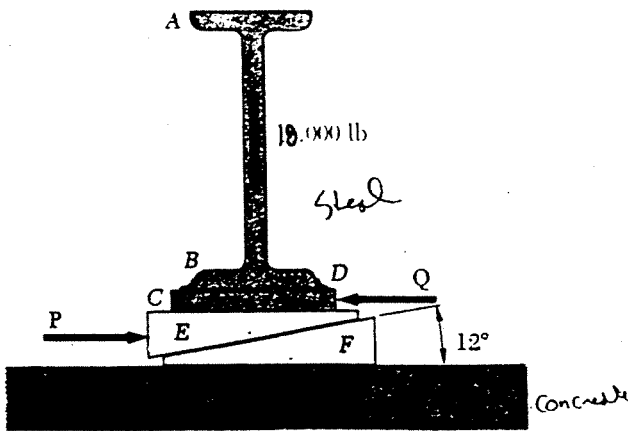
Marks

- 35 (1) For the beam and loading shown, (a) sketch the shearing-force and bending-moment diagrams and indicate all peak values; (b) write equations for the shearing force and bending moment between A and B; (c) locate the point(s) of inflection, if any.

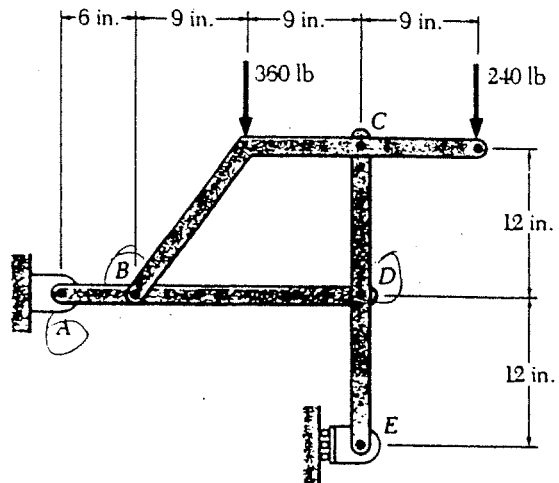


- 30 (2) The elevation of the end of a steel floor beam is adjusted by means of steel wedges E and F. The steel base plate CD has been welded to the lower flange of the beam; the end reaction of the beam is known to be 18,000 lb. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and concrete. If horizontal motion of the base plate CD is prevented by the force Q, determine (a) the force P required to raise the beam and the corresponding force Q and (b) whether the steel wedge F will move.

- 35 (3) For the frame and loading shown, determine the components of all forces acting on member ABD and show the results on a clear sketch.



Question (2)



Question (3)

Concordia University  
 Faculty of Engineering and Computer Science  
 ENGR 242/2 Statics Section V

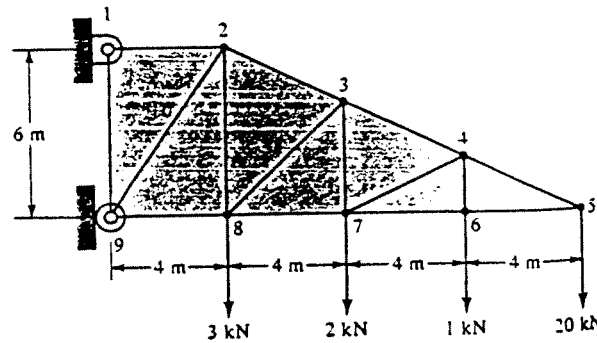
Test #3

Please, attempt all questions, only calculators permitted.

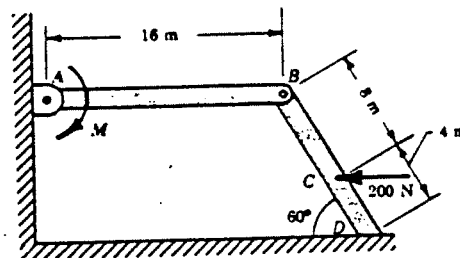
Time: 60 Minutes

Marks

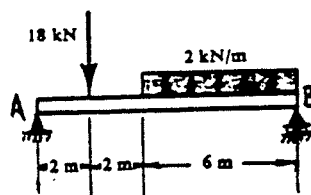
- 30 1) Determine the forces in the members 3-7 and 4-6 for the truss shown.



- 35 2) A horizontal 200-N force is applied to the sloping member BCD, whose bottom rests on a smooth horizontal plane. Its upper end is pinned at B to the horizontal member AB. What couple M must be applied to the member AB to hold the system in equilibrium? What is the magnitude of the pin reaction at B?



- 35- 3) Draw the shear force and bending moment diagrams of the simply supported beam shown.



12

4. Two thin plates shown hatched and a rod are welded together to form the structure shown in Fig. 4. Find the centre of gravity of mass of the structure.

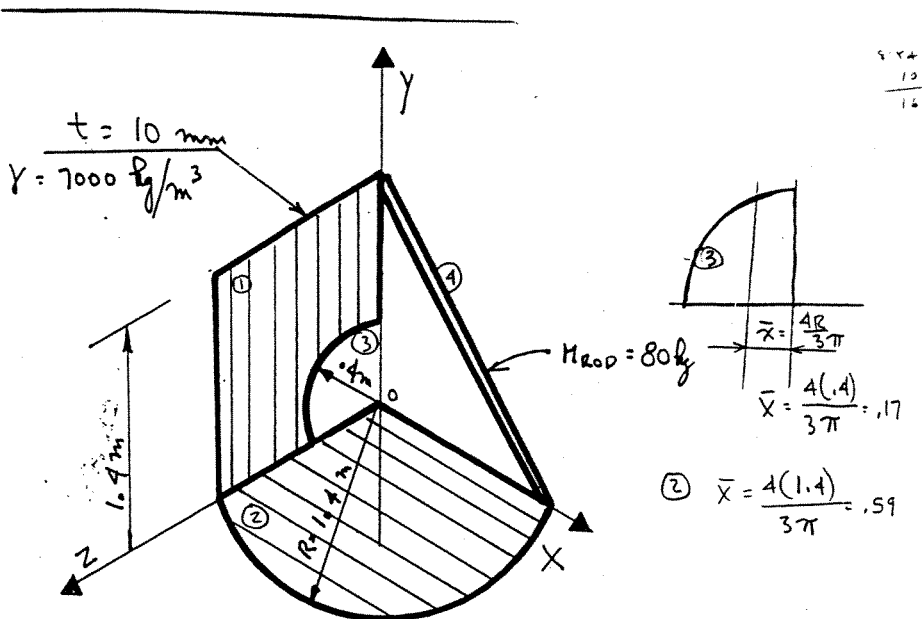


FIG. 4

$$(m_1 + m_3) = (1.4 \times 1.4)(.01)(7000) = 137.2 \text{ kg}$$

$$m_2 = \left[ \frac{\pi(1.4)^2}{4} \right] (.01)(7000) = 107.7$$

$$m_3 = \left[ \frac{\pi(.4)^2}{4} \right] (.01)(7000) = 8.8$$

ITEM	m	$\bar{x}$	$m\bar{x}$	$\bar{y}$	$m\bar{y}$	$\bar{z}$	$m\bar{z}$
1+3	137.2	0	0	.7	96	.7	96
2	107.7	.59	63.5	0	0	.59	63.5
3	8.8	0	0	.17	-1.5	.17	-1.5
4	80	.7	56	.7	56	0	0
$\Sigma$	316.1		119.5		150.5		158

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{119.5}{316.1} = .378 \text{ m}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{150.5}{316.1} = .476 \text{ m}$$

$$\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{158}{316.1} = .4998$$

4



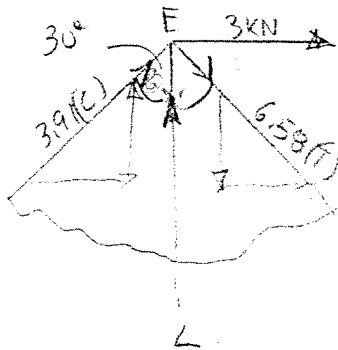
Ac 20 ①

$\sum M_{OC} = 0$  FIND  $F_{EF}$  ASSUME COMP

$$= -F_{EF} \cos 60^\circ (3.5) - 3(3.5) - (2/3.5) - 0.5(2.5) - (4(1.5) + 1.9(3))$$

$$0.75 F_{EF} = 2.60 - 4.59 - 1.25 - 2.4 + 0.7$$

$$F_{EF} = -4.94 / 0.75 = -6.58 \quad \therefore F_{EF} = 6.58 (T)$$



at pt E

$\sum F_y = 0$

FIND  $F_{EL}$  ASSUME TEN

$$-3.9 \cos 30^\circ - 6.58 \cos 30^\circ - F_{EL} = 0$$

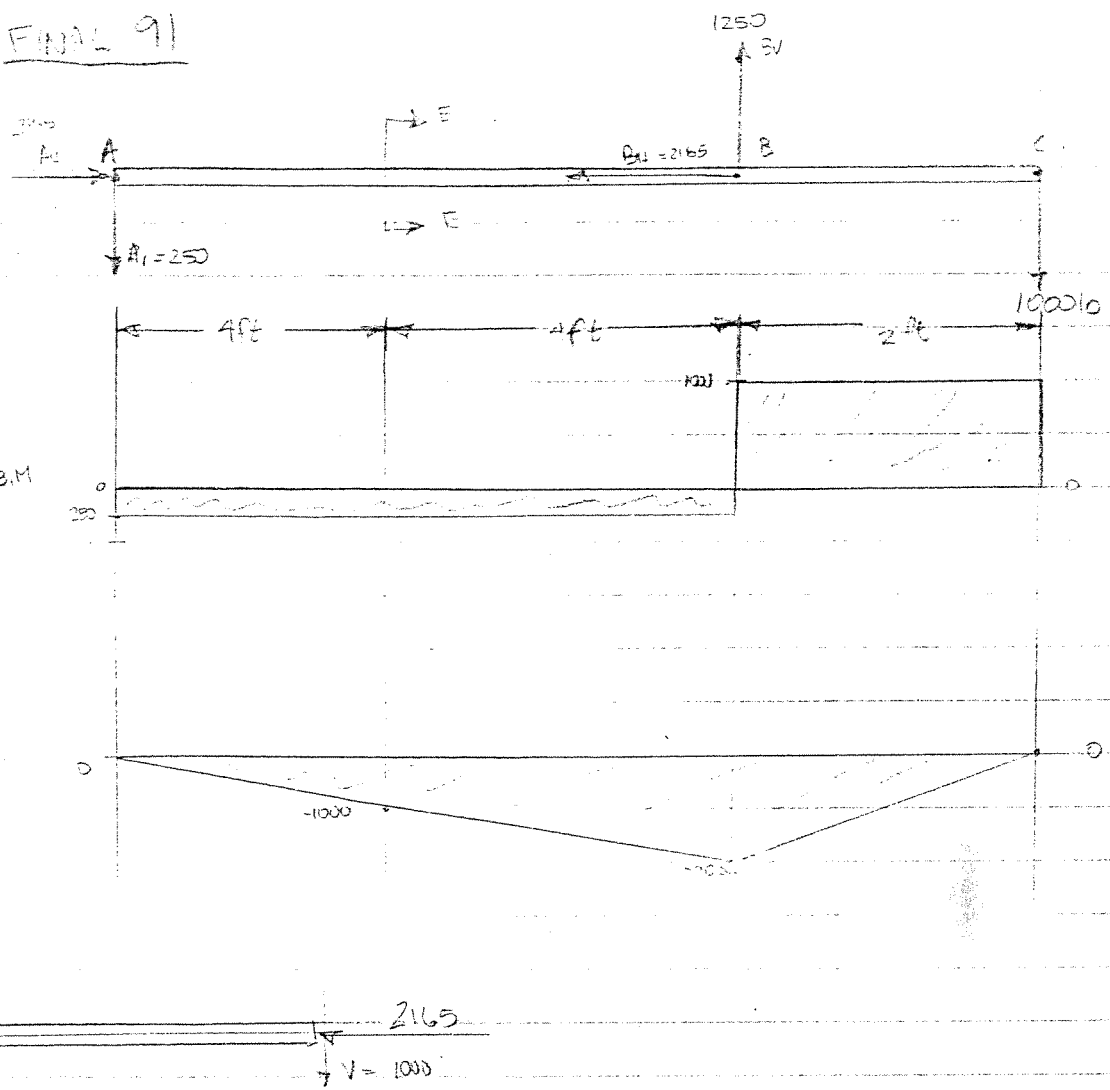
$$3.386 - 5.693 - F_{EL} = 0$$

$$F_{EL} = -2.312$$

$$\therefore F_{EL} = 2.312 (C)$$

FINAL 91

B2)



$$\sum M_{E_A} = 0 \quad 1000(10) - B_1(9) = 0 \quad \sum M_{E_B} = -A_1(3) + 100(2) = 0$$

$$B = 10000/9 = 1250 \quad A_1 = 200/3 = 250$$

tan θ =  $\frac{B}{A}$      $A = \frac{1250}{\tan 30^\circ} = 2165$

MOMENT EQUATION

$$\sum M_{E_A} = -250(4) = -1000$$

$$\sum M_{E_B} = -250(3) = -2000$$

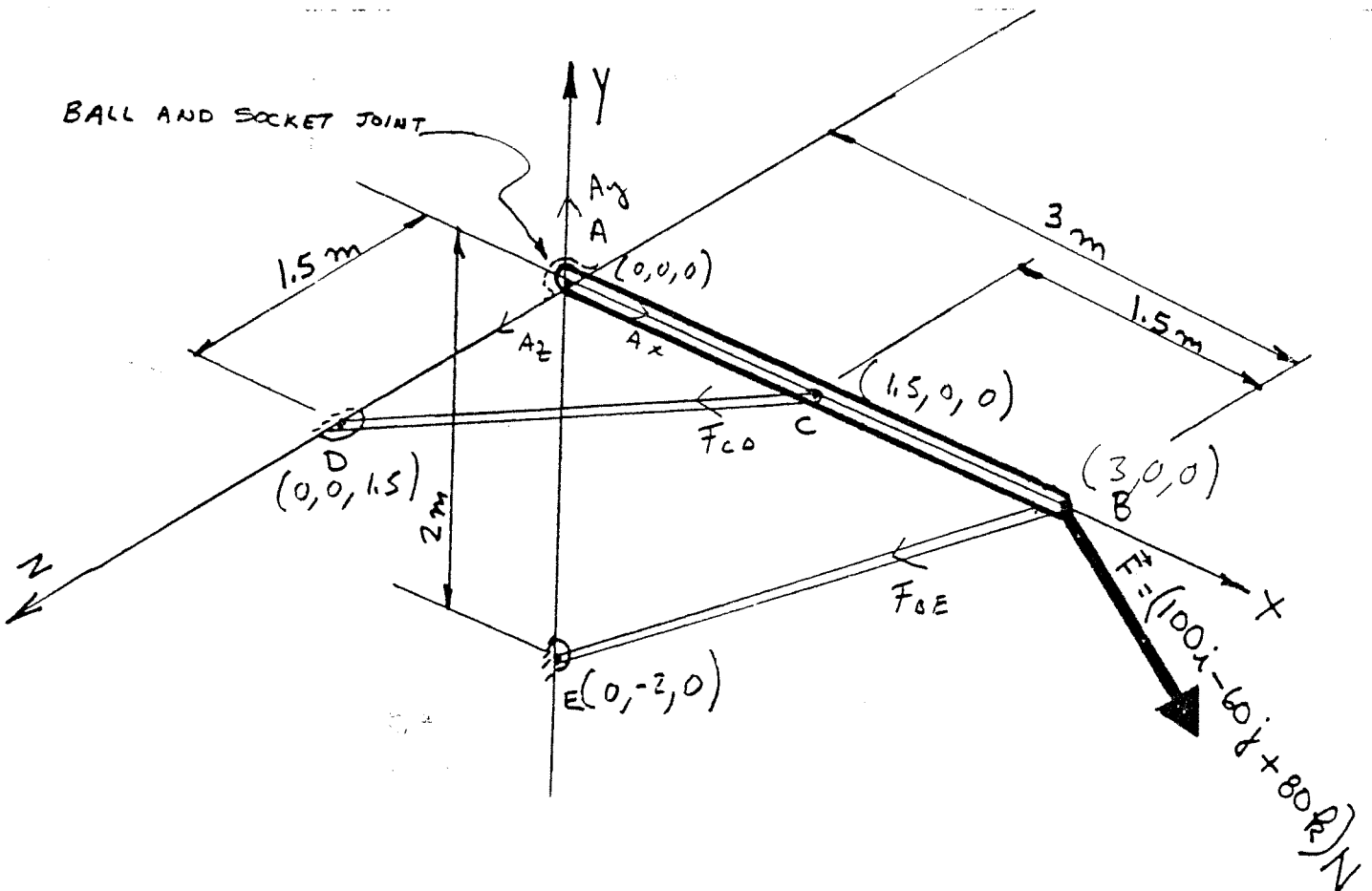
$$\sum M_{E_C} = -250(10) + 1000(2) = -2500 + 2000 = -500$$



COURSE Statics	NUMBER ENGR 242/2	SECTION T,V,X,Y,XX	
EXAMINATION Final	DATE December 16, 1988	TIME 19:00-22:00	# OF PAGES 5
INSTRUCTOR Prof. Fabrikant, Goldman, Stathopoulos, Troitsky			
MATERIALS ALLOWED: <input type="checkbox"/> NO <input checked="" type="checkbox"/> YES (PLEASE SPECIFY) Any calculating device			
SPECIAL INSTRUCTIONS: Do any five questions. All questions carry equal weight.			

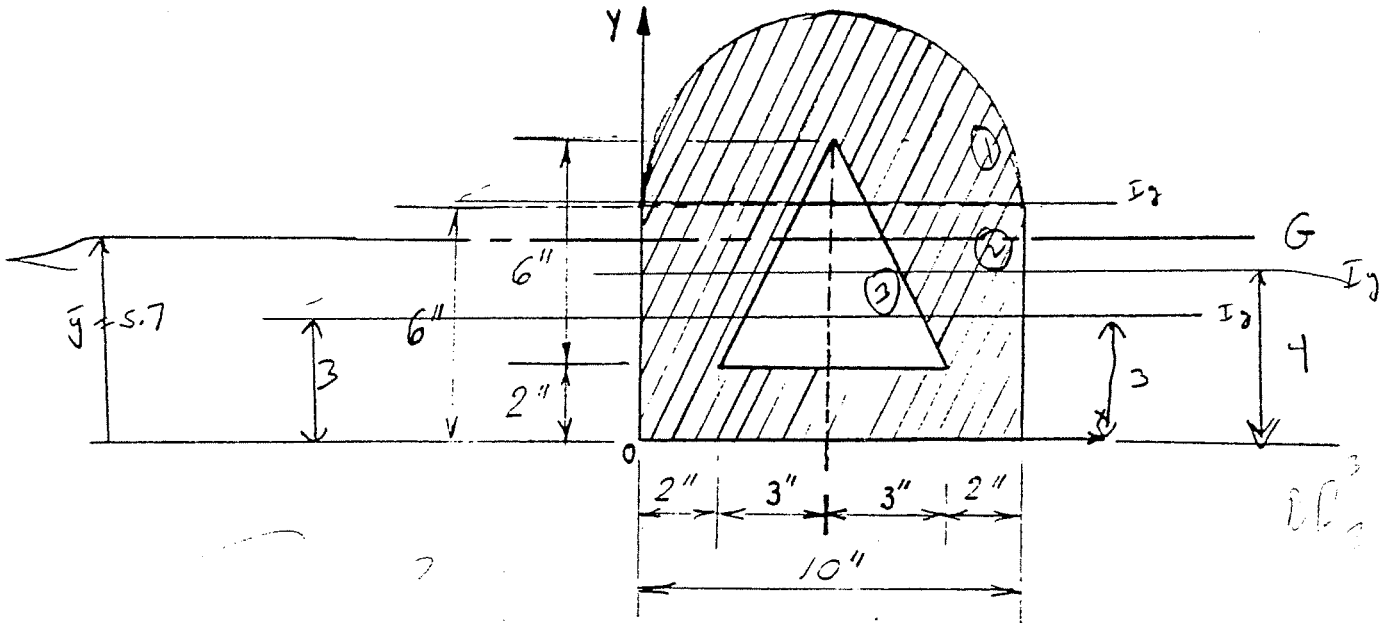
Question 1:

Calculate the forces in members CD and BE and the reactions at A for the structure shown. Note that all members can take tension or compression and that A is a ball and socket joint.



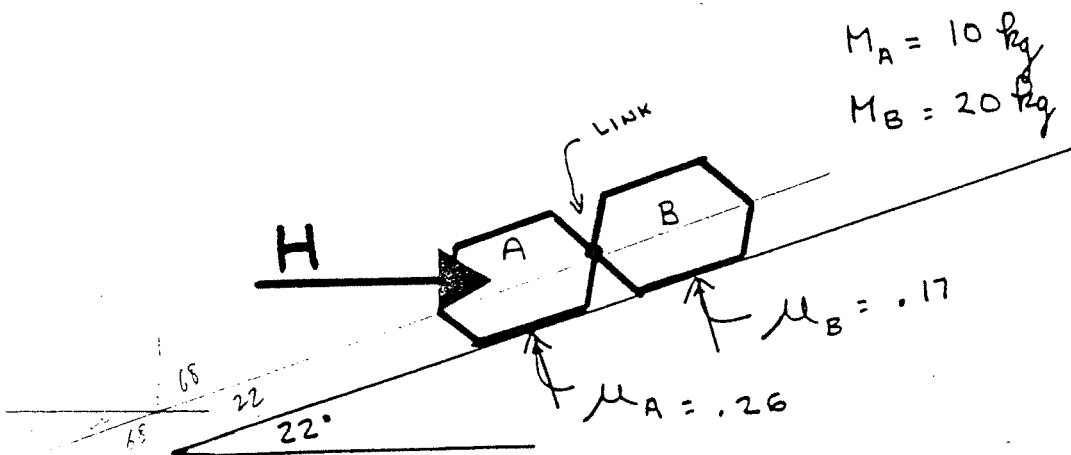
**Question 2:**

Determine the coordinates of the centroid and find the moment of inertia of the shaded area about its horizontal centroidal axis.



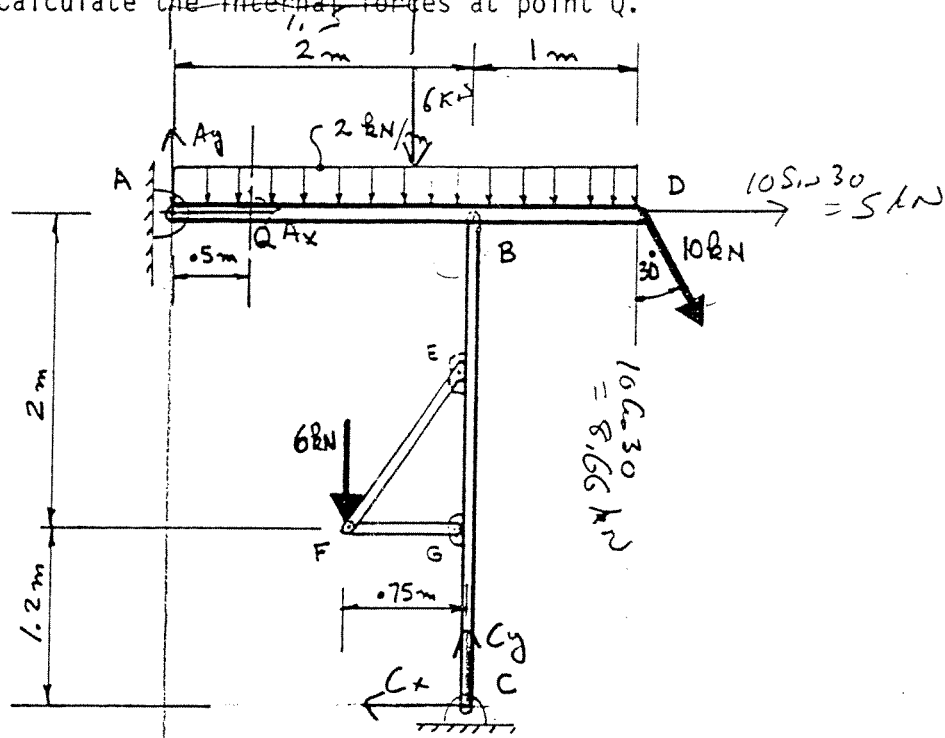
**Question 3:**

Find the horizontal force  $H$  required just to push the two blocks A and B up the plane. Block A has a mass of 10 kg and a coefficient of static friction with the plane equal to 0.26. Block B has a mass of 20 kg and a coefficient of static friction with the plane equal to 0.17. Note that the short link is parallel to the plane and  $H$  is applied to block A only.



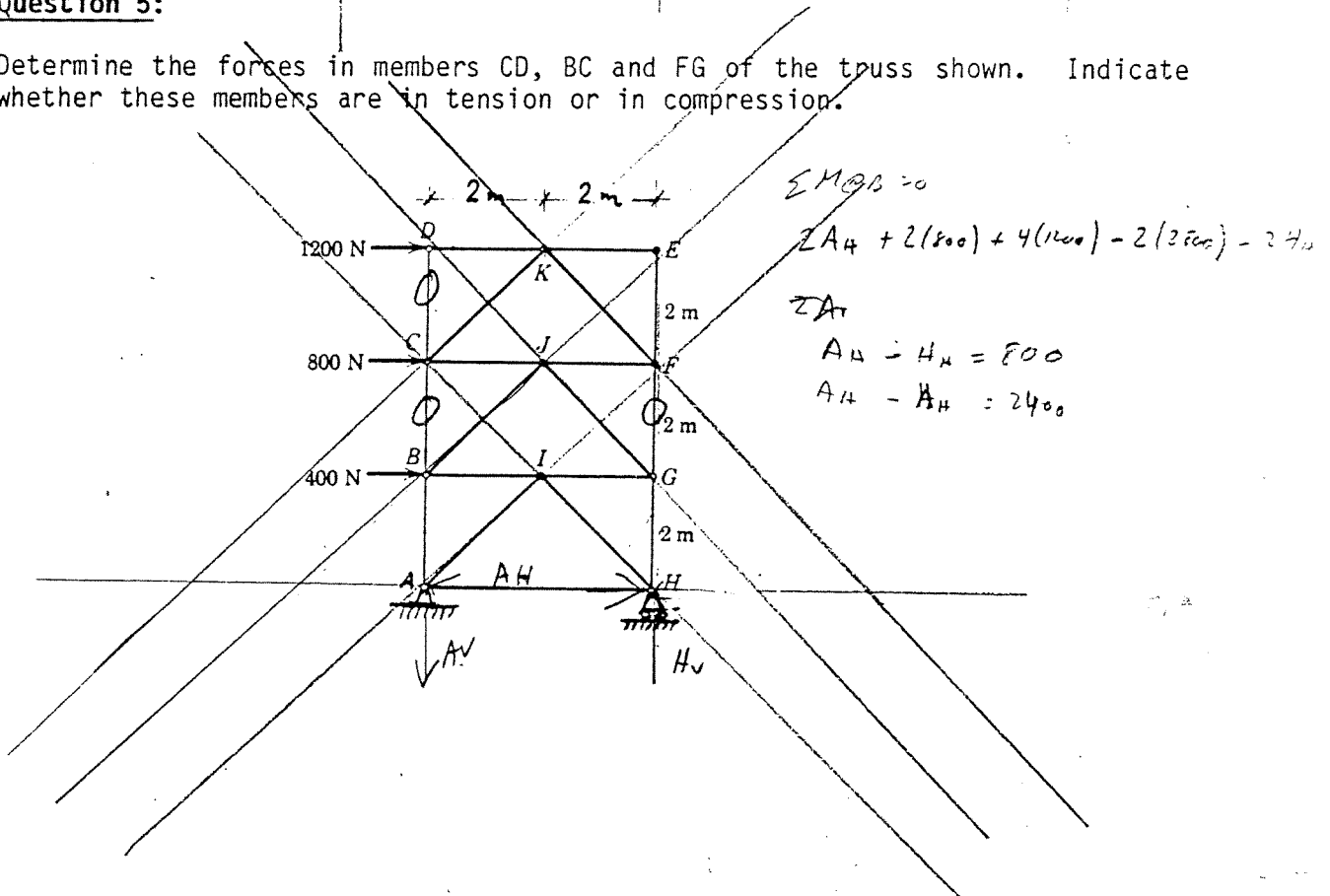
**Question 4:**

For the frame shown determine the reactions at A and C and the forces on the pin at B. Calculate the internal forces at point Q.



**Question 5:**

Determine the forces in members CD, BC and FG of the truss shown. Indicate whether these members are in tension or in compression.



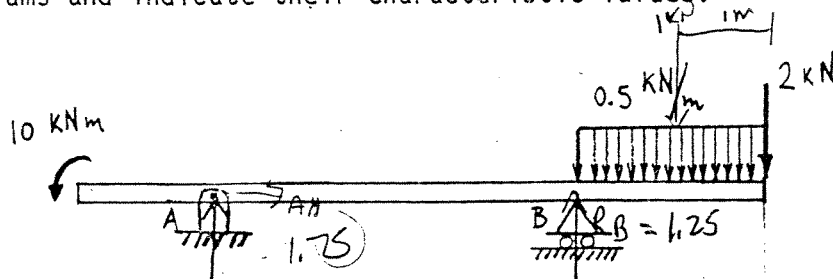
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ENGR 242/2

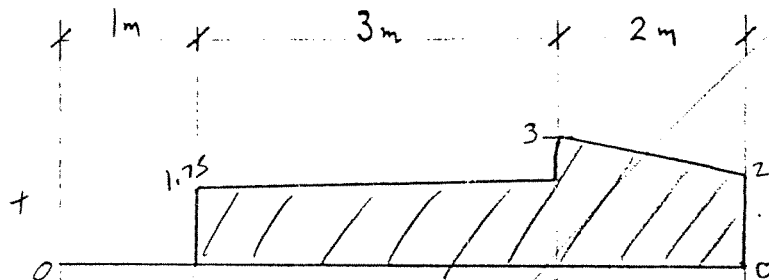
- 4 -

Question 6:

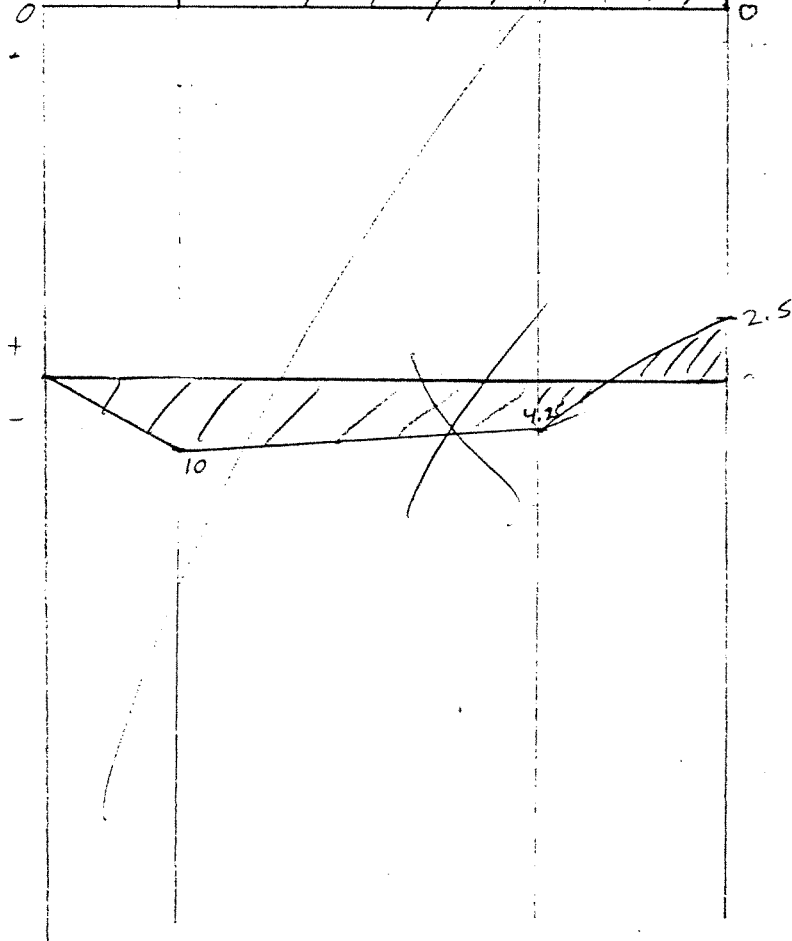
For the beam and loading shown draw the shear force and bending moment diagrams and indicate their characteristic values.



S.F.



B.M.





BASE	NUMBER	SECTION
Statics	ENGR. C242/2	01, T, V, X, Y, XX
TERMINATION	DATE	TIME
Final	December 15, 1984	9:30 - 12:30
INSTRUCTOR		# OF PAGES
s. Douglass, Goldman, Stathopoulos, Troitsky & Zielinski		3
EXAM CONDITIONS ALLOWED:		DIVISION
Any calculating device		Day and Evening

ALL INSTRUCTIONS.

Do any five questions. Only the first five problems presented will be graded.

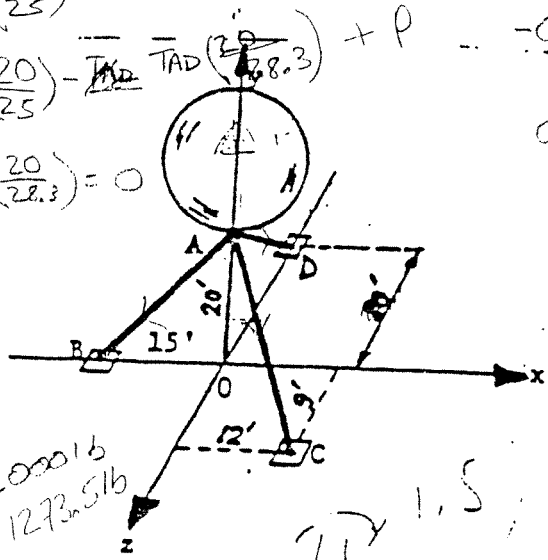
All questions carry equal weight.

1. A balloon is held by three cables as shown. If the tension of cable AC is 2500 lb., what is the total lift force provided by the balloon?

$$\sum M_x = 0 \Rightarrow T_{AC} \left(\frac{12}{25}\right) - T_{AB} \left(\frac{15}{25}\right) = 0$$

$$\sum M_y = 0 \Rightarrow -T_{AC} \left(\frac{20}{25}\right) - T_{AB} \left(\frac{20}{25}\right) - T_{AD} \left(\frac{20}{28.3}\right) + P = 0$$

$$\sum M_z = 0 \Rightarrow T_{AC} \left(\frac{9}{25}\right) - T_{AD} \left(\frac{20}{28.3}\right) = 0$$



$$0.48 T_{AC} - 0.6 T_{AB} = 0$$

$$-0.8 T_{AC} - 0.8 T_{AB} - 0.707 T_{AD} = P$$

$$0.36 T_{AC} - 0.707 T_{AD} = 0$$

$T_{AC} = 2500 \text{ lb}$

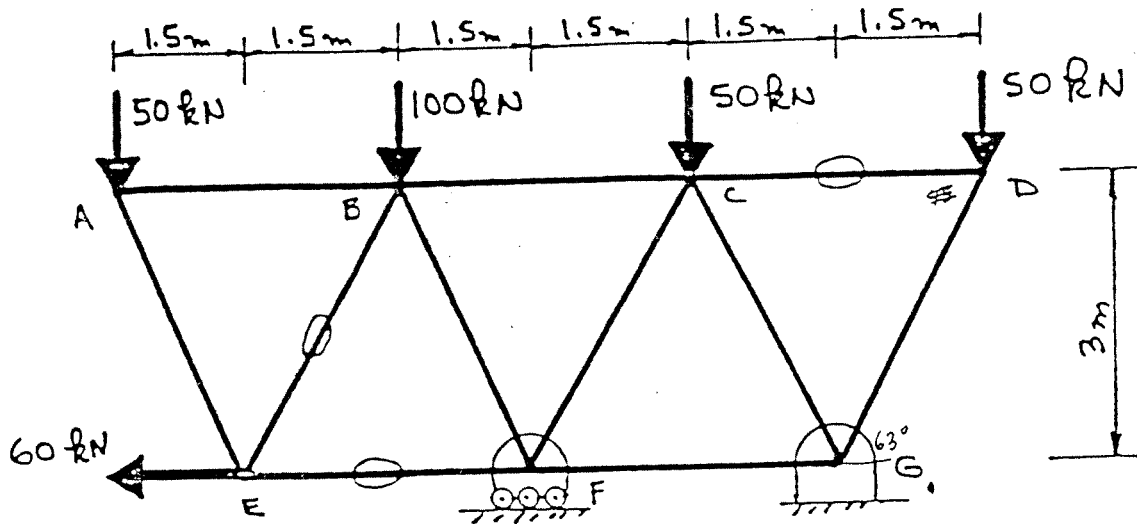
$T_{AB} = 2000 \text{ lb}$   
 $T_{AD} = 1273.5 \text{ lb}$

$AC = 25' = \frac{2500 \text{ lb} \times 0.8}{2500 \text{ lb}} = 2000 \text{ lb}$   
 $AB = 25' = \frac{2500 \text{ lb} \times 0.8}{2500 \text{ lb}} = 2000 \text{ lb}$   
 $AD = 28.3' = \frac{2500 \text{ lb} \times 0.707}{2500 \text{ lb}} = 1273.5 \text{ lb}$

$2000 + 1600 = 3600$   
 $\uparrow$   
 $14500 \text{ lb}$

$Ans = 6000 \text{ lb}$

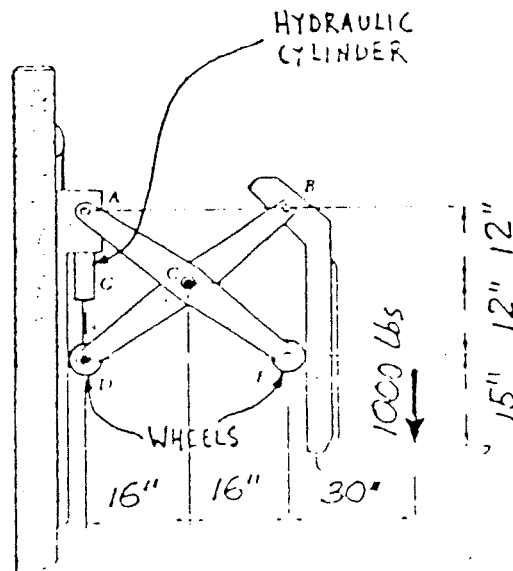
2. For the truss shown calculate the forces in members CD, BE, and EF, marked thus O. Indicate whether these members are in tension or compression.



ANS :  
 CD = 25 kN ✓ Tension  
 BE = 55 kN ✓ Tension  
 EF = 10 kN ✓ Tension

3. Two identical linkage-and-hydraulic-cylinder systems control the position of the forks for a fork-lift truck; only one system is shown. Knowing that the load supported by the one system shown is 1000 lbs, determine
- The force exerted by the hydraulic cylinder on point D,
  - The components of the force exerted on member ACE at pin C.

?



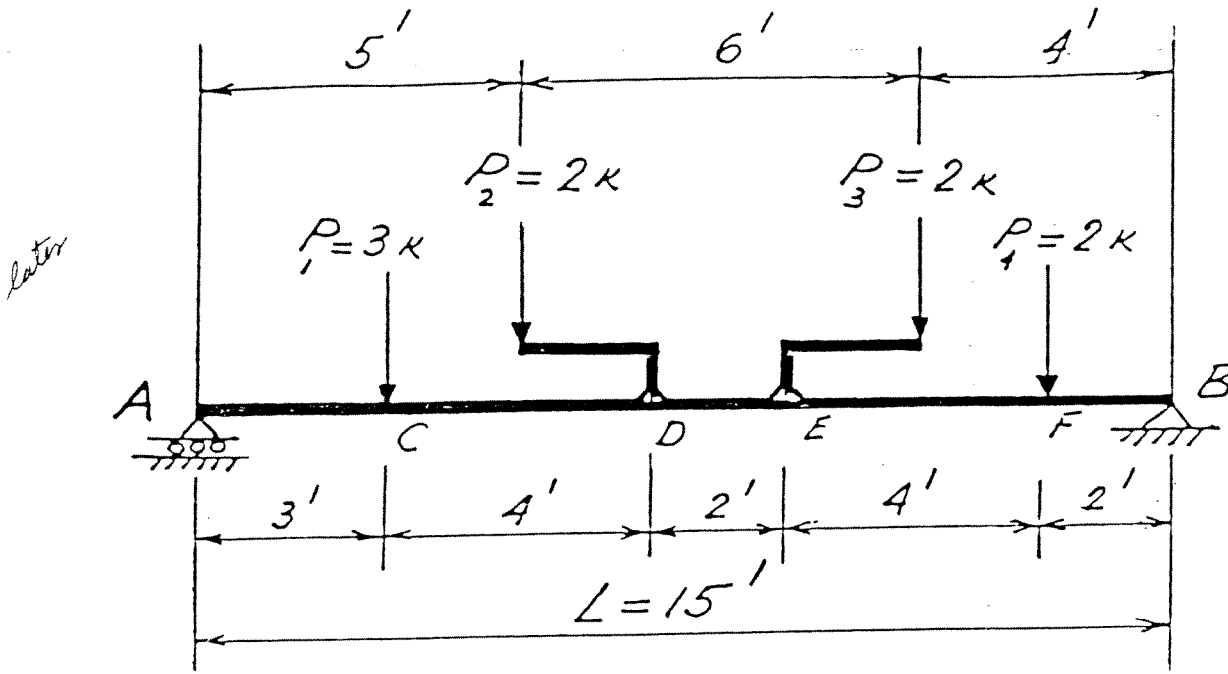
A, B and C are simple pins

$$D = A_x = 2583.3 \text{ lb}$$

$$C_x = -3833 \text{ lb}$$

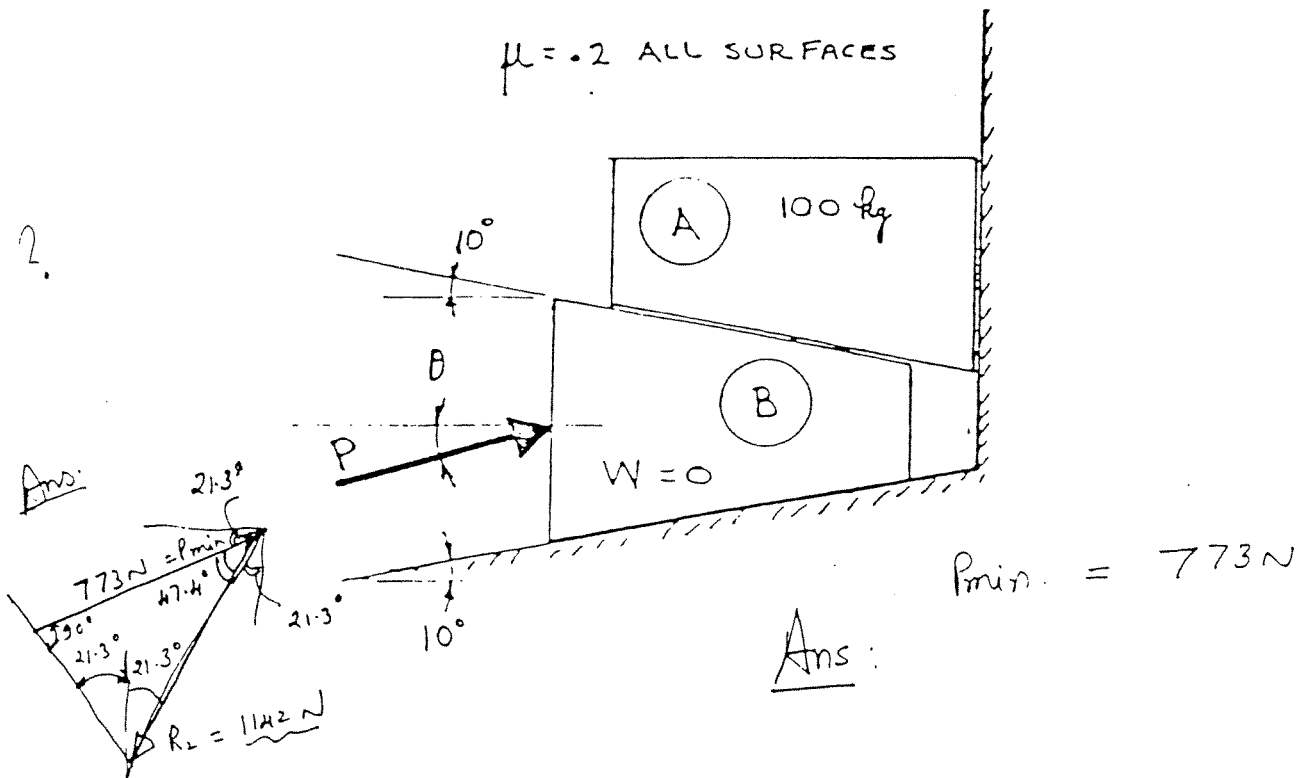
$$C_y = -1000 \text{ lb}$$

4. Draw the shear and bending-moment diagram for the beam AB.



5. For the system shown determine the minimum value of the force  $P$  to cause impending upward motion of block A having a mass of  $100\text{ kg}$ . What angle does the force  $P$  make with the horizontal?

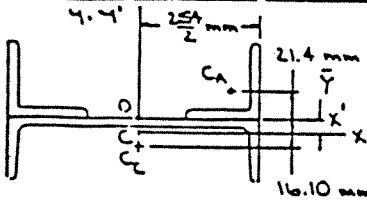
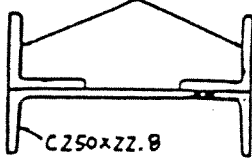
Assume that block B is weightless and that the coefficient of static friction is equal to  $0.2$  for all contact surfaces.



9.53

L76x76x6.4

GIVEN: COMBINED SECTION SHOWN  
FIND:  $\bar{I}_x$  AND  $\bar{I}_y$



ANGLE:  $A = 929 \text{ mm}^2$   
 $\bar{I}_x = \bar{I}_y = 0.516 \times 10^6 \text{ mm}^4$   
CHANNEL:  $A = 2897 \text{ mm}^2$   
 $\bar{I}_x = 0.949 \times 10^6 \text{ mm}^4$   
 $\bar{I}_y = 28.1 \times 10^6 \text{ mm}^4$

FIRST LOCATE CENTROID C OF THE SECTION

	A, mm <sup>2</sup>	$\bar{y}$ , mm	$\bar{y}A$ , mm <sup>3</sup>
ANGLE	2(929) = 1858	21.4	39761.2
CHANNEL	2897	-16.10	-46641.7
$\Sigma$	4755		-6880.5

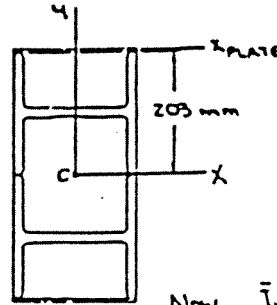
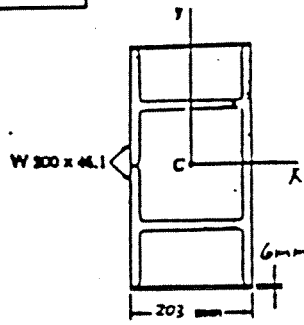
THEN..  $\bar{Y} \Sigma A = \Sigma \bar{y}A$ :  $\bar{Y}(4755 \text{ mm}^2) = -6880.5 \text{ mm}^3$   
OR  $\bar{Y} = -1.447 \text{ mm}$

NOW..  $\bar{I}_x = 2(I_x)_L + (I_x)_C$   
WHERE  $(I_x)_L = \bar{I}_x + Ad^2 = 0.516 \times 10^6 \text{ mm}^4 + (929 \text{ mm}^2)(21.4 + 1.447 \text{ mm})^2$   
 $= 1.00092 \times 10^6 \text{ mm}^4$   
 $(I_x)_C = \bar{I}_x + Ad^2 = 0.949 \times 10^6 \text{ mm}^4 + (2897 \text{ mm}^2)(16.10 - 1.447 \text{ mm})^2$   
 $= 1.57102 \times 10^6 \text{ mm}^4$   
THEN..  $\bar{I}_x = [2(1.00092) + 1.57102] \times 10^6 \text{ mm}^4$   
OR  $\bar{I}_x = 3.57 \times 10^6 \text{ mm}^4$

ALSO..  $\bar{I}_y = 2(I_y)_L + (I_y)_C$   
WHERE  $(I_y)_L = \bar{I}_y + Ad^2 = 0.516 \times 10^6 \text{ mm}^4 + (929 \text{ mm}^2)(127 - 21.4 \text{ mm})^2$   
 $= 10.8756 \times 10^6 \text{ mm}^4$   
 $(I_y)_C = \bar{I}_y$   
THEN..  $\bar{I}_y = [2(10.8756) + 28.1] \times 10^6 \text{ mm}^4$   
OR  $\bar{I}_y = 49.9 \times 10^6 \text{ mm}^4$

9.50

GIVEN: COMBINED SECTION SHOWN  
FIND:  $\bar{I}_x$  AND  $\bar{I}_y$ ;  $\bar{k}_x$  AND  $\bar{k}_y$



W SECTION:  $A = 5890 \text{ mm}^2$   
 $\bar{I}_x = 15.44 \times 10^6 \text{ mm}^4$   
 $\bar{I}_y = 45.8 \times 10^6 \text{ mm}^4$

NOTE:  $A_{\text{TOTAL}} = 2A_W + 2A_{\text{PLATE}}$   
 $= 2(5890 \text{ mm}^2) + 2(203 \text{ mm} \times 6 \text{ mm})$   
 $= 14216 \text{ mm}^2$

NOW..  $\bar{I}_x = 2(I_x)_W + 2(I_x)_{\text{PLATE}}$   
WHERE  $(I_x)_W = \bar{I}_x + Ad^2 = 15.44 \times 10^6 \text{ mm}^4 + (5890 \text{ mm}^2)(142.5 \text{ mm})^2$   
 $= 76.1203 \times 10^6 \text{ mm}^4$   
 $(I_x)_{\text{PLATE}} = \bar{I}_x + Ad^2$   
 $= \frac{1}{12}(203 \text{ mm} \times 6 \text{ mm})^3 + (1218 \text{ mm}^2)(203 + 3 \text{ mm})^2$   
 $= 51.6907 \times 10^6 \text{ mm}^4$   
THEN..  $\bar{I}_x = [2(76.1203) + 2(51.6907)] \times 10^6 \text{ mm}^4$   
 $= 255.622 \times 10^6 \text{ mm}^4$   
OR  $\bar{I}_x = 256 \times 10^6 \text{ mm}^4$

AND  $\bar{k}_x = \frac{\bar{I}_x}{A_{\text{TOTAL}}} = \frac{255.622 \times 10^6 \text{ mm}^4}{14216 \text{ mm}^2}$   
OR  $\bar{k}_x = 184.1 \text{ mm}$

ALSO..  $\bar{I}_y = 2(I_y)_W + 2(I_y)_{\text{PLATE}}$   
WHERE  $(I_y)_W = \bar{I}_y$   
 $(I_y)_{\text{PLATE}} = \frac{1}{12}(6 \text{ mm} \times 203 \text{ mm})^3 = 4.1827 \times 10^6 \text{ mm}^4$   
THEN..  $\bar{I}_y = [2(45.8) + 2(4.1827)] \times 10^6 \text{ mm}^4$   
 $= 99.9654 \times 10^6 \text{ mm}^4$   
OR  $\bar{I}_y = 100.0 \times 10^6 \text{ mm}^4$

AND  $\bar{k}_y = \frac{\bar{I}_y}{A_{\text{TOTAL}}} = \frac{99.9654 \times 10^6 \text{ mm}^4}{14216 \text{ mm}^2}$   
OR  $\bar{k}_y = 83.9 \text{ mm}$



COURSE: STATICS  
 INSTRUCTOR: ENGR 242/2  
 SECTION: T, V, X, XX, III  
 EXAMINATION: Final  
 DATE: December 15, 1988  
 TIME: 19:00 - 22:00  
 PERIODS: 3

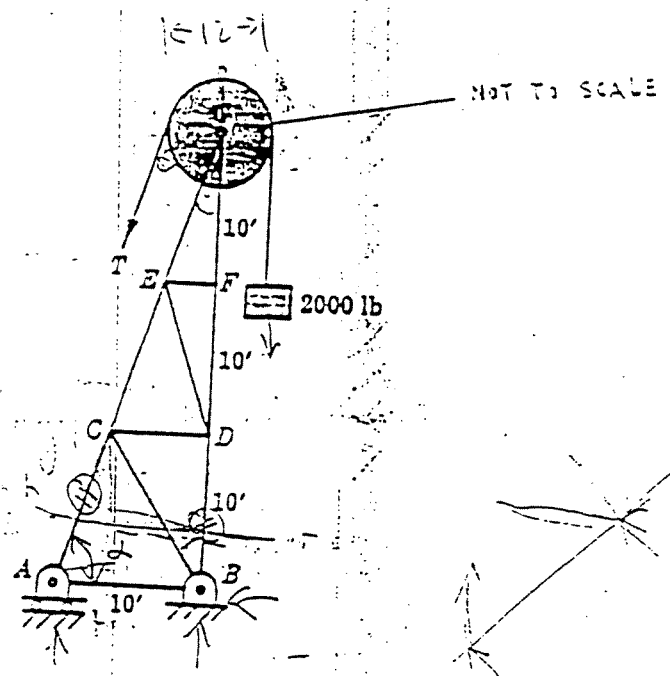
INSTRUCTORS: Profs. Bédard, Fabrikant, Goldman, Stathopoulos, Zaheeruddin  
 MATERIALS ALLOWED:  NO  YES (PLEASE SPECIFY)

Any calculating device

SPECIAL INSTRUCTIONS  
 Do any five questions:  
 All questions carry equal weight.

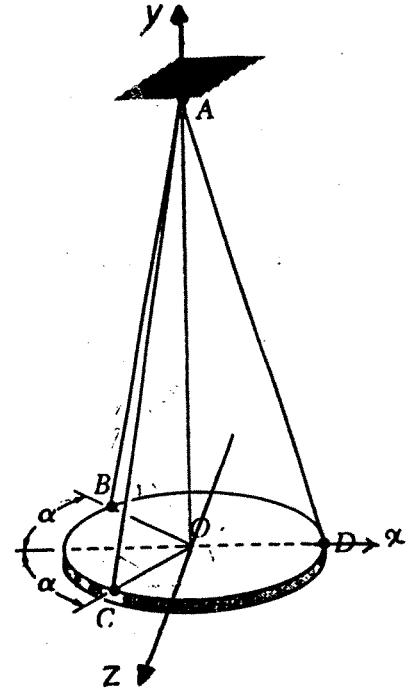
Question 1:

The steel cable is hoisting a 2000 lb weight. The diameter of the pulley is 12 inches. Determine the forces in AC and BD assuming the cable is parallel to the sides of the truss as shown.



$\alpha = 71.56$

1-(a) A 10 kg circular plate of 200-mm radius is supported as shown by three wires of equal length  $L$ . Knowing that  $\alpha = 65^\circ$ , determine the smallest permissible value of the length  $L$  if tension not to exceed 45 N in any of the wire. (30 marks).



$$\sum F_z = 0 \Rightarrow -T_{AC} \cos \theta \sin \alpha + T_{AB} \cos \theta \sin \alpha = 0$$

$$\Rightarrow T_{AC} = T_{AB} = T_1$$

$$\sum F_x = 0 \Rightarrow T_{AC} \cos \theta \cos \alpha + T_{AB} \cos \theta \cos \alpha - T_{AD} \cos \theta = 0$$

$$\cos \alpha (T_{AC} + T_{AB}) - T_{AD} = 0, (T_{AD} = T_2) \Rightarrow$$

$$T_1 = 1.183 T_2 \Rightarrow T_1 > T_2 \Rightarrow T_1 = 45 \text{ N} \Rightarrow T_2 = 38 \text{ N}$$

$$\sum F_y = 0 \Rightarrow T_1 \sin \theta + T_1 \sin \theta + T_2 \sin \theta - W = 0 \Rightarrow$$

$$\sin \theta (2T_1 + T_2) = W \Rightarrow \sin \theta = \frac{W}{2T_1 + T_2} = \frac{10 \times 9.81}{2 \times 45 + 38} = 0.766$$

$$\Rightarrow \theta = 50^\circ$$

$$\cos \theta = \frac{r}{L} \Rightarrow L = \frac{r}{\cos \theta} = \frac{0.2}{0.643} \Rightarrow \underline{\underline{L = 0.311 \text{ m}}}$$

1-(b) A container is supported by three cables and a single force P exerted at A as shown.

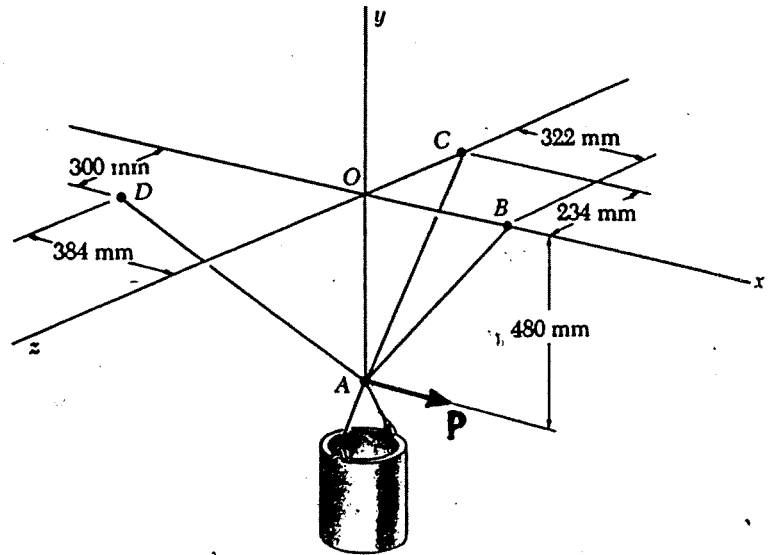
a)- Determine the angle between cables AD and AC. (10 marks)

b)- Determine the weight W of the container, knowing that the tension in cable

AB is zero ( $T_{AB}=0$ ) and  $P=250$  N. (20 marks)

$$A \begin{vmatrix} 0 \\ -0.48 \text{ m} \\ 0 \end{vmatrix} B \begin{vmatrix} 0 \\ 0 \\ -0.234 \text{ m} \end{vmatrix} C \begin{vmatrix} 0 \\ 0 \\ -0.234 \text{ m} \end{vmatrix}$$

$$D \begin{vmatrix} -0.384 \text{ m} \\ 0 \\ 0.3 \text{ m} \end{vmatrix}$$



a)

$$\vec{AD} = -0.384\vec{i} + 0.48\vec{j} + 0.3\vec{k}$$

$$\vec{AC} = 0.48\vec{j} - 0.234\vec{k}$$

$$\cos \theta = \frac{\vec{AD} \cdot \vec{AC}}{|\vec{AD}| \cdot |\vec{AC}|} = \frac{x_{AD} \cdot x_{AC} + y_{AD} \cdot y_{AC} + z_{AD} \cdot z_{AC}}{|\vec{AD}| \cdot |\vec{AC}|} = \frac{0 + 0.2304 + 0.069}{0.684 \times 0.534} = 0.439 \Rightarrow \theta = 64^\circ$$

b)

$$\vec{T}_{AD} = T_{AD} \cdot \frac{\vec{AD}}{AD} = T_{AD} (-0.561\vec{i} + 0.702\vec{j} + 0.439\vec{k})$$

$$\vec{T}_{AC} = T_{AC} \cdot \frac{\vec{AC}}{AC} = T_{AC} (0.899\vec{j} - 0.438\vec{k}), \vec{P} = 250\vec{i}, \vec{W} = -W\vec{j}$$

$$\vec{R} = \sum F = 0 \Rightarrow \vec{R} = \vec{T}_{AD} + \vec{T}_{AC} + \vec{P} + \vec{W} = 0$$

$$R = \underbrace{(-0.561T_{AD} + 250)}_{R_x} \vec{i} + \underbrace{(0.702T_{AD} + 0.899T_{AC} - W)}_{R_y} \vec{j} + \underbrace{(0.439T_{AD} - 0.438T_{AC})}_{R_z} \vec{k} = 0$$

$$R_x = 0 \rightarrow -0.561T_{AD} + 250 = 0 \Rightarrow T_{AD} = 446 \text{ N}$$

$$R_y = 0 \rightarrow 0.702T_{AD} + 0.899T_{AC} - W = 0$$

$$\Rightarrow W = 715 \text{ N}$$

$$R_z = 0 \rightarrow 0.439T_{AD} - 0.438T_{AC} = 0 \Rightarrow T_{AC} = 447 \text{ N}$$

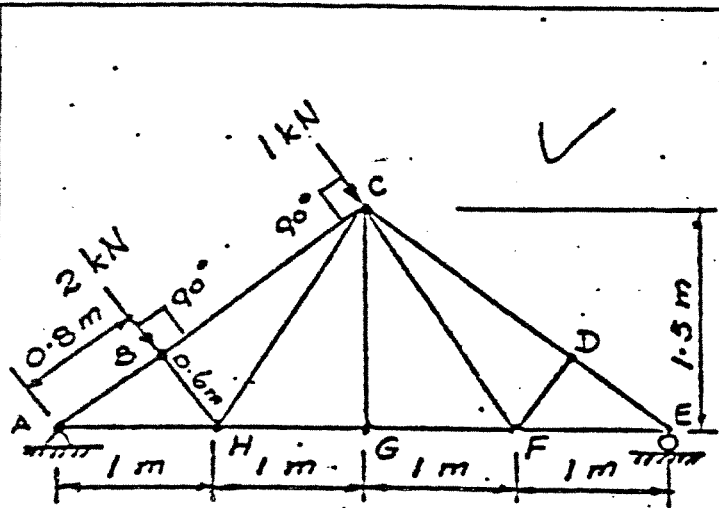


Fig. Prob. 1.

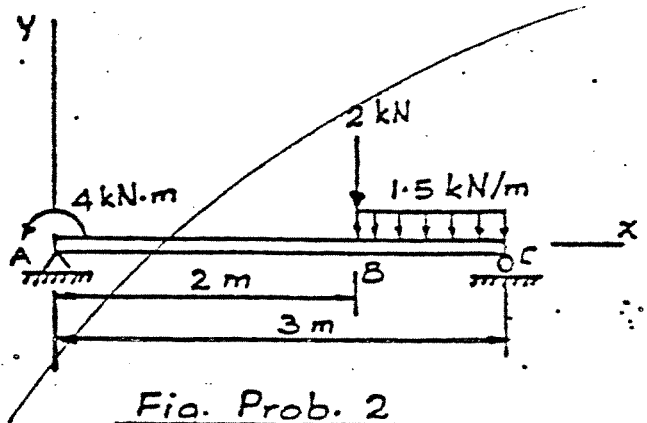


Fig. Prob. 2

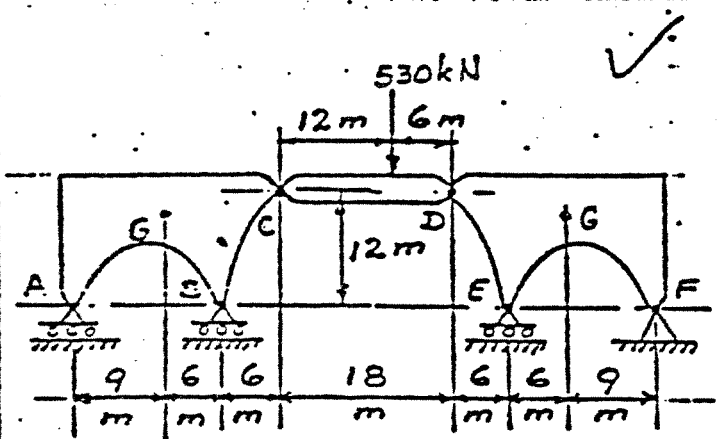


Fig. Prob. 3

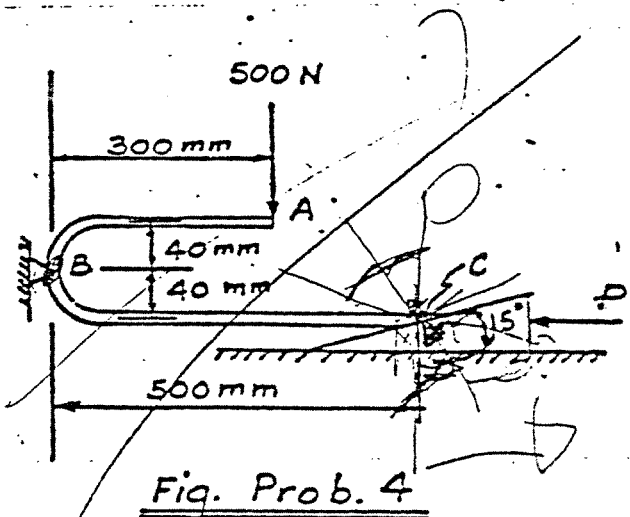


Fig. Prob. 4

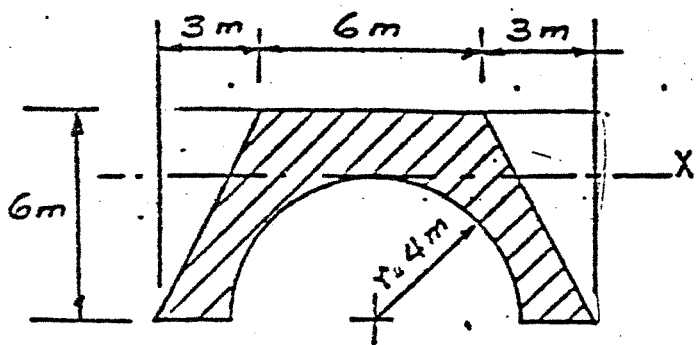


Fig. Prob. 5

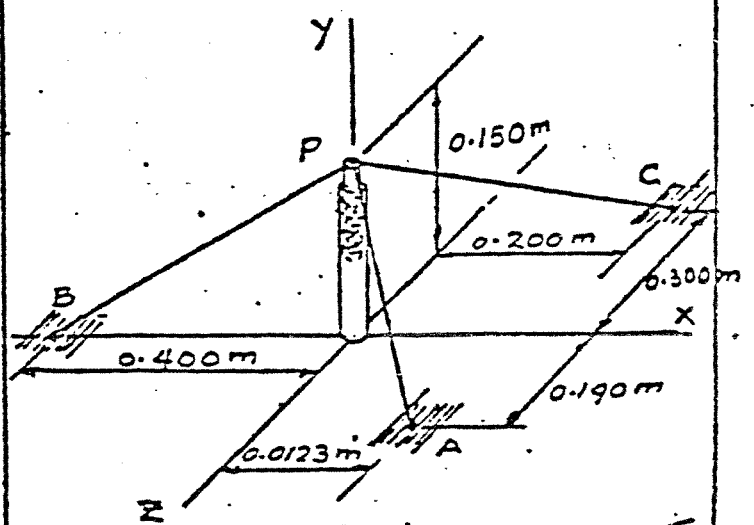


Fig. Prob. 6

Name: \_\_\_\_\_ Given Name: \_\_\_\_\_ I.D.: \_\_\_\_\_

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# STATICS

ENGR 242/2-T (1992)

## TEST # 1

Professor: Dr. Ashraf M. Ghaly

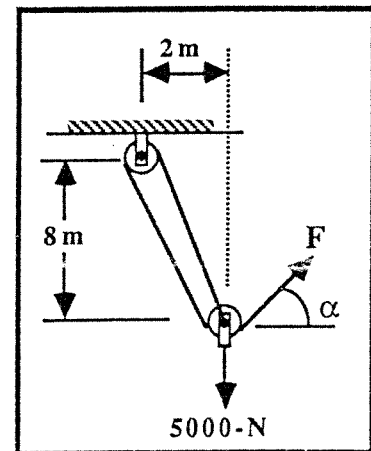
Date: October 2, 1992

Time: 60 minutes

Solve Problems (1 OR 2), 3, and 4

### Problem 1 (4 marks) [8 minutes]

A weight of 5000-N is to be supported by the rope-and-pulley arrangement as shown. Determine the magnitude and direction of the force  $F$  which should be exerted on the free end of the rope.



1996

Amy Moselhi  
Oct - 1 - 96

CONCORDIA UNIVERSITY  
Faculty of Engineering & Computer Science

①

ENGR 242/2 STATICS, Section V, Fall 1996

Test # 1

Attempt all questions; only calculators permitted.

Time: 75 minutes

Marks

- 35 (1) Struts  $AB$  and  $AC$  in Fig. 1 can transmit only axial tensile or compression forces. Determine the forces in struts  $AB$  and  $AC$  and the tension in cable  $AD$  when force  $F = 1250$  kN.

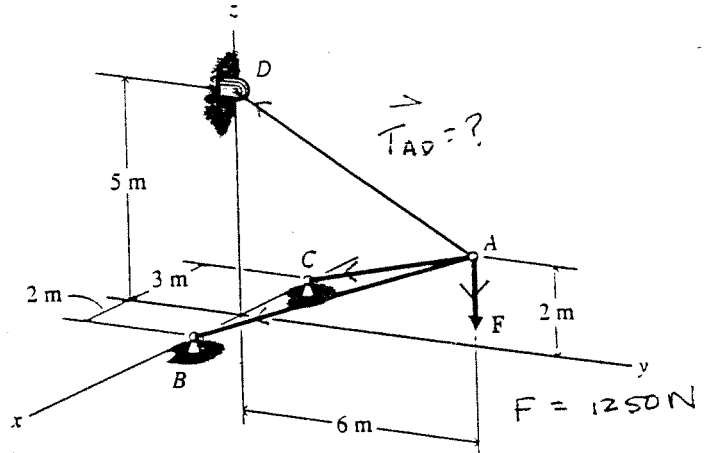


Fig. 1

- 35 (2) The tension in cable  $AB$  is 2 kN in Fig. 2. (a) What are the magnitude and direction of the moment about point  $D$  due to the force exerted by the cable? (b) What is the moment about the shaft  $CD$  due to the same force? Draw a sketch to indicate the sense of the moment about the shaft.

- 30 (3) A bracket is subjected to the force-couple system shown in Fig. 3. Determine (a) the magnitude and direction of the single equivalent force; (b) the perpendicular distance from support  $O$  to the line of action of the equivalent force.

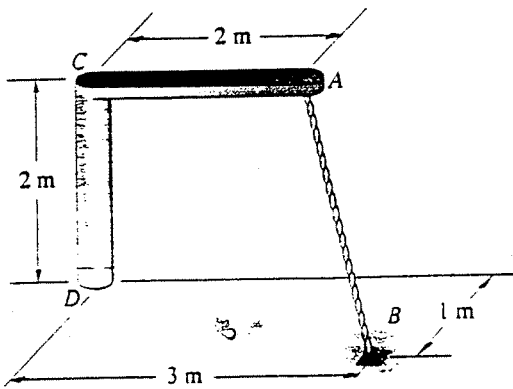


Fig. 2

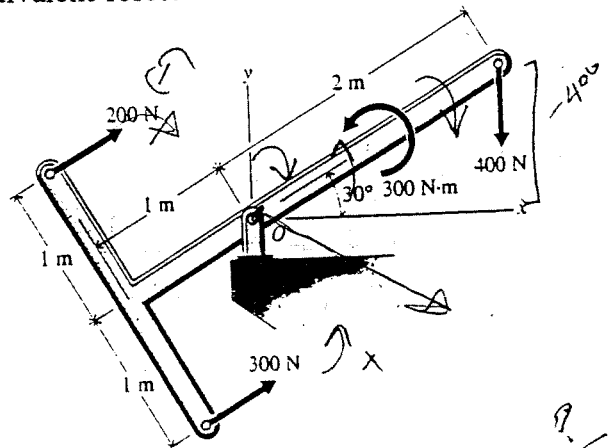
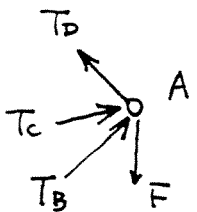


Fig. 3

SOA  
 $\sin 30 = \frac{1}{2}$   
-CAM  
TOA

Test # 1, ENGR 242/V, 1996

1.



$$\vec{AD} = 0\vec{i} - 6\vec{j} + 3\vec{k} \quad \therefore AD = \sqrt{45}$$

$$\vec{BA} = -2\vec{i} + 6\vec{j} + 2\vec{k} \quad BA = \sqrt{44}$$

$$\vec{CA} = 3\vec{i} + 6\vec{j} + 2\vec{k} \quad CA = 7$$

$$\sum \vec{F} = 0$$

$$x: \quad -\frac{2}{\sqrt{44}} T_B + \frac{3}{7} T_C = 0$$

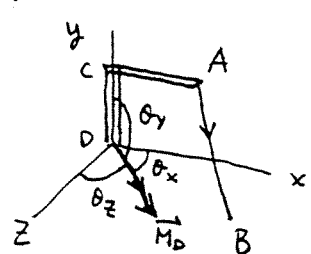
$$y: \quad -\frac{6}{\sqrt{45}} T_D + \frac{6}{\sqrt{44}} T_B + \frac{6}{7} T_C = 0$$

$$z: \quad \frac{3}{\sqrt{45}} T_D + \frac{2}{\sqrt{44}} T_B + \frac{2}{7} T_C = 1250$$

$$\therefore T_D = 1677 \text{ kN}, \quad T_B = 995 \text{ kN}, \quad T_C = 700 \text{ kN} \quad \Delta$$

2.

(a)



$$\vec{r} = \vec{DB} = 3\vec{i} + \vec{k}$$

$$\vec{F} = 2 \vec{AB} / AB = 2(\vec{i} - 2\vec{j} + \vec{k}) / \sqrt{1^2 + (-2)^2 + 1^2}$$

$$= \frac{2}{\sqrt{6}} (\vec{i} - 2\vec{j} + \vec{k})$$

$$\vec{M}_D = \vec{r} \times \vec{F} = \frac{2}{\sqrt{6}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

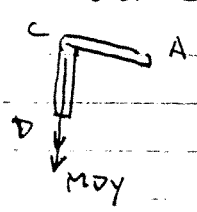
$$= \frac{4}{\sqrt{6}} (\vec{i} - \vec{j} - 3\vec{k}) \quad (\text{kN}\cdot\text{m})$$

Magnitude:  $M_D = \frac{4}{\sqrt{6}} \sqrt{1^2 + (-1)^2 + (-3)^2} = \frac{4}{\sqrt{6}} \sqrt{11} = 5.42 \text{ (kN}\cdot\text{m)} \quad \Delta$

Direction:  $\theta_x = \cos^{-1}(\frac{1}{\sqrt{11}}) = 72^\circ$ ;  $\theta_y = \cos^{-1}(\frac{-1}{\sqrt{11}}) = 108^\circ$ ;  $\theta_z = \cos^{-1}(\frac{-3}{\sqrt{11}}) = 155^\circ \quad \Delta$

(b) about CD:  $M_{Dy} = -\frac{4}{\sqrt{6}} = -1.63 \text{ (kN}\cdot\text{m)} \quad \Delta$

Sense:  $\downarrow \quad \Delta$

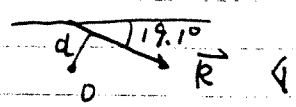


3. (a)

$$R_x = (200 + 300) \cos 30^\circ = 433 \quad \text{N}$$

$$R_y = (200 + 300) \sin 30^\circ - 400 = -150 \quad \text{N}$$

$$\therefore R = \sqrt{R_x^2 + R_y^2} = 458.3 \text{ (N)} \quad \Delta$$

$$\theta = \tan^{-1}(\frac{R_y}{R_x}) = -19.1^\circ$$


(b)  $\vec{M}_D = -300(1) + 200(1) + 400(2 \cos 30^\circ) - 300$

$$= 292.8 \text{ (N}\cdot\text{m)}$$

$$= d R$$

$$\therefore d = 292.8 / 458.3 = 0.64 \text{ (m)} \quad \Delta$$

CONCORDIA UNIVERSITY  
Faculty of Engineering & Computer Science

STATICS

ENGR 242/2-T (1992)

TEST # 3

Professor: Dr. Ashraf M. Ghaly  
Date: November 27, 1992

Time: 100 minutes

**SOLVE ALL PROBLEMS**

Family Name: ~~XXXXXXXXXX~~

Given Name: ~~XXXXXXXXXX~~

I.D.: ~~XXXXXXXXXX~~

Full Mark. 30 20 30 -  
Less Mark  $\cdot 3X_1$   $X_2$   $X_3$   
Wght 1 .33 .33 .42  
Wght 2 .33 .25 .42

$$\frac{25}{30}$$

QFT 1  $YH = \left( \frac{3}{2} X_1 + \frac{3}{2} X_2 + X_3 \right) / 3 = \frac{26.8}{3}$

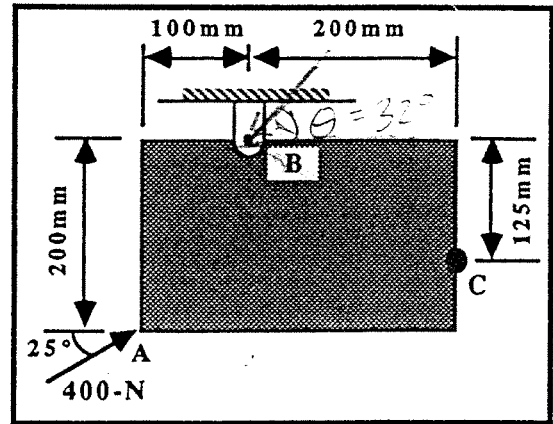
QFT 2  $YH = \frac{3}{2} X_1 (.33) + \frac{3}{2} X_2 (.25) + X_3 (.42) = \frac{9.91}{3}$   
~~12.5~~ + 4.5 + 10.5 = ~~27.5~~ 30

Good Luck!

Name: \_\_\_\_\_ Given Name: \_\_\_\_\_ I.D.: \_\_\_\_\_

**Problem 2 (4 marks) [8 minutes]**

A 400-N force is applied at A as shown. Determine:  
(a) the moment of the 400-N force about B,  
(b) the smallest force applied at B which creates the same moment about C.



a)  $M_B = F \cdot d$   
 $= F \cdot r \cdot \sin \theta$   
 $= \vec{r} \times \vec{F}$

$$F_x = \cos 25^\circ F = 362.5 \text{ N}$$

$$F_y = \sin 25^\circ F = 169.0 \text{ N}$$

$$\vec{r} = -0.1 \hat{i} - 0.2 \hat{j}$$

$$M_B = (-0.1 \hat{i} - 0.2 \hat{j}) \times (365.2, 169.0)$$

$$= -16.9 \hat{k} + 73.0 \hat{k} = \underline{56.1 \text{ Nm (k)}}$$

3

b)  $56.1 = F \cdot d$

$$d = \sqrt{(0.2)^2 + (0.1)^2}$$

$$F = \frac{56.1}{0.24} = \underline{238 \text{ N}}$$

$$= 0.24 \text{ m}$$

$$\theta = 32^\circ \text{ (see diagram)}$$

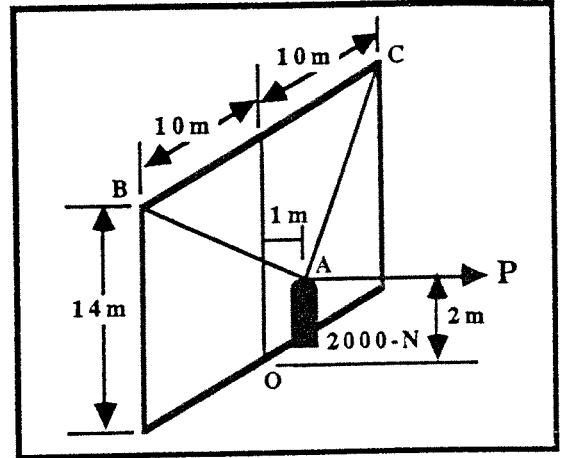
Name: \_\_\_\_\_

Given Name: \_\_\_\_\_

I.D.: \_\_\_\_\_

**Problem 3 (8 marks) [30 minutes]**

A 2000-N cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of P and the tension in each cable. Consider point O as origin.



$$W = 2000 \text{ N}$$

$$\overline{AB} = \overline{AC} = 15.65 \text{ m}$$

$$T_{ABx} + T_{ACx} = P$$

$$T_{ABy} + T_{ACy} = W = 2000 \text{ N}$$

$$T_{ABz} = T_{ACz}$$

$$T_{ABz} = \frac{10}{15.65} T_{AB} = 0.64 T_{AB}$$

$$\therefore T_{AB} = T_{AC}$$

$$T_{ABx} = \frac{1}{15.65} T_{AB} = 0.064 T_{AB}$$

$$P = 0.64 T_{AB} + 0.64 T_{AC}$$

$$T_{ABy} = \frac{12}{15.65} T_{AB} = 0.77 T_{AB}$$

$$2000 = 0.77 T_{AB} + 0.77 T_{AC}$$

$$T_{ACy} = 0.77 T_{AC}$$

$$P = 1.28 T_{AB}$$

$$T_{ACx} = 0.064 T_{AC}$$

$$2000 = 1.54 T_{AB}$$

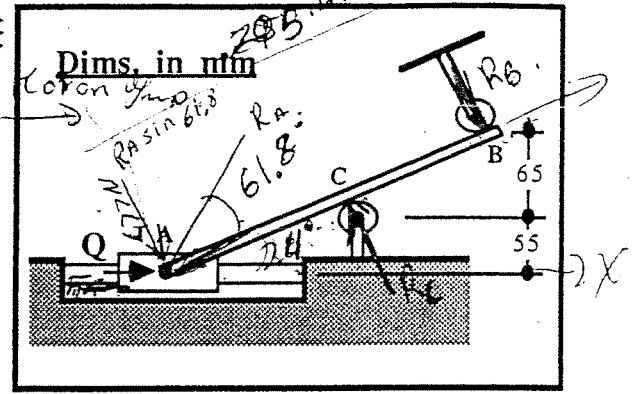
$$T_{ACz} = 0.64 T_{AC}$$

$$T_{AB} = T_{AC} = \underline{1299 \text{ N}} \checkmark$$

$$P = (1.28)(1299) = \underline{1662 \text{ N}} \checkmark$$

**Problem 4 (10 marks) [25 minutes]**

A slender rod AB 295 mm long attached to a collar at A and resting on wheels B and C. The coefficient of static friction between the collar and the horizontal rod is 0.35. Neglecting the friction at the wheels and knowing that the maximum friction force at A is 77 N when motion impending to the right, determine the value of the force Q and the reactions at A, B, and C.



$\mu_{sA} = 0.35$   
 $F_m = 77 \text{ N}$

$F_m = \mu N = 0.35 N \quad N = \frac{77}{0.35} = 220 \text{ N}$

*Handwritten work:*

$\sum F_x = 0$   
 $Q - F_A \sin 24 = 0$   
 $F_A = \frac{220}{\sin 24} = 541 \text{ N}$  ok.

$\sum F_y = 0$   
 $Q = 77 \text{ N} + \cos 24 (541) = 571 \text{ N}$  →

$R_A = 541 \text{ N}$  at  $24^\circ$  from horizontal → ok.

$R = \sqrt{77^2 + 220^2} = 223.1 \text{ N}$   
 $\theta = \tan^{-1} \mu = 19.3^\circ$

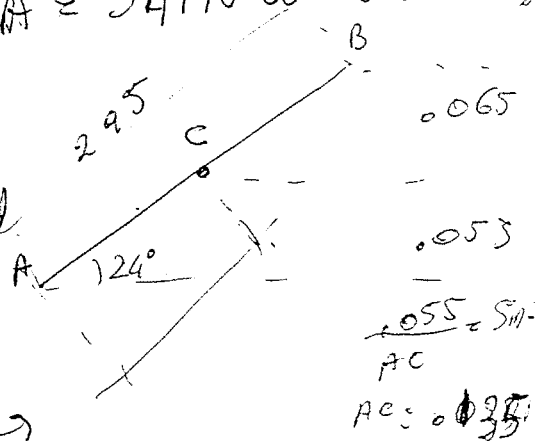
$109.3^\circ$

meatness please

$\frac{541}{\sin 109.3} = \frac{571}{\sin \theta} = \frac{223.1}{\sin \phi}$   
 $\theta = 85.8^\circ$

$R_A = 541 \text{ N}$  at  $85.8^\circ$

$\sum M_C = 0$   
 $477 (\cdot 0.35) = R_B (160) \cdot R_B = 402 \text{ N}$



SRS →

## Class Examples

#1-

**6.100** Water pressure in the supply system exerts a downward force of 30 lb on the vertical plug at A. Determine the tension in the fusible link DE and the force exerted on member BCE at B.

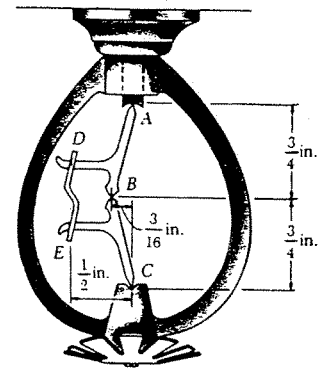


Fig. P6.100

#2-

**6.114** The tool shown is used to crimp terminals onto electric wires. Knowing that a worker will apply forces of magnitude  $P = 135 \text{ N}$  to the handles, determine the magnitude of the crimping forces which will be exerted on the terminal.

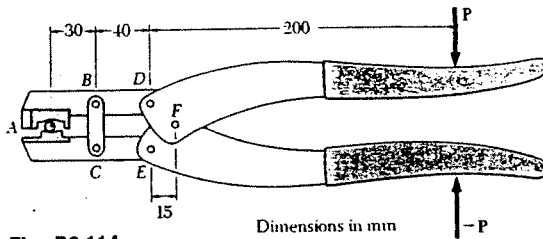
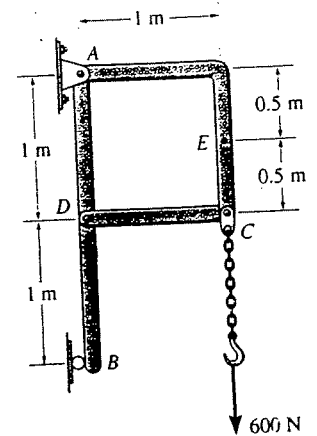


Fig. P6.114

#3-

Determine the normal force, shear force, and bending moment acting at point E of the frame loaded as shown in Fig. 7-8a.



Class Examples

2

# 1

A bar is bent and loaded as shown in Fig. 4-21. Determine

- The moment of force  $F$  about point  $O$ .
- The perpendicular distance  $d$  from point  $O$  to the line of action of the force.

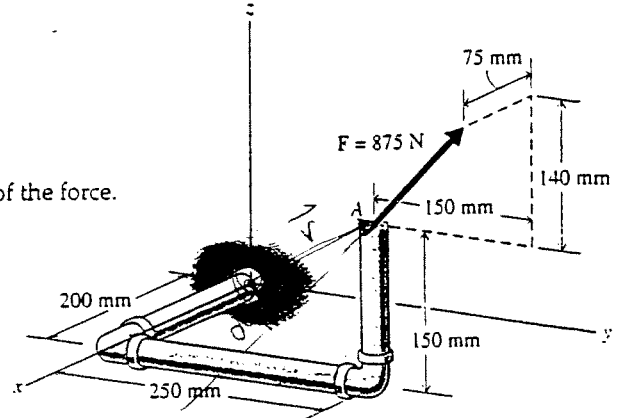


Fig. 4-21

# 2

The force  $F$  in Fig. 4-27 can be expressed in Cartesian vector form as

$$F = 60i + 100j + 120k \text{ lb}$$

Determine the moment of force  $F$  about line  $BC$ .

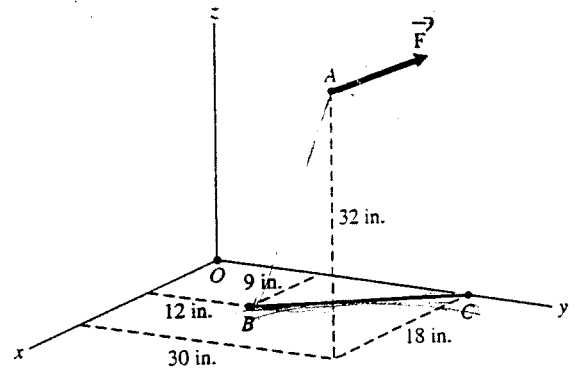
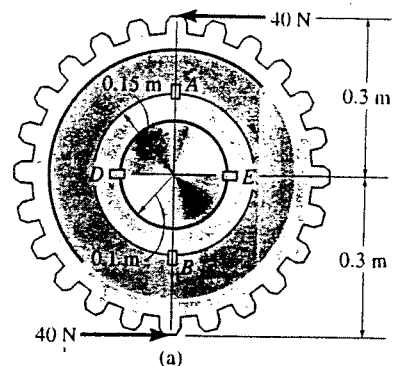


Fig. 4-27

# 3

A couple acts on the gear teeth as shown in Fig. 4-31a. Replace it by an equivalent couple having a pair of forces that act through (a) points  $A$  and  $B$ , and (b) points  $D$  and  $E$ .

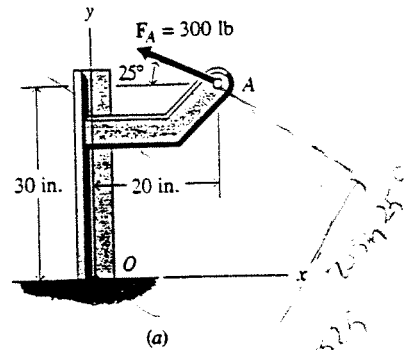


(a)

Class Examples 31

# 1

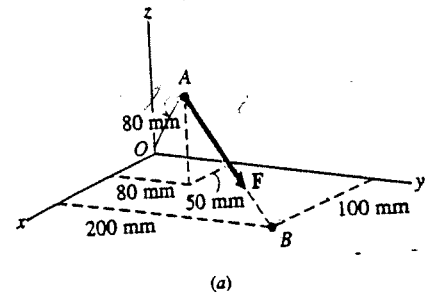
A 300-lb force  $F_A$  is applied to a bracket at point A as shown in Fig. 4-37a. Replace the force  $F_A$  by a force  $F_O$  and a couple C at point O.



# 2

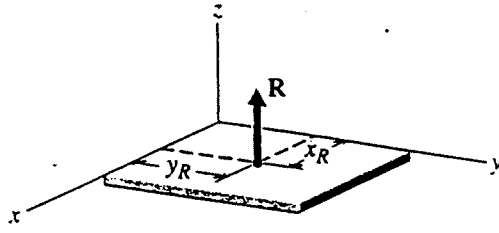
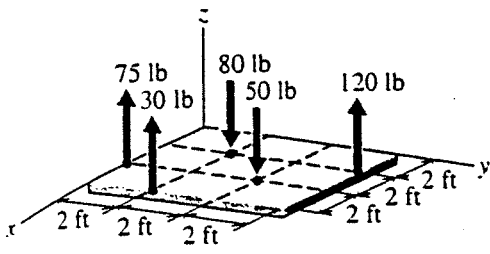
The force F shown in Fig. 4-38a has a magnitude of 763 N. Replace the force F by a force  $F_O$  at point O and a couple C.

Express the force  $F_O$  and the couple C in Cartesian vector form. Determine the direction angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  associated with the couple vector.



# 3

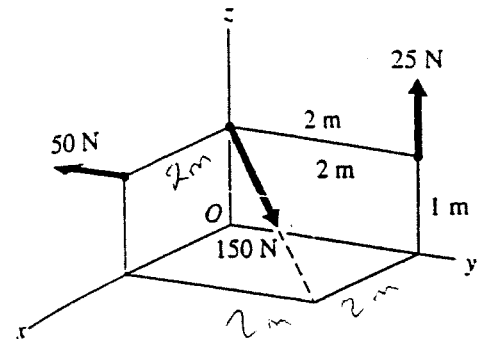
Determine the resultant of the parallel force system shown in Fig. 4-44a and locate the intersection of the line of action of the resultant with the xy-plane.



# 4

Three forces are applied to a rigid body as shown in Fig. 4-50a.

- Reduce the forces to a wrench.
- Determine the intersection of the wrench with the xy-plane.



Class Examples

# 1

Locate the center of gravity for the six particles shown in Fig. 5-5 if  $W_A = 50$  lb,  $W_B = 25$  lb,  $W_C = 30$  lb,  $W_D = 35$  lb,  $W_E = 20$  lb, and  $W_F = 40$  lb.

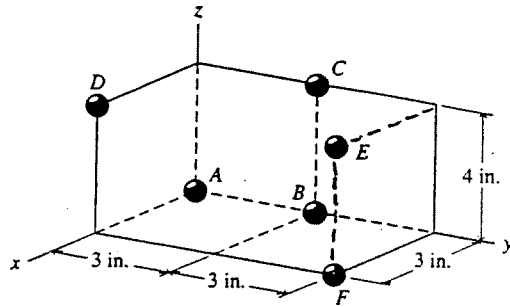


Fig. 5-5

# 2

Locate the center of gravity  $G$  of the homogeneous right circular cone shown in Fig. 5-11a, which has an altitude  $h$  and radius  $r$  and is made of a material with a specific weight  $\gamma$ .

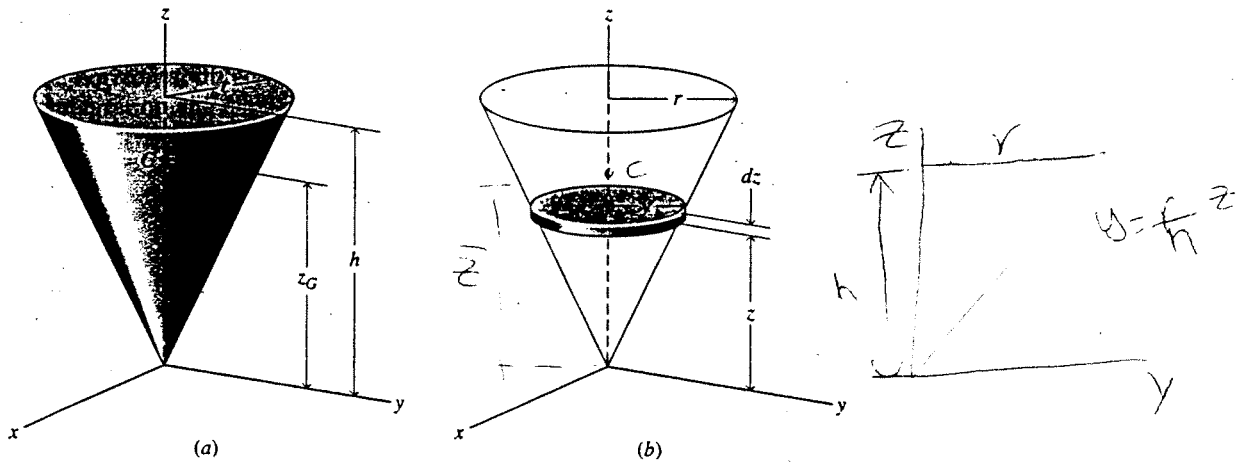


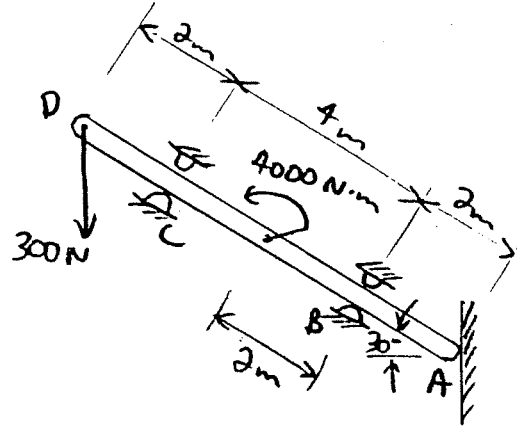
Fig. 5-11

Haja  
Rjaffal@hotmail.com

Class Examples

#1

A uniform beam is subjected to a force and a couple. If the beam is supported at A by a smooth wall and at B and C either at the top or bottom by smooth contacts, find the reactions at these supports.

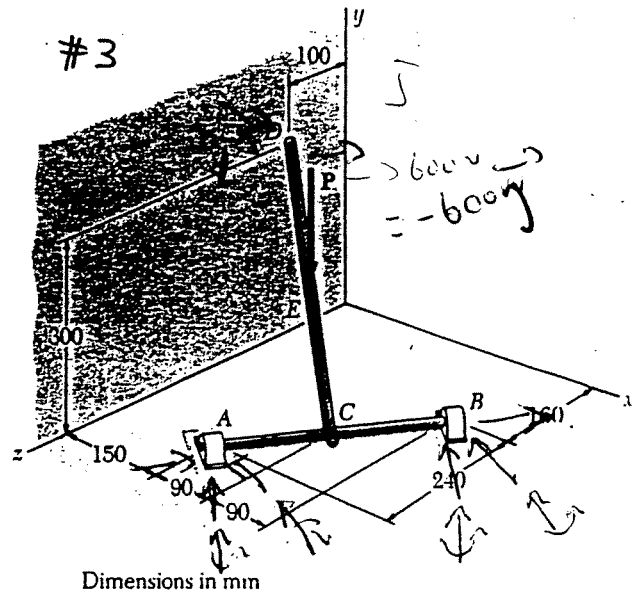
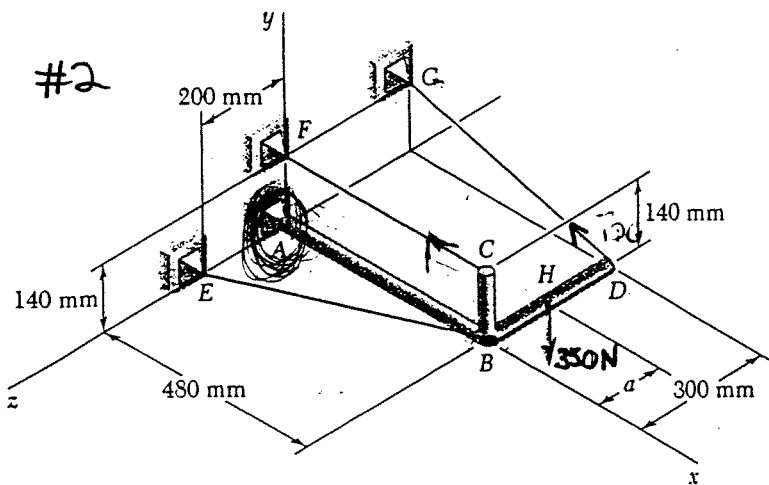


#2

Frame ABCD is supported by a ball-and-socket joint at A and by three cables. For  $a = 150$  mm, determine the tension in each cable and the reaction at A.

#3

Two rods are welded together to form a T-shaped lever which leans against a frictionless vertical wall at D and is supported by bearings at A and B. A vertical force P of magnitude 600 N is applied at the midpoint E of rod DC. Determine the reaction at D.



Class Examples

#1

2.91 Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 300\text{ N}$  and  $Q = 400\text{ N}$ .

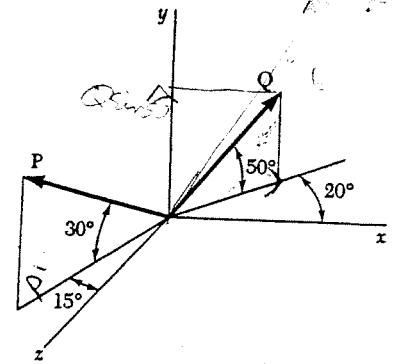
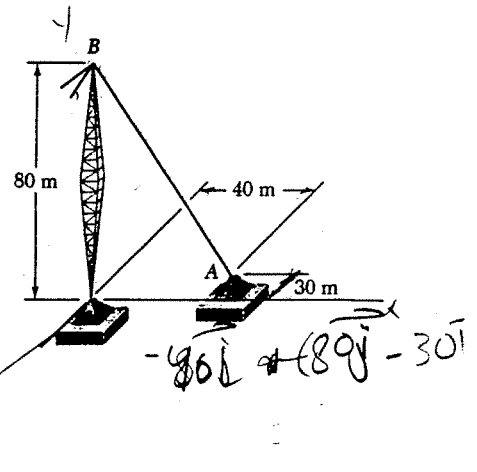


Fig. P2.91 and P2.92

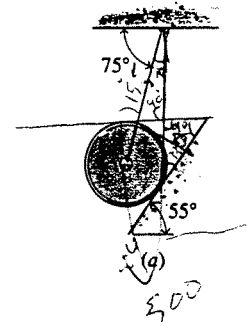
#2

A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine (a) the components  $F_x, F_y, F_z$  of the force acting on the bolt, (b) the angles  $\theta_x, \theta_y, \theta_z$  defining the direction of the force.



#3

③ A homogeneous steel cylinder weighing 500 lb is supported by a flexible cable and a smooth inclined plane as shown in Fig. 3-3a. Determine the tension  $T$  in the cable and the force  $R$  exerted by the inclined plane on the cylinder.



#4

⑥ Four forces act on a small airplane in flight, as shown in Fig. CE3-2: its weight, the thrust provided by the engine, the lift provided by the wings, and the drag resulting from its motion through the air. Is it possible for the plane to be in equilibrium under the action of these four forces?

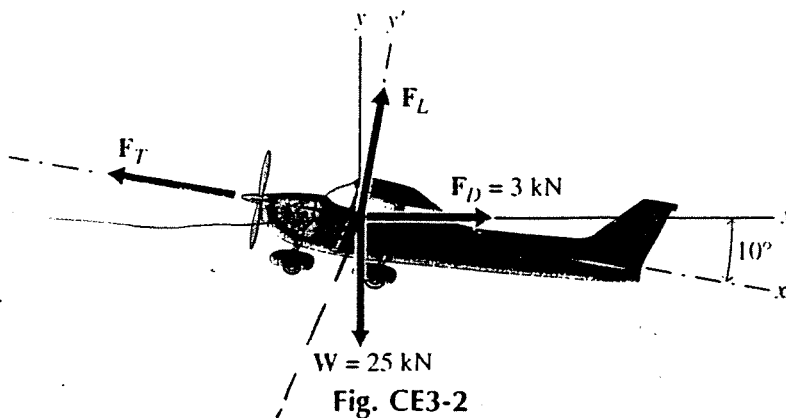
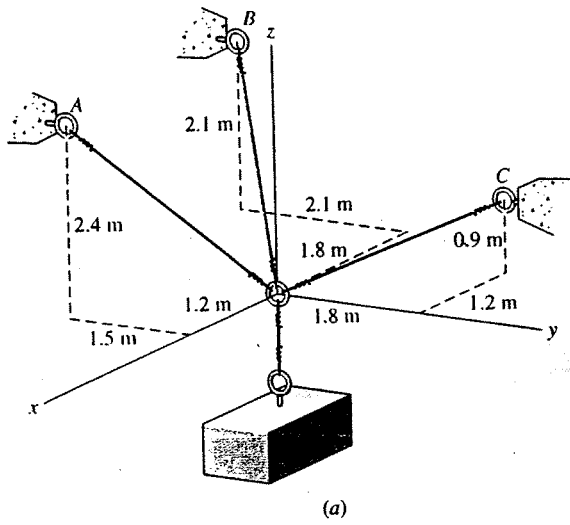


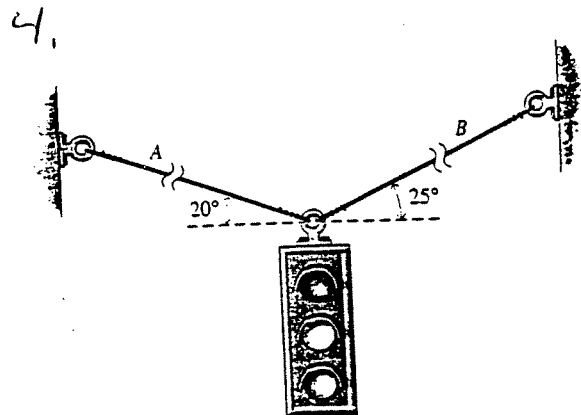
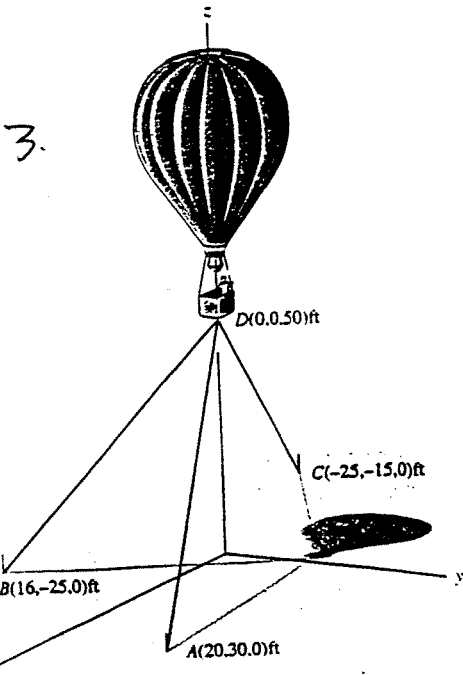
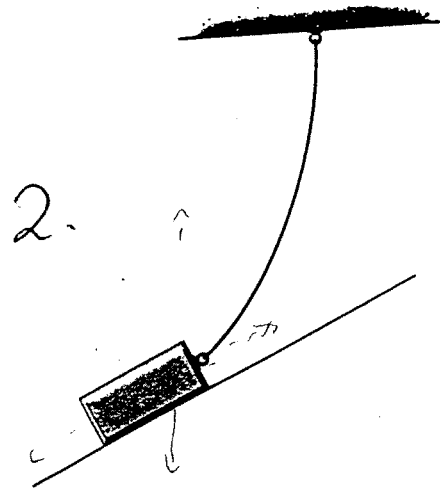
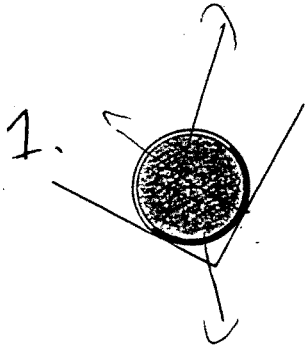
Fig. CE3-2

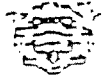
#5

A block is supported by a system of cables as shown in Fig. 3-6a. The weight of the block is 500 N. Determine the tensions in cables A, B, and C.



F.B.D. Exercise

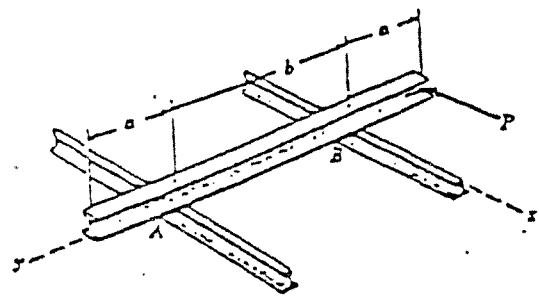




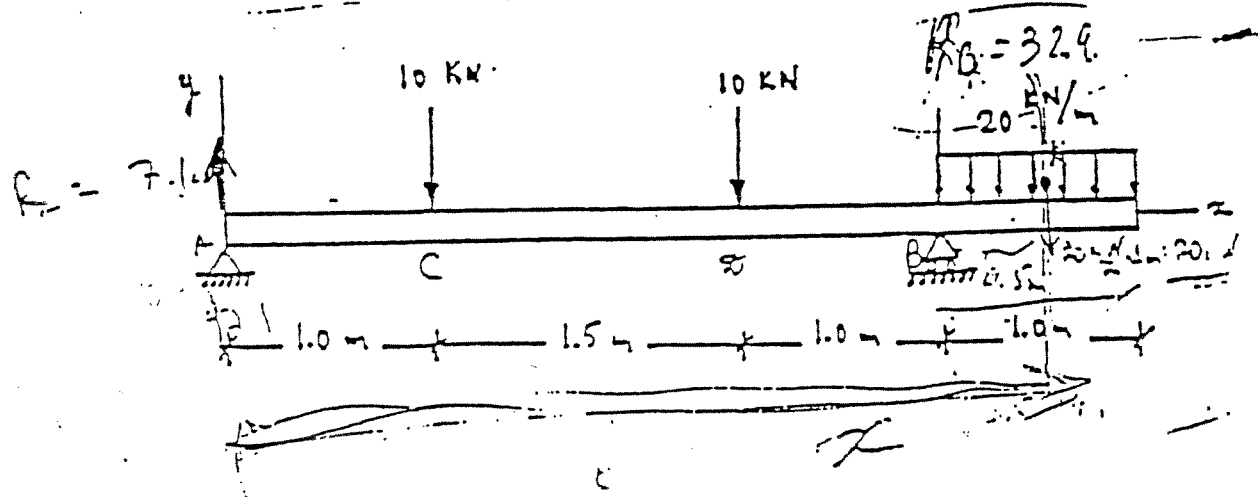
COURSE	STATICS	NUMBER	ENGR C 242/2	SECTION	01, V, W, X, AA
DESCRIPTION	SUPPLEMENTAL	DATE		TITLE	
STAFF	Prof. Goldman, Stathopoulos			DIVISION	Day and Evening
MATERIALS ALLOWED	Any Calculating Device.				

**SPECIAL INSTRUCTIONS**  
 Attempt all problems.  
 All questions carry equal weight.

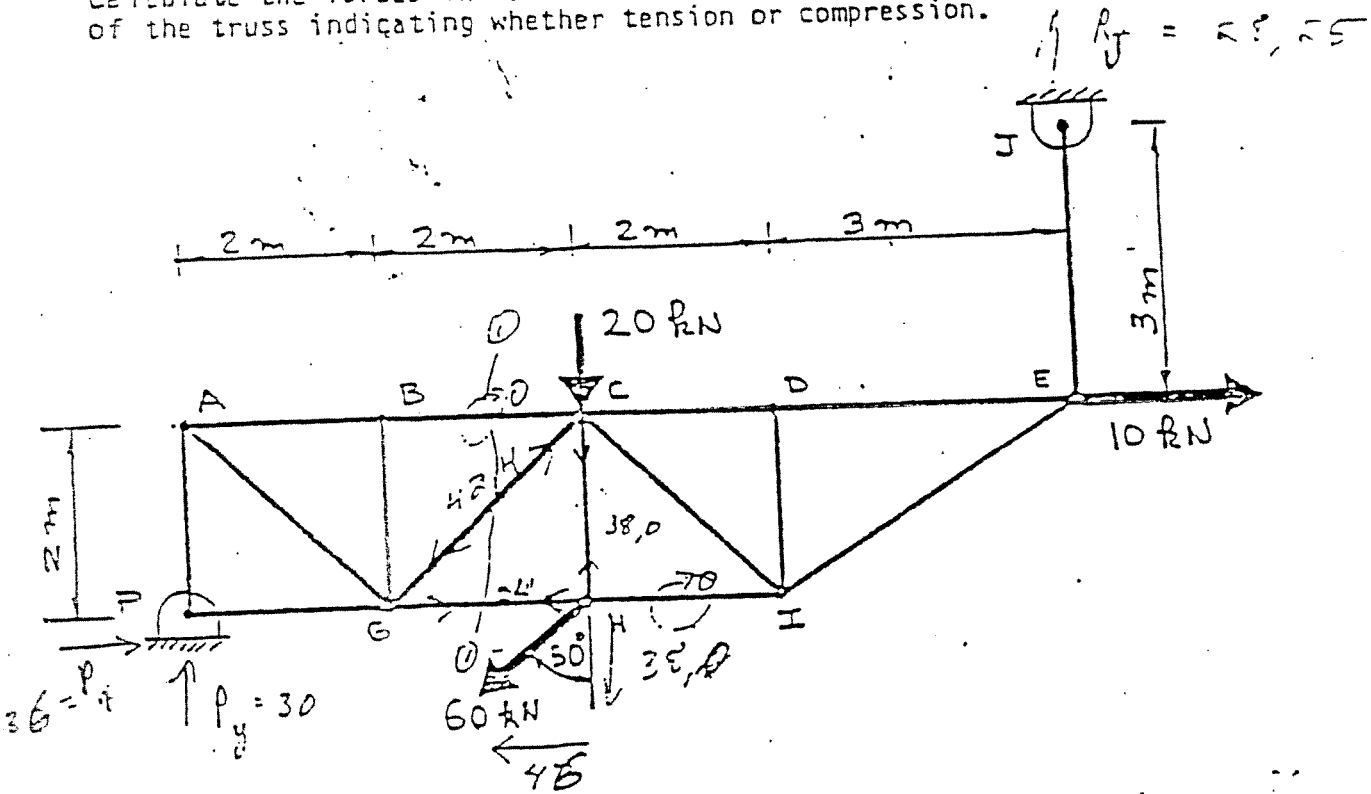
1.- An I-beam of mass  $m$  is supported by two fixed horizontal rails as shown. Compute the applied load  $P$  that is just sufficient to cause the beam to slip and determine the corresponding friction force at A as slippage begins. The coefficient of friction between the beam and the rails is  $f$ .



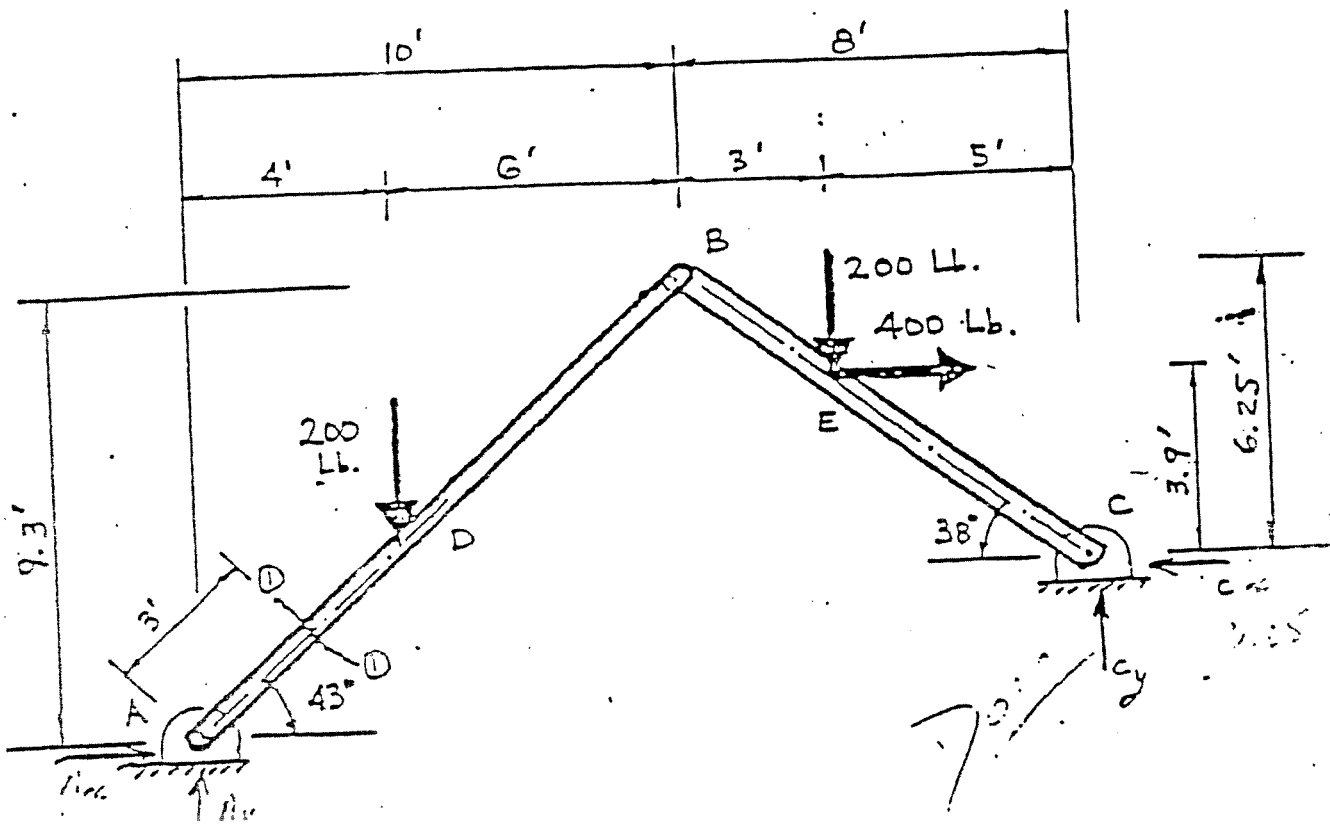
2.- For the beam shown:  
 a.- Draw the shear and bending moment diagrams.  
 b.- Write the shear and bending moment equations for  $3.5 < x \leq 4.5$



- For the truss shown, calculate the reactions at J and P. Calculate the forces in members BC, CH, CG and HI. Show the values on the sketch of the truss indicating whether tension or compression.

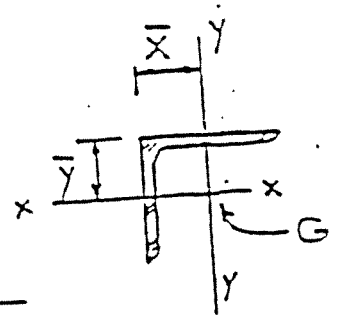
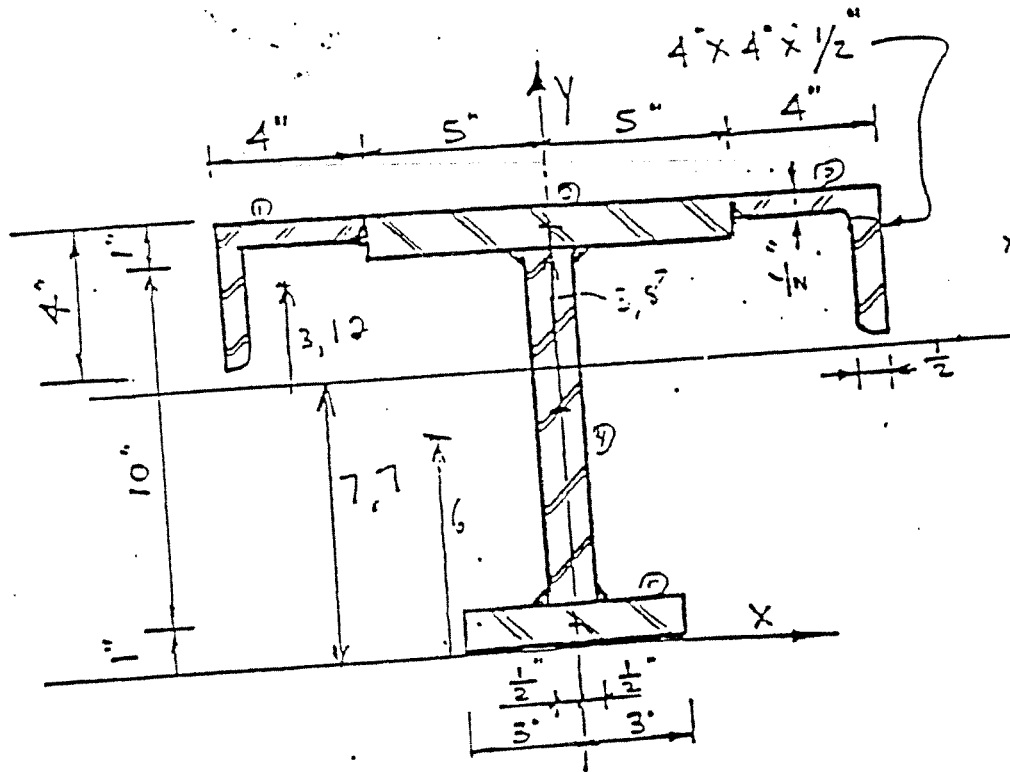


4. For the frame shown obtain the reactions at A and C. Check if the frame is in equilibrium. At section 1-1, obtain axial and shear forces and bending moment.



For the cross-section of the beam indicated:

- Obtain the location of the centroid.
- Obtain the moment of inertia of the area with respect to an axis parallel to the X axis and passing through the centroid.
- Obtain the radius of gyration of the area with respect to the same axis as in b.- above.



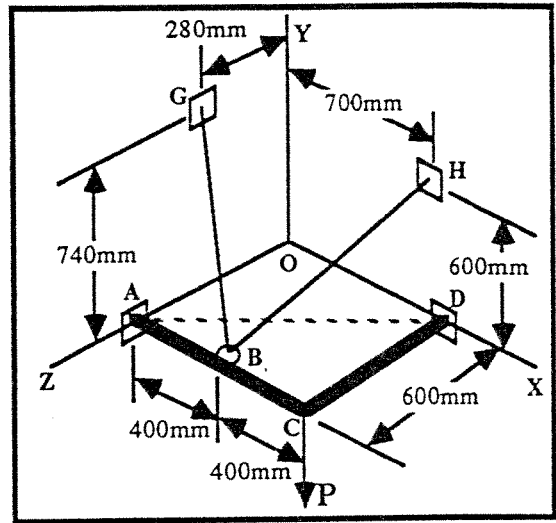
$AREA = 3.75 \text{ in}^2$   
 $I_x = 5.6 \text{ in}^4 =$   
 $\bar{X} = \bar{Y} = 1.18 \text{ in.}$

$I_{xx} = \frac{BD^3}{12}$

Name: \_\_\_\_\_ Given Name: \_\_\_\_\_ I.D.: \_\_\_\_\_

**Problem 4 (8 marks) [22 minutes]**

The frame ACD is hinged at A and D and supported by a cable which passes through a ring at B and is attached to hooks at G and H. The tension in the cable is 1125 N, determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.



$$\overline{BH} = 900 \text{ mm}$$

$$T_{BHx} = 375 \text{ N}$$

$$T_{BHy} = 750 \text{ N}$$

$$T_{BH z} = -750 \text{ N}$$

$$\overline{AD} = \sqrt{(800)^2 + (600)^2}$$

$$= 1000 \text{ mm} = 1 \text{ m}$$

$$\lambda_L = \frac{800}{1000} = 0.8 \quad \lambda_j = 0 \quad \lambda_k = -0.6$$

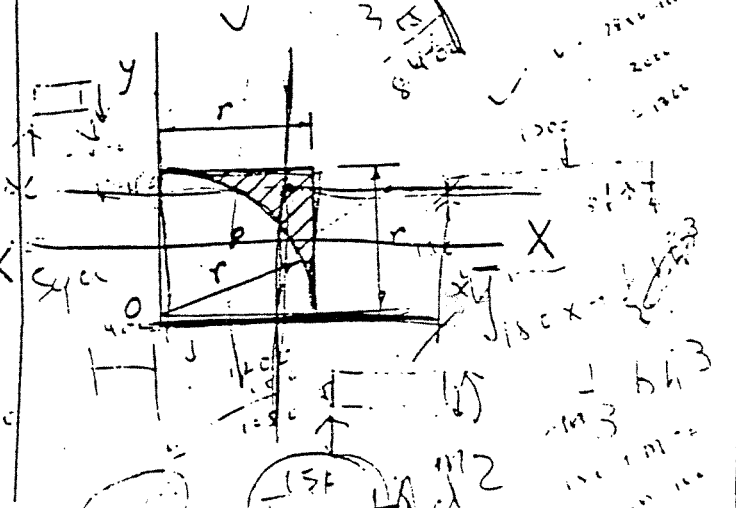
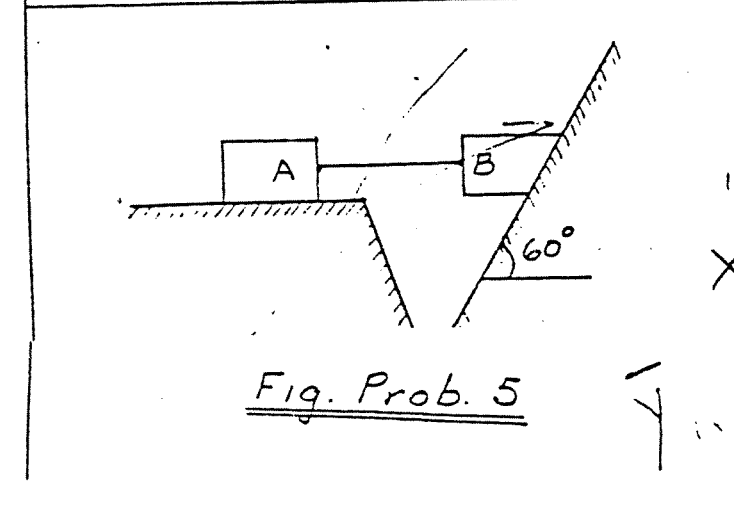
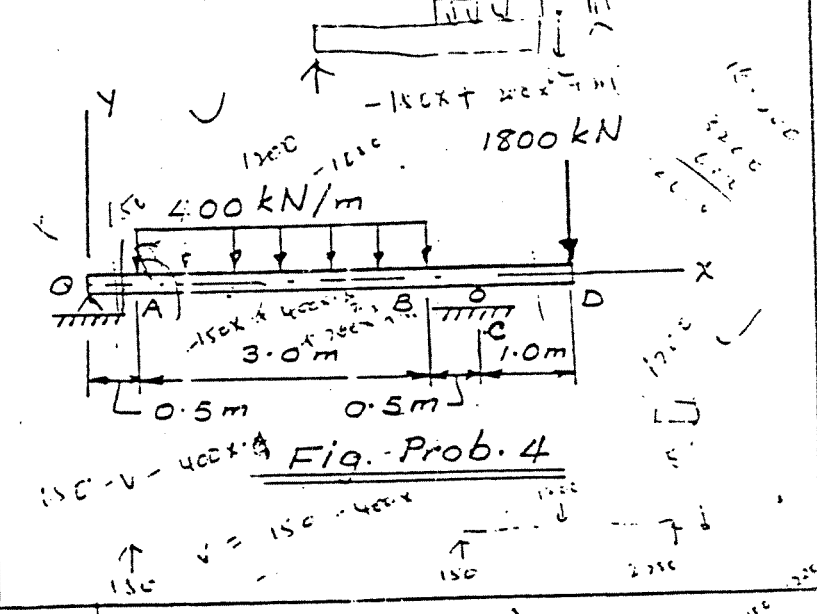
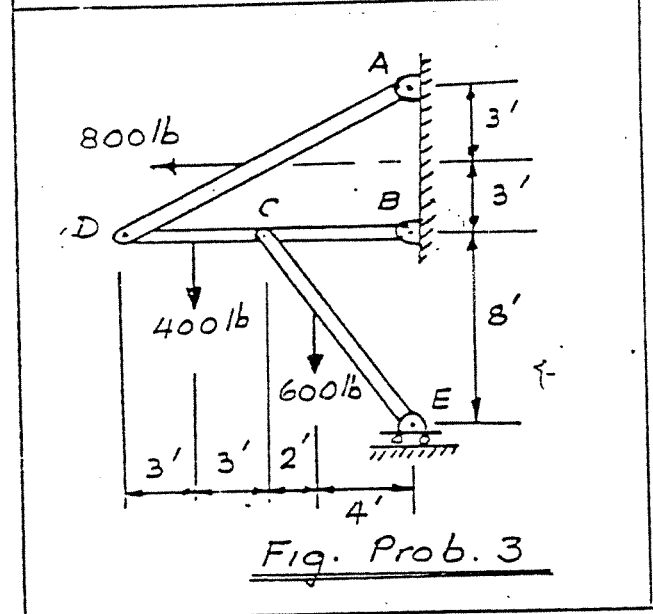
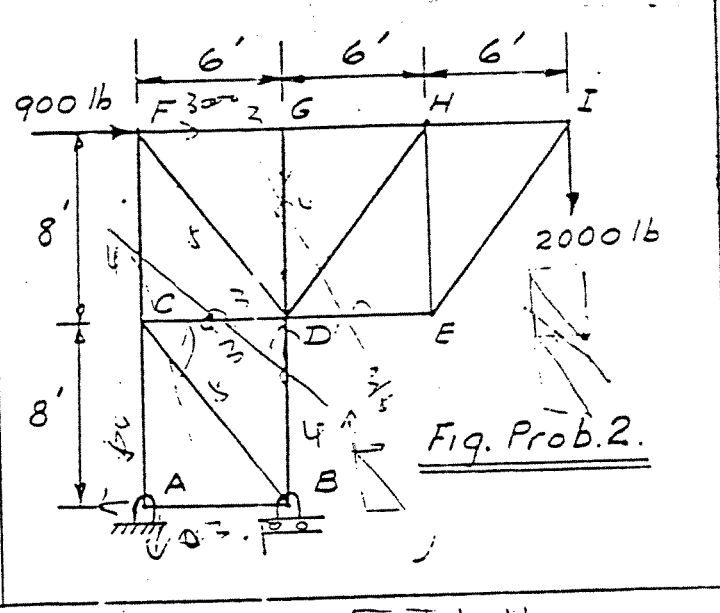
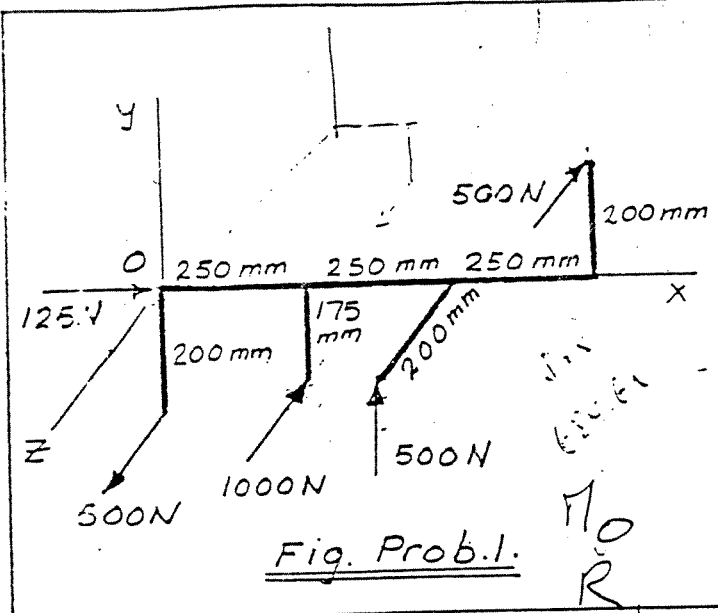
$$r_{A/B} = 0.4 \text{ m } i + 0j - 0.4k$$

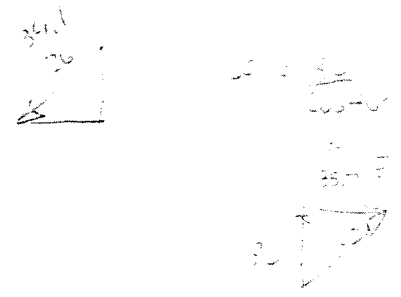
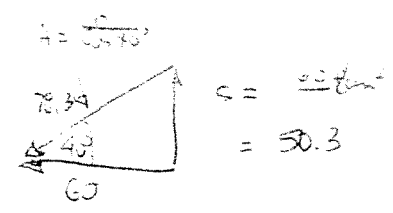
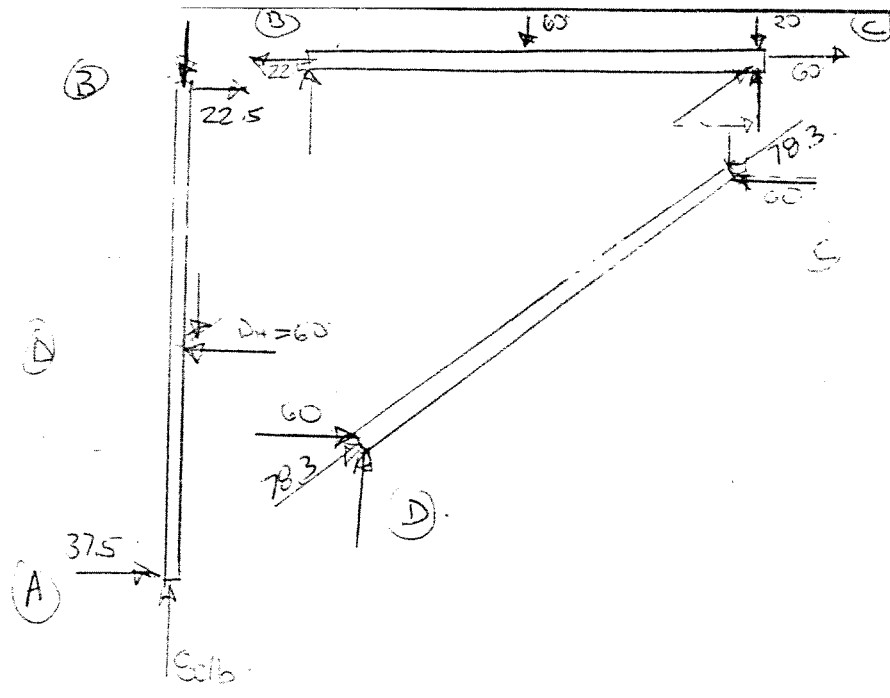
$$M_{AD} = \begin{vmatrix} 0.8i & 0j & -0.6k & 0.4i & 0 \\ 0.4 & 0 & 0 & 0.4 & 0 \\ 375 & 750 & -750 & 375 & 750 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} = -180 \text{ k}$$

$$M_{AD} = \underline{\underline{-180 \text{ kNm}}}$$

### Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic span-drel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	$\alpha r^2$
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	$\pi r$
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$





EMBER BDA

$$\sum M_B^+ = 0$$

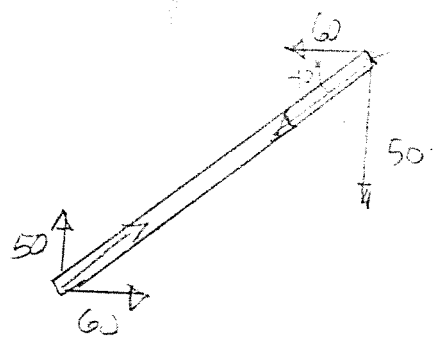
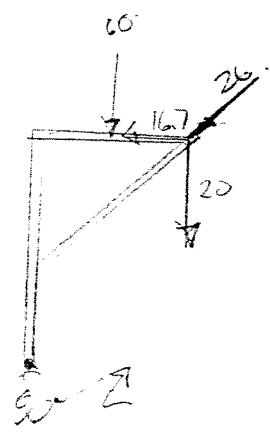
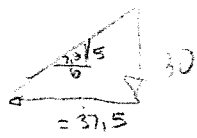
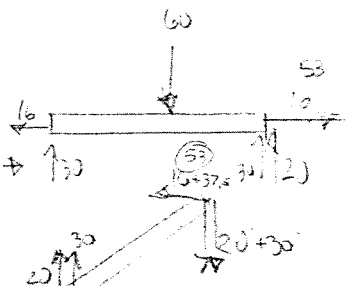
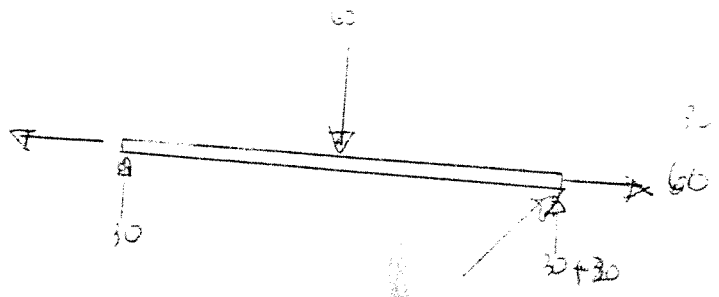
$$+D_H(5) - 37.5(9) = 0$$

$$D_H = 300/5 = 60$$

$$\sum M_C^+ = 0$$

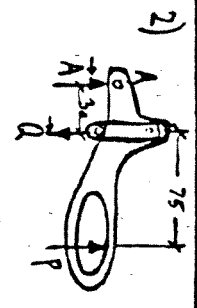
$$B_H(5) - 37.5(3) = 0$$

$$B_H = 112.5/5 = 22.5$$

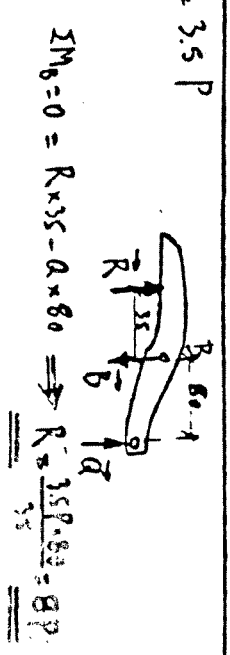


ΣF

1) Joint J :  $\underline{S_{Jx}} = 0$  ; Joint G :  $\underline{S_{Gx}} = -4 \text{ kN (C)}$  ; Vertical section through EH yields :  
 $S_{EH} \frac{4}{5} - 4 - 4 = 0 \Rightarrow \underline{S_{EH}} = 10 \text{ kN (T)}$



2)  $\sum M_A = 0 = P \times 105 - Q \times 30 \Rightarrow Q = 3.5 P$



$\sum M_B = 0 = R \times 35 - Q \times 80 \Rightarrow \underline{R} = \frac{35P \cdot 80}{35} = 8P$

3) See solution of assigned problem # 7.66

C. Goldman

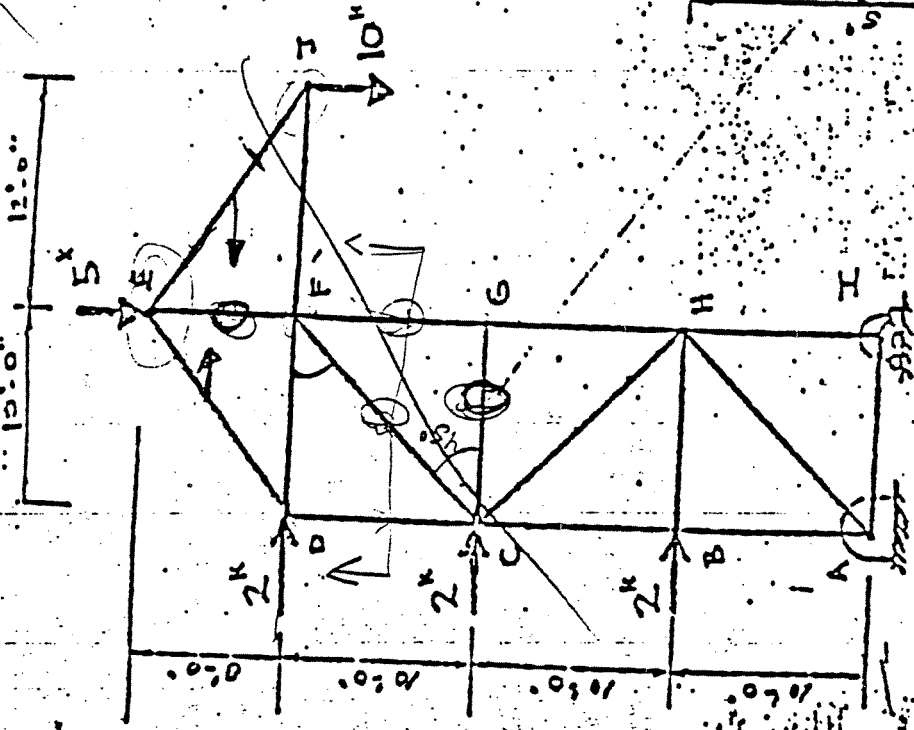


FIG. 1

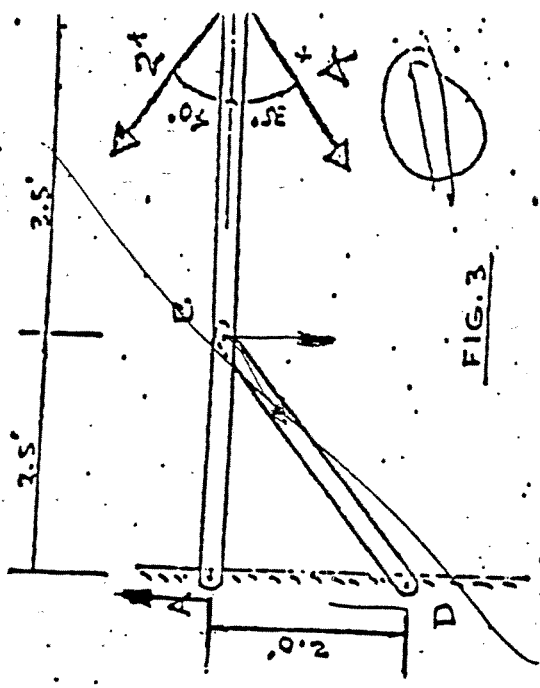


FIG. 3

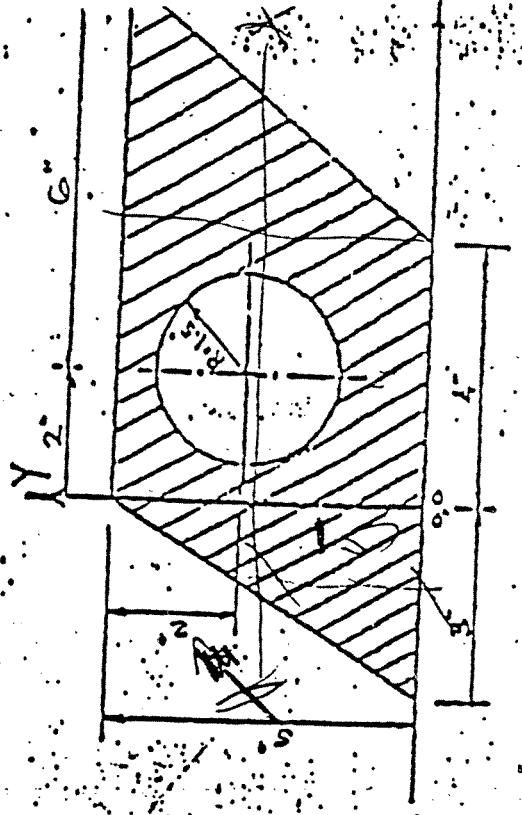
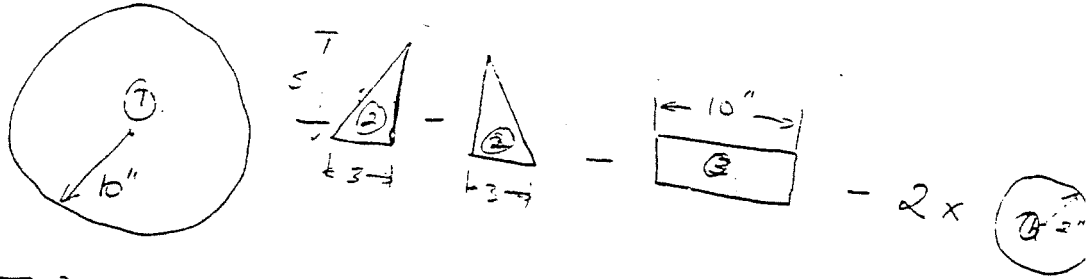


FIG. 40

2) a)



$$A_1 = \pi r^2 = 100\pi$$

$$2 \times A_2 = \frac{1}{2}bh$$

$$A_3 = b(h)$$

$$2 \times A_4 = \pi r^2$$

$$I_{x1} = \frac{\pi r^4}{4}$$

$$I_{x2} = 2 \times \frac{Ah^2}{18}$$

$$I_{x3} = \frac{1}{3}bh^3$$

$$I_{x4} = \frac{\pi r^4}{4} \times 2$$

	Area (in <sup>2</sup> )	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$	$I_x$
①	314.16	10	10	3141.6	3141.6	7653.98
②	7.5	1.5	1.67	11.25	12.525	10.42
③	7.5	1.5	1.67	11.25	12.525	10.42
④	30	5	1	150	30	26.67
⑤	12.57	2	2	25.14	25.14	12.57
⑥	12.57	2	2	25.14	25.14	12.57
$\Sigma A$	244.02			2918.82	3036.27	

$$\bar{X} = \frac{2918.82}{244.02} = \underline{11.96}$$

$$\bar{Y} = \frac{3036.27}{244.02} = \underline{12.44}$$

$$\bar{I}_x = \underline{7781.33 \text{ in}^4}$$

$$R_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{7781.33 \text{ in}^4}{244.02 \text{ in}^2}} = \underline{5.65 \text{ in}}$$