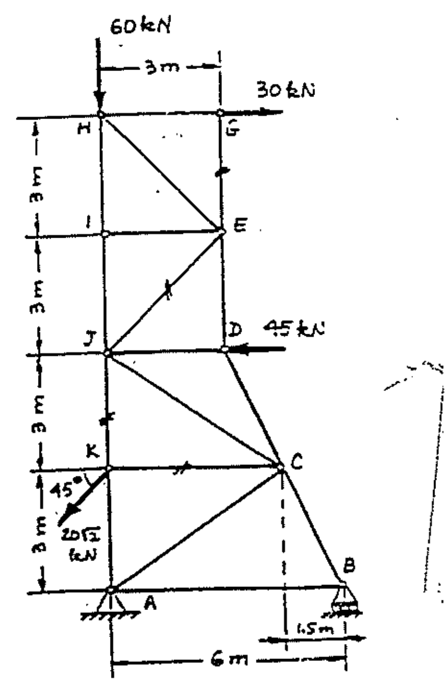
CONCORDIA UNIVERSITY

DEPARTMENT OF MECHANICAL ENGINEERING

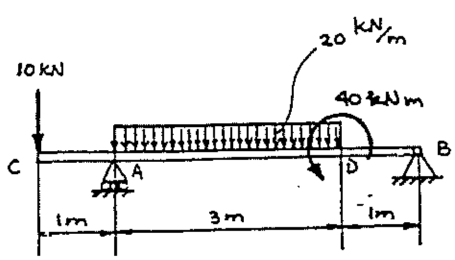
41

COURSE:	NUMBER:	SECTION:
STATICS	ENGR 242/2	T
EXAMINATION:	**DATE & TIME:**	**# OF PAGES:**
MIDTERM 2	November 25, 1999, 8:30 – 9:45 pm	3
INSTRUCTOR: Dr. Eliza M. Haseganu		
MATERIAL ALLOWED:	☒ NO ☐ YES	
CALCULATORS ALLOWED:	☐ NO ☒ YES	(non-programmable)
SPECIAL INSTRUCTIONS:	Answer questions 1 and 2 for 35 marks each, and question 3 OR 4, for 30 marks. Question 2 should be answered on the separate sheet provided.	

1. Determine the forces in members GE, EJ, JK, and KC of the truss shown, and indicate whether these members are in tension or compression. Use the method of joints combined with the method of sections. Indicate with justification the zero force members.



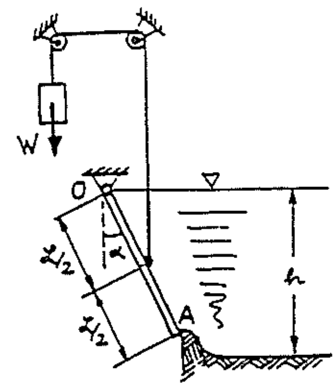
2. Draw the shear force and bending moment diagrams for the beam loaded as shown; indicate peak values. Write an equation for the shear force and the bending moment for section AD. Also, locate the points of zero moment on the beam. Answer this question on the separate sheet provided.

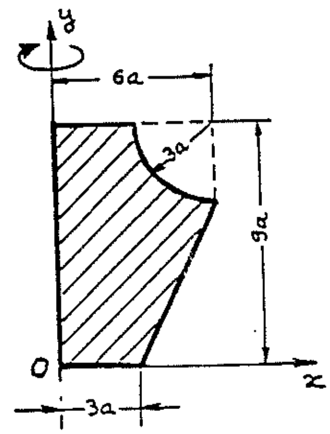


5.00

3. The gate OA is located at the end of a 2m wide channel and is supported by hinges along its top edge. The gate is held in its closed position by a cable and a weight W, as shown. Determine the minimum magnitude of the weight W necessary to keep the gate closed when the water level riches O.

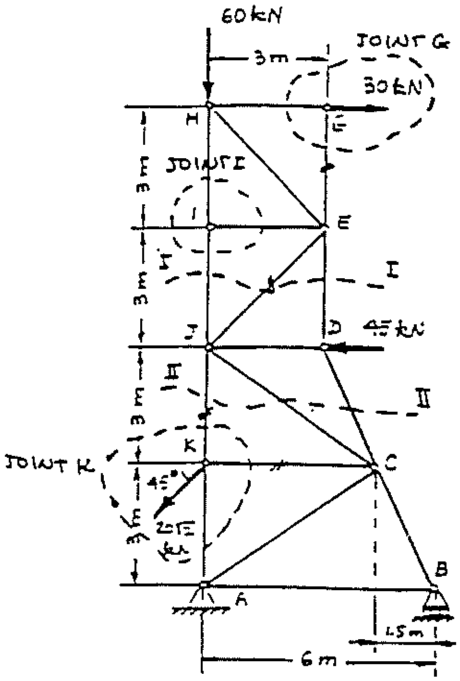
Consider the density of water $\rho = 1000$ kg/m^3, $L = 3$ m, $\alpha = 30^0$ and $h = 3$ m. Neglect the weight of the gate.





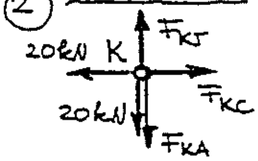
4. For the figure shown determine:
   (1) the location of the centroid of the area;
   (2) the volume generated by a complete rotation of the shaded area about the y axis.
   (3) If $a = 0.1$ m, and the mass density of the material is $\rho = 100$ kg/m^3, determine the mass and the weight of the body of revolution obtained at (2).

1.



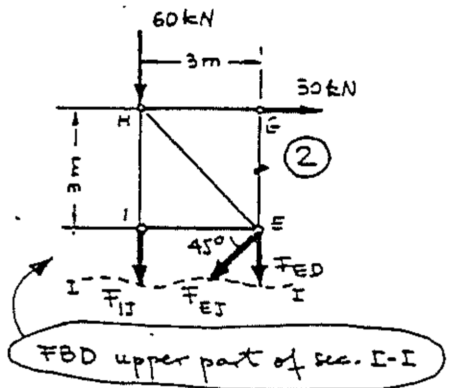
- **METHOD of JOINTS:**

(2) FBD JOINT K:



$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{KC} - 20 = 0$ $\boxed{F_{KC} = 20\,k}$ (6) TENSION

- **METHOD of SECTIONS:**



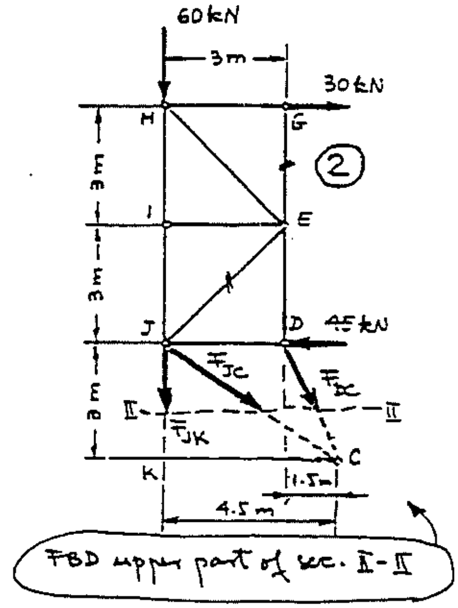
FBD upper part of sec. I-I

$\overset{+}{\rightarrow}\Sigma F_x = 0;$

$-F_{EJ}\frac{\sqrt{2}}{2} + 30 = 0$

$\rightarrow \boxed{F_{EJ} = 42.43\,kN}$ (6)

TENSION



FBD upper part of sec. II-II

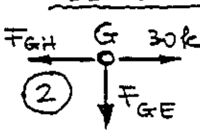
$\overset{\curvearrowleft}{+}\ \Sigma M_C = 0;$

$F_{JK}(4.5) + 60(4.5) - 30(9) + 45(3) = 0$

$\rightarrow F_{JK} = -30\,kN$ $\boxed{F_{JK} = 30\,kN}$

COMPRESSION

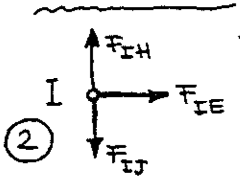
FBD JOINT G:



$\overset{+}{\downarrow}\ \Sigma F_y = 0;$

$\rightarrow \boxed{F_{GE} = 0}$ (4)

FBD JOINT I:

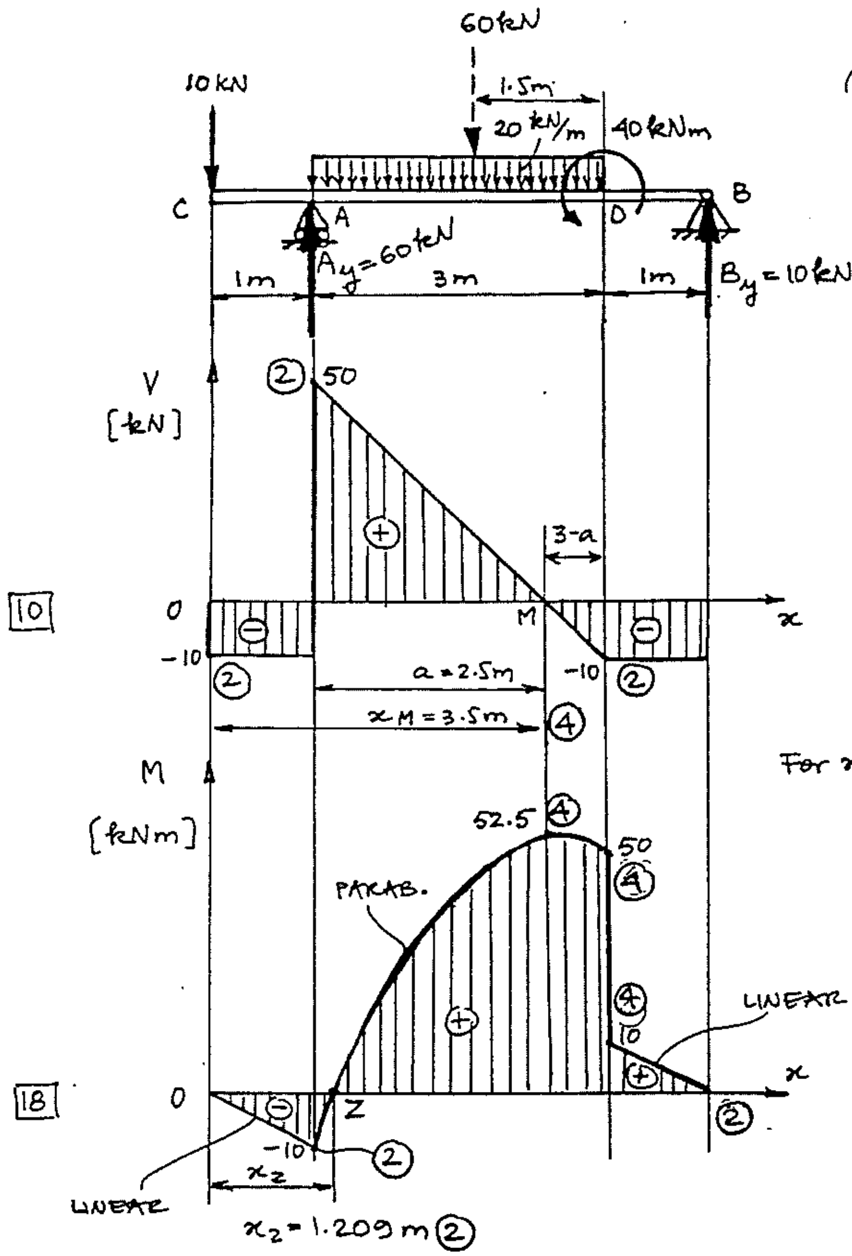


$\overset{+}{\rightarrow}\Sigma F_x = 0;$

$\rightarrow \boxed{F_{IE} = 0}$ (3)

$\boxed{\Sigma\ 35}$

2. Draw the shear force and bending moment diagrams for the beam loaded as shown; indicate peak values. Write an equation for the shear force and the bending moment for section AD. Also, locate the points of zero moment on the beam. Answer this question on this sheet.



• REACTION FORCES

$\circlearrowright^{+} \Sigma M_B = 0;\ 4A_y - 10(5) - 60(2.5) - 40 = 0$

$\rightarrow\ \underline{A_y = 60 kN}$ (2)

$\circlearrowleft^{+} \Sigma M_A = 0;\ 4B_y + 40 - 60(1.5) + 10(1)$

$\rightarrow\ \underline{B_y = 10 kN}$ (2)

Check: $\uparrow^{+} \Sigma F_y = 0;\quad 60 + 10 - 10 - 60 \equiv$

• SHEAR FORCE

$V_{C\ominus} = 0;\ V_{C\oplus} = -10 kN$

$V_{A\ominus} = -10 kN;\ V_{A\oplus} = -10 + 60 = 50 k$

$V_D = -10 + 60 - 60 = -10 kN = V_{B\ominus}$

$V_{B\oplus} = 0$

For x_M: $V_M = 60 - 10 - 20(x_M - 1) = 0$

$\rightarrow\ \underline{x_M = 3.5 m}$

or: similarity in $\triangle$ $\quad \frac{a}{50} = \frac{3-a}{10}\ \rightarrow\ \underline{a = 2.5}$

$\rightarrow\ x_M = a + 1 = 3.5 m$

• BENDING MOMENT

$M_C = 0;\ M_A = -10 kNm$

$M_M = 60(2.5) - 10(3.5) - \frac{20(2.5)^2}{2} = 52.5$ kNm

$M_{D\ominus} = 60(3) - 10(4) - 60(1.5) = 50 k$

$M_{D\oplus} = 50 - 40 = 10 kNm$

$M_B = 0$

$$V = \left[\oplus\uparrow \Sigma F_y \text{ LEFT of } (1) \right]$$

$$M = \left[\circlearrowright^{+} \Sigma M_{(1)} \text{ LEFT of } (1) \right]$$

• INTERVAL AD

(1) $V = -10 + 60 - 20x = -20x + 50$ kN

(2) $M = -10x + 60(x-1) - 20\frac{(x-1)^2}{2}$ kNm

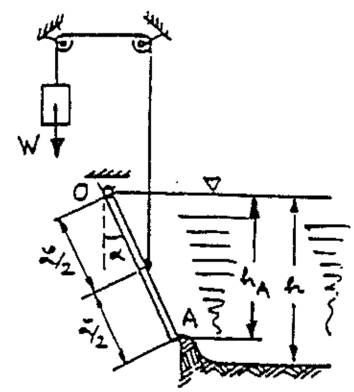
• POINTS of ZERO MOMENT

$M_Z = -10x_z + 60(x_z - 1) - 20\frac{(x_z-1)^2}{2} = 0$

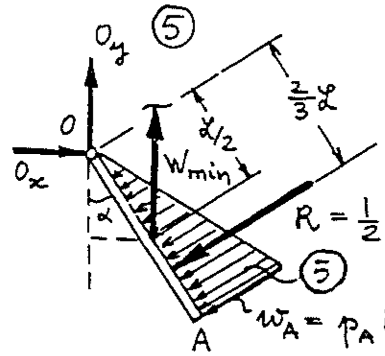
$x_z^2 - 7x_z + 7 = 0 \rightarrow x_z = \cancel{5.79 m}$, $\underline{1.209 m}$

$\boxed{\Sigma\ 35}$

3.



FBD: Since W_{min} is required, reaction force at support A is zer



$R = \frac{1}{2} w_A \mathcal{L} = \frac{1}{2}\, 50.9743\,(3) = \underline{76.46\ kN}$

$w_A = p_A b = \overbrace{\rho g}^{\gamma} h_A b$

$= 1000\,(9.81)\,\frac{3\sqrt{3}}{2}\, 2 = 50.9743\ \frac{kN}{m}$ (5)

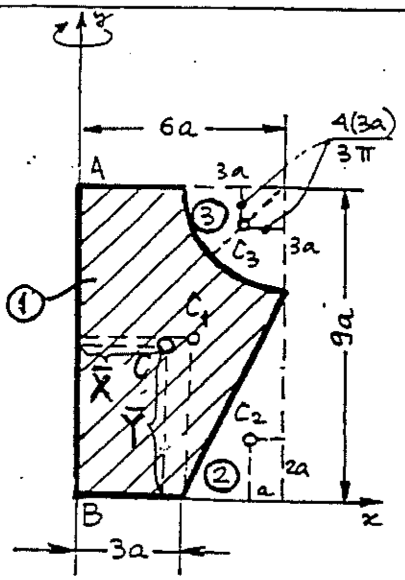
$h_A = \mathcal{L}\cos\alpha = \mathcal{L}\cos 30^\circ = \mathcal{L}\frac{\sqrt{3}}{2}$

$\circlearrowleft + \ \Sigma M_o = 0 \qquad W_{min}\frac{\cancel{\mathcal{L}}}{2}\sin\alpha - R\,\frac{2}{3}\cancel{\mathcal{L}} = 0$

$\underline{W_{min}} = \frac{8R}{3} = \frac{8(76.46)}{3} = \underline{203.89\ kN}$ (10)

$\boxed{\Sigma \mid 30}$

4.



PART	$\bar{x}$	$\bar{y}$	A	$\bar{x}A$	$\bar{y}A$
① (9a × 6a)	3a	4.5a	$54a^2$	$162a^3$	$243a^3$
② (−)	5a	2a	$-9a^2$	$-45a^3$	$-18a^3$
③ (−)	$6a-\frac{4(3a)}{3\pi}$	$9a-\frac{4(3a)}{3\pi}$	$-\frac{9\pi a^2}{4}$	$-33.41a^3$	$-54.62a^3$
Σ	—	—	$37.93a^2$	$83.59a^3$	$170.38a^3$

$\bar{X}\,\Sigma A = \Sigma \bar{x} A \qquad \bar{X}\, 37.93a^2 = 83.59a^3 \rightarrow \underline{\bar{X} = 2.2a}$ $\boxed{5}$

$\bar{Y}\,\Sigma A = \Sigma \bar{y} A \qquad \bar{Y}\, 37.93a^2 = 170.38a^3 \rightarrow \underline{\bar{Y} = 4.49a}$ $\boxed{5}$

$V = 2\pi \bar{X} A = 2\pi\, \Sigma \bar{x} A = 2\pi(83.59a^3) = \underline{525.2a^3}$ $\boxed{5}$

$m = \rho V = 100\,(525.2)(0.1)^3 = \underline{52.52\ kg}$ $\boxed{5}$

$W = mg = \underline{515.22\ N}$ $\boxed{1}$

$\boxed{\Sigma \mid 30}$

$\boxed{1} + \boxed{2} + \boxed{3}/\boxed{4}$ TOTAL: 100

Example 4.1

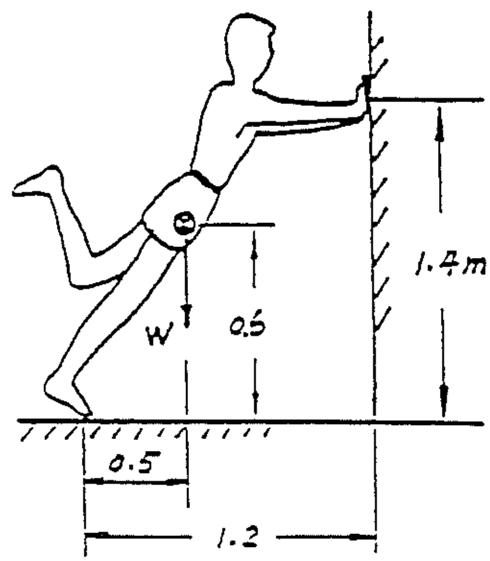
A man maintained his inclined body position statically by hands pushing against a wall with one foot standing on the ground. Assume that the man's body weight is 80 kg. The location of COG of whole body is shown in the figure. The force applied to the toe of the foot along the ground was measured by the a force platform and was known as R_x = 200 N. Find

1). The forces acting on the hand and the vertical force applied to the toe of foot

2). The minimum value of coefficient of static friction between the wall and hand for maintaining the hand in the position.

(1). Force Table

Force	F_x	F_y	T_h
W	0	-80×9.8	784×0.7
R_x	200	0	200×1.4
N_g	0	N_g	$-N_g\times1.2$
N_h	$-N_h$	0	0
F_h	0	F_h	0



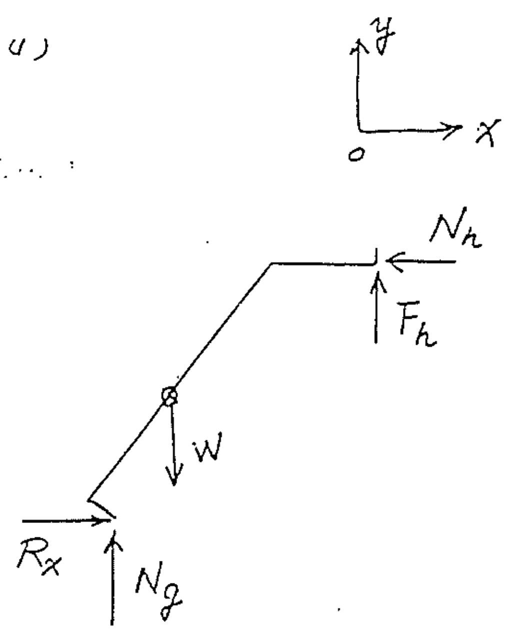
$\Sigma F_x = 0 \qquad 200 - N_h = 0, \quad N_h = 200\ N \qquad (1)$

$\Sigma F_y = 0$

$\Sigma T_h = 0$

From (3) $\qquad N_g =$

From (2). $\qquad F_h =$



(2)

$$\mu_s = \frac{F_h}{N_h}$$

Example 4.2

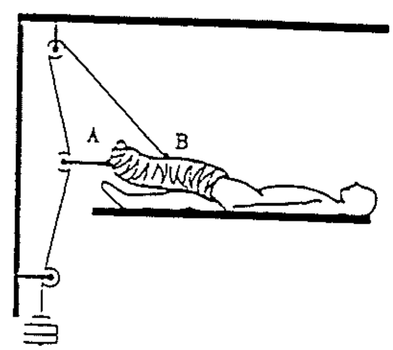
Traction Devices are designed to maintain parts of human body in particular positions for healing purposes. As shown in the figure a leg traction pulls the leg towards the left by applying force on the leg. A weight pan is suspended on a long cable that passes over pulleys. The cable is attached to the leg at B where the COG of the leg is located and the angle β between the cable and leg is 45 degrees. A second cable is connected to the shaft of one of the pulleys and to the leg at point A, such that point A and the center of the pulley lie on a horizontal line, and the cable pulled by the pulley has an angle α of 60 degrees with the horizontal line. The total weight of the leg and the cast, W_1 = 200 N. The distance between A and the COG of the leg is 0.5 m. The weight of the patient's upper body, W_u = 60 kg . The weight of the pan is 20 kg.

(1) Analyze the forces acting on the patient and the pulley.
(2) Determine the minimum static friction coefficient for maintaining the patient's body in the bed.

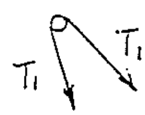
(1). Known data

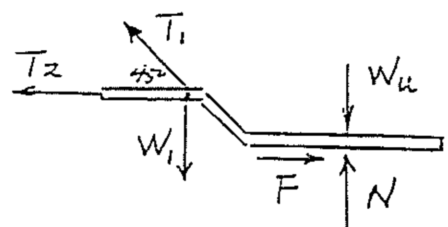
$W_1 = 200\,N,\ W_u = 60 \times 9.8 = 588\,N$

$W_p = 20 \times 9.8 = 196\,N$



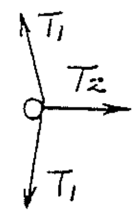
Force	F_x	F_y	T
W_1	0	$-W_1$	—
W_u	0	$-W_u$	—
N	0	N	—
F	F	0	—
T_1	$-T_1 \cos 45°$	$T_1 \sin 45°$	—
T_2	$-T_2$	0	—





$\Sigma F_x = 0 \qquad F - T_1 \cos 45° - T_2 = 0 \qquad (1)$

$\Sigma F_y = 0 \qquad -W_1 - W_u + N + T_1 \sin 45° = 0 \qquad (2)$



$T_1 = W_p = 196\,N$

$\Sigma F_x = 0 \qquad T_2 - 2T_1 \cos 60° = T_2 - T_1 = 0$

$T_2 = T_1 = 196\,N$

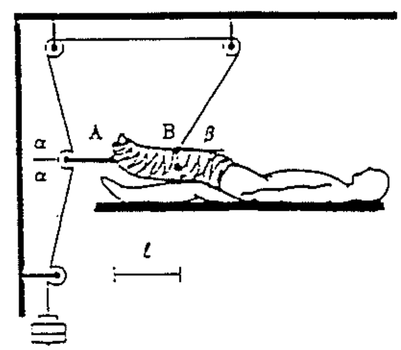
From (1) $\qquad F = T_1 \cos 45° + T_2$

Example 4.3

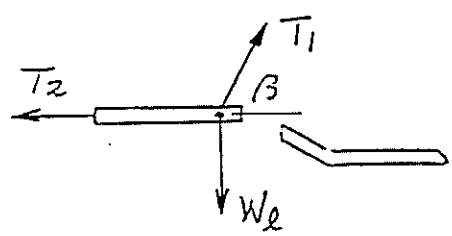
For patient's comfort the traction device can be designed as shown in the figure for minimizing the friction force between the patient's body and the bed. A weight pan is suspended on a long cable that passes over four pulleys. The cable is attached to the leg at B where the COG of the leg is located. A second cable is connected to the shaft of one of the pulleys and to the leg at point A, such that point A and COG of the pulley lie on a horizontal line, and the cable pulled by the pulley has an angle with the horizontal line, $\alpha = 75.5\,°$. The distance between A and the COG of the leg is 0.5 m. The total weight of the leg and the cast is $W_l = 200$ N.

If the friction between the patient and the bed should be minimized to zero, determine (1) the angle (β) between the cable attached to the leg at B and the horizontal line; (2) the weight W of the pan required to maintain the leg in the position shown.

(i). Consider a free body diagram of leg isolated from the patient's body.

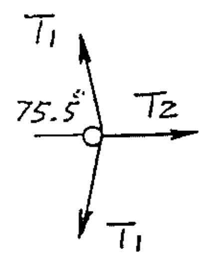


Force	F_x	F_y
W_l	0	$-W_l$
T_1	$T_1 \cos\beta$	$T_1 \sin\beta$
T_2	$-T_2$	0



$\Sigma F_x = 0 \qquad T_1 \cos\beta - T_2 = 0 \qquad (1)$

$\Sigma F_y = 0 \qquad T_1 \sin\beta - W_l = 0 \qquad (2)$



From picture: $\quad T_2 - 2T_1 \cos 75.5^\circ = 0$

$$T_2 = 2T_1 \cos 75.5^\circ = 0.5\,T_1$$

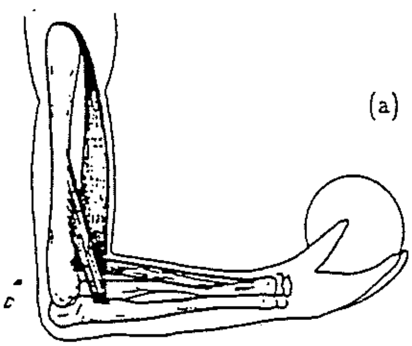
Example 4.4

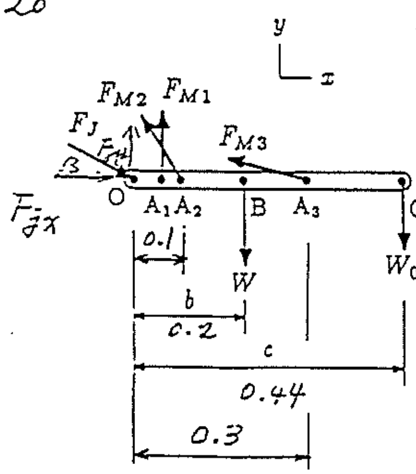
Consider the flexed position of the arm in load lifting as shown in the figure. The forces exerted on the forearm by the biceps, the brachialis, and the brachioradialis muscles are presented by F_{M1}, F_{M2}, F_{M3} with attachments at A_1, A_2, and A_3, respectively. Let θ_1, θ_2, and θ_3 be the angles the biceps, the brachialis, and the brachioradialis muscles make with the long axis of the forearm. Given the following data:

- The weight of the load $W_0 = 100$ N
- The weight of the forearm with hand $W = 15$ N
- The distance between the elbow joint and load, $OC = 0.44$ m
- The distance between the elbow joint and COG, $OB = 0.2$ m
- The distances between elbow joint and muscle attachments at A_1, A_2, and A_3
  $OA_1 = 0.05$ m, $OA_2 = 0.1$ m, $OA_3 = 0.3$ m
- $\theta_1 = 90$, $\theta_2 = 60$, and $\theta_3 = 20$.
- Assume $F_{M1} = 8 F_{M2} = 8 F_{M3}$

(1) Analyze the forces acting on the forearm.
(2) Determine the muscle force and joint reaction forces at elbow joint.

Force	F_x	F_y	T_o
W_o	0	$-W_o$	$-W_o \times 0.44$
W	0	$-W$	$-W \times 0.2$
F_{M1}	0	F_{M1}	$F_{M1} \times 0.05$
F_{M2}	$-F_{M2}\cos 60^\circ$	$F_{M2}\cdot\sin 60^\circ$	$F_{M2} \times 0.1 \times \sin 60^\circ$
F_{M3}	$-F_{M3}\cdot\cos 20^\circ$,	$F_{M3}\sin 30^\circ$,	$F_{M3} \times 0.3 \sin 20^\circ$
F_{jx}	F_{jx}	0	0
F_{jy}	0	F_{jy}	0
or F_j	$F_j \cdot \cos\beta$	$-F_j \sin\beta$	0





$\Sigma F_x = 0.$

$\Sigma F_y = 0$

$\Sigma T_c = 0$

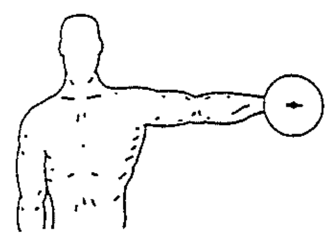
Example 4. 5

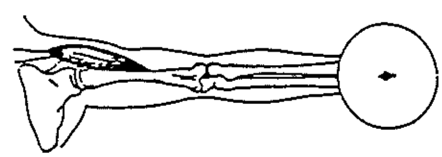
A person is strengthening the shoulder muscles by means of dumbbell exercises. As shown in the figure the arm is abducted to horizontal. O is the axis of the shoulder joint, A is where the deltoid muscle is attached to the humerus, B is the COG of the entire arm, and C is the COG of the dumbbell. F_M is the tension in the deltoid muscle, W is the weight of the arm, and W_o is the weight of the dumbbell. The deltoid muscle force makes an angle θ with the horizontal. With the following data

- The weight of the arm W = 40 N
- The weight of the dumbbell W_o = 60 N
- The distance between the shoulder joint and muscle attachment point A, OA = 0.15 m
- The distance between the shoulder joint and the COG of the arm, OB = 0.3 m
- The distance between the shoulder joint and the COG of dumbbell, OC = 0.6 m
- The angle θ = 15°

Determine (1) the magnitude of the force exerted by the deltoid muscle to hold the arm at the position shown; (2) the magnitude and direction of the reaction force at the shoulder joint.

Force	F_x	F_y	T_o
W_o	0	$-W_o$	$-W_o \times 0.6$
W	0	$-W$	$-W \times 0.3$
F_m	$-F_M \cos 15^\circ$	$F_M \sin 15^\circ$	$F_M \times 0.15 \times \sin 15^\circ$
F_j	$F_j \cos\beta$	$-F_j \sin\beta$	0

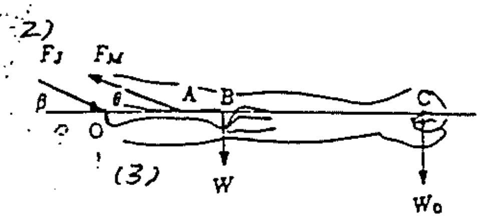




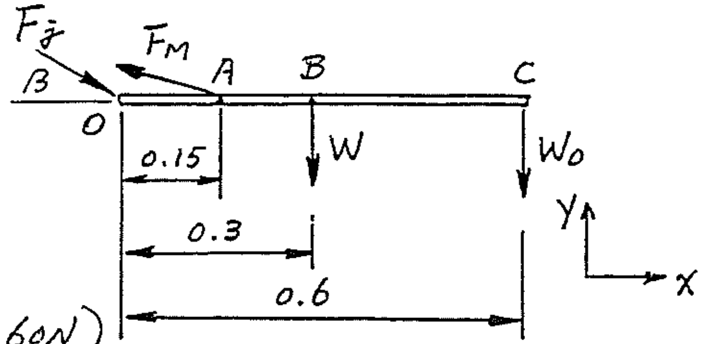
$\Sigma F_x = 0 \quad -F_M \cos 15^\circ + F_j \cos\beta = 0 \quad (1)$

$\Sigma F_y = 0$ (2)

$\Sigma T_o = 0$ (3)



From (3) : F_M



(F_M is 20 times greater than the weight of dumbbell $W_o = 60N$)

LIST OF PRIMARY DYNAMICS FORMULAE FOR ENGR 243

Sherif

$a_{av} = \frac{\Delta V}{\Delta t}$

$a_{inst} = \frac{dv}{dt}$

$V_{av} = \frac{\Delta X}{\Delta t}$

$V_{inst} = \frac{dx}{dt}$

$a = f(t)$

$-V_0 = \int_0^t f(t)\,dt$

$f(x)$

$v^2 - \frac{1}{2}v_0^2 = \int_{x=0}^{x} f(x)\,dx$

$\theta \to X$

$\omega \to V$

$\alpha \to a$

$r = 2\pi \text{ rad} = 360^\circ = 60 \text{ rpm}$

$= A^2 + B^2 - 2AB\cos\theta$

$\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$

$\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$

$= F_t + F_n$

$= m\frac{dv}{dt} + m\frac{v^2}{r}$

$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Kinematics of Particles

$a = v\frac{dv}{dx}$ $\quad V = V_0 + at$ $\quad a = C$ Uniform acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad V = C$, $a = 0$ Uniform velocity

$v^2 = v_0^2 + 2a(x - x_0)$

$x_B = x_A + x_{B/A}$

$v_t = \frac{dr}{dt} \qquad v = \frac{ds}{dt}$

$a = \frac{dv}{dt}e_t + \frac{v^2}{\rho}e_n \qquad a_t = \frac{\Delta V}{\Delta t} \qquad a_n = \frac{V^2}{r}$

$v = \dot{r}e_r + r\dot{\theta}e_\theta$

$a = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})e_\theta$

Kinematics of Rigid Bodies

$v = \frac{dr}{dt} = \omega \times r \qquad \Delta y = r\theta$

$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \alpha \times r + \omega \times (\underbrace{\omega \times r}_{v})$

$\omega = \omega k = \dot{\theta}k$

$\alpha = \alpha k = \dot{\omega}k = \ddot{\theta}k$

$v = \omega k \times r \qquad \underline{v = \omega r}$

$a_t = \alpha k \times r \qquad a_t = r\alpha$

$a_n = -\omega^2 r \qquad a_n = r\omega^2$

$* \; v_B = v_A + \omega \times r_{B/A}$

$a_B = a_A + \alpha \times r_{B/A} + \omega \times (\omega \times r_{B/A})$

$v_P = v'_P + v_{P/F}$

$a_P = a'_P + a_{P/F} + a_C$

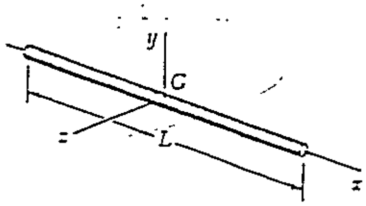
$a_C = 2\Omega \times v_{P/F}$

Kinetics Particles of Particles - F & a

$L = mV$

$\sum F = ma \qquad \sum F = \dot{L}$

Mass Moments of Inertia



Slender rod

$I_y = I_z = \frac{1}{12}mL^2$

$H_0 = r \times mv \qquad \sum M_0 = \dot{H}_0$

$F = G\frac{Mm}{r^2} \qquad GM = gR^2$

$\dot{H}_G = I\dot{\omega} = I\alpha = \sum M_G$

Kinetics of Particles - Energy Methods

$dU = F \cdot dr \qquad U_{1\to2} = \int_{A_1}^{A_2} F \cdot dr$

$U_{1\to2} = -\int_{y_1}^{y_2} W\,dy = Wy_1 - Wy_2$

$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

$U_{1\to2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$

$T_1 + U_{1\to2} = T_2$

$\text{Power} = \frac{dU}{dt} = F \cdot v$

$U_{1\to2} = V_1 - V_2 \qquad T_1 + V_1 = T_2 + V_2$

$mv_1 + \int_{t_1}^{t_2} F\,dt = mv_2$

$v'_B - v'_A = e(v_A - v_B)$

Kinetics of Rigid Bodies - F & a

$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y$

$\sum M_G = \bar{I}\alpha$

Kinetics of Rigid Bodies - Energy Methods

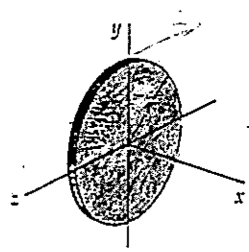
$U_{1\to2} = \int_{\theta_1}^{\theta_2} M\,d\theta = Fr\theta \qquad y = r\theta$

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$\text{Power} = \frac{dU}{dt} = \frac{M\,d\theta}{dt} = M\omega$

$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$

$I = k^2 m$



Thin disk

$I_z = \frac{1}{2}mr^2$

$I_x = I_z = \frac{1}{4}mr^2$

LIST OF PRIMARY DYNAMICS FORMULAE FOR ENGR 243

$\lambda_{av} = \frac{\Delta V}{\Delta t}$

$\lambda_{inst} = \frac{dv}{dt}$

$\lambda_{av} = \frac{\Delta X}{\Delta t}$

$\lambda_{inst} = \frac{dx}{dt}$

$\lambda = f(t)$

$-V_0 = \int_0^t f(t)\,dt$

$f(x)$

$\lambda^2 - \frac{1}{2}V_0^2 = \int_{x=0}^{x} f(x)\,dx$

$g \to X$

$J \to V$

$C \to a$

$r = 2\pi$ rad $= 360° = 60$ rpm

$= A^2 + B^2 - 2AB\cos\theta$

$\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$

$\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$

$= F_t + F_n$

$= m\frac{dv}{dt} + m\frac{v^2}{r}$

$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Kinematics of Particles

$a = v\frac{dv}{dx}$ $\quad V = V_0 \pm at \quad a = c$ Uniform acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad V = C,\ a = 0$ Uniform velocity

$v^2 = v_0^2 + 2a(x - x_0)$

$x_B = x_A + x_{B/A}$

$v_t = \frac{dr}{dt} \qquad v = \frac{ds}{dt}$

$a = \frac{dv}{dt}e_t + \frac{v^2}{\rho}e_n \qquad a_t = \frac{\Delta V}{\Delta t} \quad a_n = \frac{V^2}{r}$

$v = \dot{r}e_r + r\dot{\theta}e_\theta$

$a = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})e_\theta$

Kinematics of Rigid Bodies

$v = \frac{dr}{dt} = \omega \times r \qquad \Delta y = r\theta$

$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \alpha \times r + \omega \times \underbrace{(\omega \times r)}_{v}$

$\omega = \omega k = \dot{\theta}k$

$\alpha = \alpha k = \dot{\omega}k = \ddot{\theta}k$

$v = \omega k \times r \qquad v = \omega r$

$a_t = \alpha k \times r \qquad a_t = r\alpha$

$a_n = -\omega^2 r \qquad a_n = r\omega^2$

$* \; v_B = v_A + \omega \times r_{B/A}$

$a_B = a_A + \alpha \times r_{B/A} + \omega \times (\omega \times r_{B/A})$

$v_P = v'_P + v_{P/F}$

$a_P = a'_P + a_{P/F} + a_C$

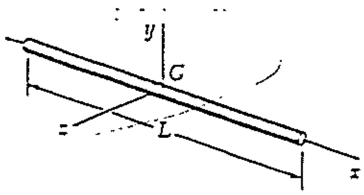
$a_C = 2\Omega \times v_{P/F}$

Kinetics Particles of Particles - F & a

$L = mv$

$\sum F = ma \qquad \sum F = \dot{L}$

Mass Moments of Inertia



Slender rod

$I_y = I_z = \frac{1}{12}mL^2$

$H_0 = r \times mv \qquad \sum M_0 = \dot{H}_0$

$F = G\frac{Mm}{r^2} \qquad GM = gR^2$

$H_G = I\dot{\omega} = I\alpha = \Sigma M_G$

Kinetics of Particles - Energy Methods

$dU = F \cdot dr \qquad U_{1\to2} = \int_{A_1}^{A_2} F \cdot dr$

$U_{1\to2} = -\int_{y_1}^{y_2} W\,dy = Wy_1 - Wy_2$

$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

$U_{1\to2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$

$T_1 + U_{1\to2} = T_2$

$\text{Power} = \frac{dU}{dt} = F \cdot v$

$U_{1\to2} = V_1 - V_2 \qquad T_1 + V_1 = T_2 + V_2$

$mv_1 + \int_{t_1}^{t_2} F\,dt = mv_2$

$v'_B - v'_A = e(v_A - v_B)$

Kinetics of Rigid Bodies - F & a

$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y$

$\sum M_G = \bar{I}\alpha$

Kinetics of Rigid Bodies - Energy Methods

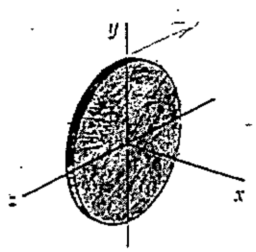
$U_{1\to2} = \int_{\theta_1}^{\theta_2} M\,d\theta = Fr\theta \qquad y = r\theta$

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$\text{Power} = \frac{dU}{dt} = \frac{M\,d\theta}{dt} = M\omega$

$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$

$I = k^2 m$



Thin disk

$I_x = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$

LIST OF PRIMARY DYNAMICS FORMULAE FOR ENGR 243

$a_{av} = \frac{\Delta V}{\Delta t}$

$a_{inst} = \frac{dv}{dt}$

$V_{av} = \frac{\Delta X}{\Delta t}$

$V_{inst} = \frac{dx}{dt}$

$a = f(t)$

$v - V_0 = \int_0^t f(t)\, dt$

$= f(x)$

$v^2 - \frac{1}{2}V_0^2 = \int_{x=0}^{x} f(x)\, dx$

$\theta \to X$

$\omega \to V$

$\alpha \to a$

$r = 2\pi \text{ rad} = 360^\circ = 60 \text{ rpm}$

$= A^2 + B^2 - 2AB\cos\theta$

$\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$

$\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$

$= F_t + F_n$

$= m\frac{dv}{dt} + m\frac{v^2}{r}$

$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Kinematics of Particles

$a = v\frac{dv}{dx}$ $\quad V = V_0 + at$ $\quad a = c$ Uniform acceleration

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad V = C, a = 0$ Uniform velocity

$v^2 = v_0^2 + 2a(x - x_0)$

$x_B = x_A + x_{B/A}$

$v_t = \frac{dr}{dt} \qquad v = \frac{ds}{dt}$

$a = \frac{dv}{dt}e_t + \frac{v^2}{\rho}e_n \qquad a_t = \frac{\Delta V}{\Delta t} \qquad a_n = \frac{V^2}{r}$

$v = \dot{r}e_r + r\dot{\theta}e_\theta$

$a = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})e_\theta$

Kinematics of Rigid Bodies

$v = \frac{dr}{dt} = \omega \times r \qquad \Delta y = r\theta$

$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \alpha \times r + \omega \times (\omega \times r)$

$\omega = \omega k = \dot{\theta}k$

$\alpha = \alpha k = \dot{\omega}k = \ddot{\theta}k$

$v = \omega k \times r \qquad v = \omega r$

$a_t = \alpha k \times r \qquad a_t = r\alpha$

$a_n = -\omega^2 r \qquad a_n = r\omega^2$

$* \; v_B = v_A + \omega \times r_{B/A}$

$a_B = a_A + \alpha \times r_{B/A} + \omega \times (\omega \times r_{B/A})$

$v_P = v'_P + v_{P/F}$

$a_P = a'_P + a_{P/F} + a_C$

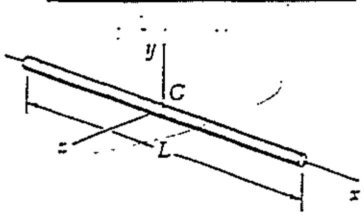
$a_C = 2\Omega \times v_{P/F}$

Kinetics Particles of Particles - F & a

$L = mV$

$\sum F = ma \qquad \sum F = \dot{L}$

Mass Moments of Inertia



Slender rod

$I_y = I_z = \frac{1}{12}mL^2$

$H_0 = r \times mv \qquad \sum M_0 = \dot{H}_0$

$F = G\frac{Mm}{r^2} \qquad GM = gR^2$

$\dot{H}_G = I\dot{\omega} = I\alpha = \sum M_G$

Kinetics of Particles - Energy Methods

$dU = F \cdot dr \qquad U_{1\to2} = \int_{A_1}^{A_2} F \cdot dr$

$U_{1\to2} = -\int_{y_1}^{y_2} W dy = Wy_1 - Wy_2$

$U_{1\to2} = -\int_{x_1}^{x_2} kx dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

$U_{1\to2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2} dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$

$T_1 + U_{1\to2} = T_2$

$\text{Power} = \frac{dU}{dt} = F \cdot v$

$U_{1\to2} = V_1 - V_2 \qquad T_1 + V_1 = T_2 + V_2$

$mv_1 + \int_{t_1}^{t_2} F dt = mv_2$

$v'_B - v'_A = e(v_A - v_B)$

Kinetics of Rigid Bodies - F & a

$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y$

$\sum M_G = \bar{I}\alpha$

Kinetics of Rigid Bodies - Energy Methods

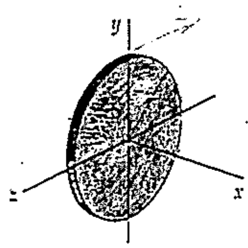
$U_{1\to2} = \int_{\theta_1}^{\theta_2} M d\theta = Fr\theta \qquad y = r\theta$

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$\text{Power} = \frac{dU}{dt} = \frac{Md\theta}{dt} = M\omega$

$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$

$I = k^2 m$



Thin disk

$I_z = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$

Solutions

Question I

(1) To determine how much energy will be used to bring outdoor air at -5°C/80%rh to indoor conditions of 23°C, 40% rh, the amount of air to be conditioned has to be determined first. Given the ventilation rate of 1/3 air change per hour, the volume of the ventilated air is:

$$V = 2(6 \times 15 \times 2.5) \times 1/3 = 150\ m^3/hour$$

The density of the dry air at indoor conditions of 23°C, 40% rh

$$\rho_a = \frac{P_a}{R_a \cdot T}$$ (refer to chapter 6, "Building science")

$$P_a = P_t - P_w$$

where, P_t — atmospheric pressure, assume $P_t = 101300\ Pa$

P_w — pressure of water vapor $P_w = P_{s(23°C)} \times RH$

From Table 5.1 (Building Science book)

at $T = 23°C$, $P_s = 2.809\ kPa$; then

$$P_a = P_t - P_w = 101300 - 2809 \times 40\% = 100176.4\ Pa$$

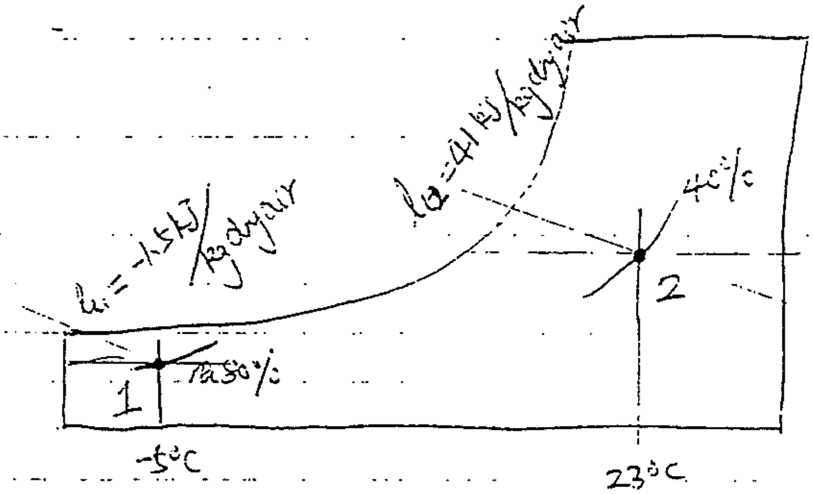
Therefore, the indoor air density:

$$\rho_a = \frac{100176.4}{287.1 \times (273+23)} = 1.1788\ kg/m^3$$

The amount of dry air to be conditioned shall be:

$$M_a = V \cdot \rho_a = 150 \times 1.1788 = 176.82\ kg/hour$$

From the Psychrometric Chart (shown in the following figure), the enthalpy of moist air at outdoor and indoor condition is:



$h_1 = -1.5$ kJ/kg dry air $h_2 = 41$ kJ/kg dry air

Then the energy needed shall be

$$Q = M_a(h_2 - h_1) = 176.82\ \text{kg/hour}\ (41 + 1.5)\ \text{kJ/kg} = 7472.4\ \text{kJ/hour}$$
$$= 2.08\ \text{kW}$$

(2) The amount of water to be evaporated:

$$W = \frac{0.622\, P_w}{P_t - P_w}$$ (equation 6.6 in "Building Science" Book)

From Table 5.1, $P_s = 401.5\ Pa$ at $t = -5°C$

$P_s = 2809\ Pa$ at $t = 23°C$

Then,

$$\omega_1 = \frac{0.622\, P_{s_1} \times Rh_1}{P_t - P_{s_1} \times Rh_1} = \frac{0.622 \times 401.5 \times 80\%}{101300 - 401.5 \times 80\%} = 1.978 \times 10^{-3}\ \text{kg/kg dry}$$

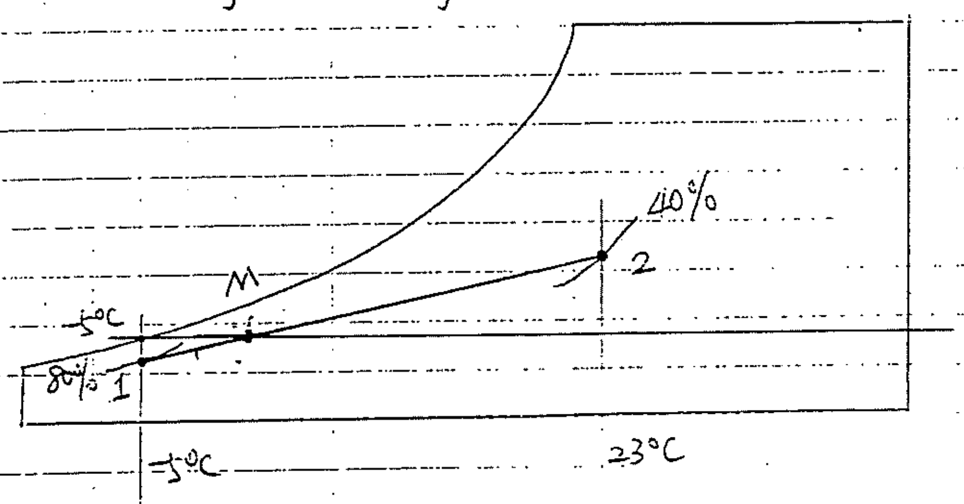
$$\omega_2 = \frac{0.622\, P_{s_2} \times Rh_2}{P_t - P_{s_2} \times Rh_2} = \frac{0.622 \times 2809 \times 40\%}{101300 - 2809 \times 40\%} = 6.976 \times 10^{-3}\ \text{kg/kg dry air}$$

$$W = M_a \cdot (\omega_2 - \omega_1) = 176.82\ \text{kg/hour} * (6.976 - 1.978) \times 10^{-3} \times 24\ \text{hour/day}$$

Therefore, about 21 Liters water would have to be evaporated.

Notes: In solving this question, it should be noticed that all of the values to be obtained from the psychrometric chart are based on unit kg dry air. So, the amount of the dry air should be determined. The moist air is composed of dry air and water vapour.

2. The key to avoid the condensation on the sheathing is to ensure that the dew point temperature of the mixed air should not be higher than -5°C (Assume the shealthing temperature is at -5°C). This question can be solved directly on the psychrometric Chart as follows:



(1) find the outdoor condition and indoor condition on the Psychrometric chart. point 1 for outdoor and point 2 for indoor

(2) Draw a straight line to connect points 1 and 2.

(3) Find the intersection between line $\overline{12}$ and the horizontal line which represents (cross) dew point of -5°C; point M. M shall be the condition after mixing between indoor air and outdoor air

(4) Since the final enthalpy and moisture content following mixing are the weighted average of the values for the two conditions 1, and 2, the state point for the mixture will be found on the straight line joining the two conditions and at a position dividing the distance between them in inverse proportion to the mass at each condition. Therefore,

$$\frac{m_1}{m_2} = \frac{\overline{M2}}{\overline{M1}} = \frac{52}{14} = 3.71$$

The amount of outdoor air should be at least 3.71 times of the indoor air in order that the mixture would not condense on the sheathing.

3.(1) The properties of materials used can be found from Table 8.1 In Building Science Book.

Element	Conductance (C)	Resistance (R)	Temp. diff. (K)	Temperature (°C)
				−10.3
Outdoor surf. f_o	34	0.029	0.35	
				−9.95
Face brick 100mm 1.32/0.1	13.2	0.0758	0.91	
				−9.04
Air Space 25mm	4.8	0.208	2.51	
Building Paper				−6.53
psum sheathing	12.5	0.08	0.96	
				−5.57
glass fiber 90mm	0.52	1.923	23.16	
				17.59
Gypsum Board 12.5mm	12.5	0.08	0.96	
				18.55
Inside surface f_i	8.3	0.1205	1.45	
		$\Sigma R = 2.516$	$\Sigma T = 30.3°C$	20

(2) The solar radiation should be considered in the calculation. From Table 9.7 In Building Science Book, for a light-colored wall facing west, the sol-air temperature is maximum at 16:00, which is $41/2 = 20.5°C$ above the outdoor air temperature. To repeat

the calculation in question (1), then

lement	Conductance (C)	Resistance (R)	Temp. diff. (K)	Temperature (°C)
				46.5°C
Outdoor surf. fo	17	0.059	0.63	
				45.9
Face Brick 100mm, 1.32/0.1	13.2	0.0758	0.81	
				45.1
Air space 25mm	6.8	0.147	1.57	
Building Paper				
				43.5
Gypsum sheathing 12.5mm	12.5	0.08	0.85	
				42.7
Glass fiber 90mm	0.52	1.923	20.51	
				22.2
Gypsum Board 12.5mm	12.5	0.08	0.85	
				21.3
Inside surf. fi	8.3	0.1205	1.285	20.0
		$\Sigma R = 2.485$	$\Sigma \Delta T = 26.5°C$	

4. To avoid condensation on the wall sheathing or on the back of the brick veneer due to air exfiltration, the surface temperature on sheathing or on the brick veneer shall be above the dew point of the indoor air.

(1) The calculation in question 3 showed that the temperature of the interior surface of the sheathing is $-5.6°C$. Using psychrometric chart to find the relative humidity which corresponding to dew point temp. of $-5.6°C$ and dry bulb temp of $20°C$, which gives about 16%. To obtain more precise result, calculation can be done as following:

From Table 5.1, the $P_s = 381.7\ Pa$ at $t = -5.6°C$;

$P_s = 2337\ Pa$ at $t = 20°C$;

Then the relative humidity $RH = \frac{381.7}{2337} \times 100\% = 16.3\%$

(2) Same procedure applies for the brick veneer. The temp. calculated in Question 3 for brick veneer is $-9.0°C$.

From Table 5.1, $P_s = 283.7\ Pa$ at $t = -9°C$

$$RH = \frac{283.7}{2337} \times 100\% = 12.1\%$$

The relative humidity of the indoor air should be below 12.1% in order to avoid condensation on the brick veneer due to exfiltration

(3) To avoid condensation on the sheathing due to exfiltration when the indoor air at 20°C and 50% r.h. the surface temp of the interior sheathing shall be above the dew point of the indoor air, which is 9°C according to the psychrometric chart.

To elevate the sheathing temperature from 5.6°C to 9°C a certain thickness of exterior insulation should be added. Since the temp. gradient across the wall assembly is proportional to the thermal resistance of that specific material, then assume the total thermal resistance after adding exterior insulation as R_t, the thermal resistance from interior surface until the interior sheathing is $= R_{f_i} + R_{gypsum} + R_{glassfiber}$

$$= 0.1205 + 0.08 + 1.923$$
$$= 2.1235 \ m^2 \cdot °C/W$$

Then:

$$\frac{R_{f_i \to glassfiber}}{R_t} = \frac{20-9}{20-(-10.3)}$$

$$R_t = \frac{30.3 * 2.1235}{20-9} = 5.849 \ m^2 \cdot °C/W$$

Before adding the exterior insulation $R_{total} = 2.516 \ m^2 \cdot °C/W$ (from 3(1))

Thus, the thermal resistance of the exterior insulation $R_{add} = R_t - R_{total}$

$$= 5.849 - 2.516$$
$$= 3.333 \ m^2 \cdot °C/W$$

The thickness should be

$$l = k * R = 0.04 \times 3.333 = 133 mm$$

5. To determine the vapor pressure distribution across the assembly and the relative humidity at each interface, the temperature distribution shall be determined first. Follow the same procedure used in Question 3 and the results are shown below.

Element	C_i ($W/m^2 \cdot °C$)	R_i ($m^2 \cdot °C/W$)	ΔT (°C)	T_i (°C)	$P_{v.sat.}$
				21.0	2486
Interior	8.3	0.1205	(0.36) 0.64 (1.4)		
				20.36	2396
Concrete 0.15/	k=1.8 l=0.15 12.0	0.083	(0.25) 0.44 (0.96)		
				19.92	2323
Type 4 xps	k=0.029 l=0.075 0.39	2.564	(7.61) 13.52 (29.6)		
				6.4	961
Air space	5.9	0.169	(0.5) 0.89 (1.95)		
				5.51	903
Brick	k=1.32 l=0.09 14.7	0.068	(0.2) 0.36 (0.79)		
				5.15	881
Exterior	17 34.0	0.0588 0.029	(0.08) 0.15 (0.33)		
		$\Sigma R = 3.034$	$\Sigma T = 16°C$	5°C	871.9

~~$\Sigma R = 3.063$~~

$\Delta T = 35°C$

$\Delta T = (30-21) = 9°C$

Vapor pressure distribution: the vapor resistance of interior and exterior surfaces are ignored. The vapor permeability of still air is 185ng/Pa·s·m.

Element	μ (ng/Pa·s·m)	l (m)	M (ng/Pa·s·m^2)	R (Pa·s·m^2/ng)	ΔP_v (Pa)	P_v (Pa)	$P_{v,sat}$ (Pa)	RH (%)
						1243	2486	50
Concrete	2.6	0.15	17.3	0.0577	302.6			
						940.4	2323	40.5
Type 4 XPS	2.0	0.075	26.7	0.0375	196.7			
						743.7	961	77.4
Air Space	185	0.025	7400	0.00014	0.734			
						743	903	82.3
Brick	10.0	0.090	111.1	0.0090	47.21			
Exterior				$\Sigma R = 0.104$	$\Sigma \Delta p =$ 545.5	697.5	881	79.2

From the above table, it shows that the vapor pressure at the interface is below the saturation pressure, therefore, no condensation will form through the assembly under this conditions.

2) when outdoor condition is -14°C 80% r.h. repeat the procedure used in question (1). The results are shown below.

Element	R ($Pa \cdot s \cdot m^2/kg$)	ΔP (Pa)	P_v (Pa)	T_i (°C)	$P_{v,sat}$ (Pa)	P_v' (Pa)	RH(%)
			1243	21.0	2486	1243	50%
Interior							
				19.6	2280.6		
Concrete	0.0577	609.2				Δp=631.0	
			633.8	18.64	2142.8	612	28.6
Type 4 XPS	0.0375	395.9				Δp=410.3	
			237.9	-10.96	237.6	201.7	84.8
Air Space	0.00014	1.478				Δp=1.53	
			236.4	-12.91	200.3	200.2	100
Brick	0.0090	95.03			Σp=1042.7 Σc=0.0753		
			141.4	-13.7	193.2		
Exterior	ΣR=0.104	$\Sigma \Delta P$=1098.1		-14	181.1	144.9	80

From the calculation, it showed that at the back of brick, the vapor pressure is above the saturation pressure, where condensation would occur. Therefore a re-calculation is needed and results are shown as "P_v'" in the Table shown above.

The amount of condensation on the back of the brick:

The vapor flow from interior surface to the airspace

$$W_1 = \frac{\Delta P_1}{R_1} = \frac{1243-200.3}{(0.0577+0.0375+0.00014)} = 10941.2\ ng/s \cdot m^2$$

The vapor flow from the interior surface of the brick to the exterior,

$$W_2 = \frac{\Delta P_2}{R_2} = \frac{200.3-144.9}{0.0090} = 6155.6\ ng/s \cdot m^2$$

Thus, the condensation rate is:

$$\Delta W = W_1 - W_2 = 10941.2 - 6155.6 = 4785.6\ ng/s \cdot m^2$$

3) When the outdoor condition is 30°C, 80% rh, calculate the heat flow to determine the temperature distribution first, and then calculate the pressure distribution to ensure that no condensation is formed. The results are listed as below

Element	R ($Pa \cdot s \cdot m^2/ng$)	ΔP	P_v (Pa)	T_i (°C)	$P_{v,sat}$ (Pa)	RH (%)
			1243	21	2486	50
Interior						
				21.36	2548.8	
Concrete	0.0577	1193.6				
			2436.6	21.61	2580.2	94.4
Type4 XPS	0.0375	775.7				
			3212.3	29.22	4058.1	79.2
Air Space	0.00014	2.896				
			3215.2	29.72	4176.6	77.0
Brick	0.009	186.2				
			3401.4	29.92	4224	80.5
	$\Sigma R = 0.104$	$\Sigma \Delta P = 2151.4$	3394.4	30	4243	80

4)

Repeat the calculation and consider the vapor resistance of the vinyl paper, the results are shown as below

Element	R ($Pa \cdot s \cdot m^2/ng$)	ΔP (Pa)	P_v (Pa)	T_i (°C)	$P_{v,sat}$ (Pa)	P_v' (Pa)	RH(%)
			1243	21	2486		50
Interior							
				21.36			
vinyl wall paper	0.082	946.9					
			2189.9	21.36	2548.8		
concrete	0.0577	666.3					
			2856.2	21.61	2580.2		
Type 4 XPS	0.0375	433.1					
			3289.3	29.22	4058.1		
Airspace	0.00014	1.617					
			3290.9	29.72	4176.6		
Brick	0.009	103.9					
			3392.9	29.92	4224		
	$\Sigma R = 0.863$	$\Sigma \Delta P = 2151.4$	3394.4	30.0	4243		80

From the calculation, it can be seen that at the exterior surface of the concrete, the vapor pressure is greater than the corresponding saturation vapor pressure at the temp., so there will be condensation to be formed at this surface. This calculation shows that in summer time

Condensation can formed on the exterior surface of some layer of the wall assemblies, especially when a low permeable material is installed on the interior. Therefore, in hot and humid climate vapor retardar shall be installed on the exterior side.

6. When the warm and humid air exfiltrates through the wall assembly, it condenses on the surfaces where the temperature is lower than the dewpoint of the indoor air (10°c). Since the outdoor air is at conditions of -5°c, 80% rh, the warm and humid air will be cooled and dehumidified until it reaches outdoor condition during the exfiltration process. Therefore the amount of condensation is the difference of the humidity ratio between indoor condition state and outdoor state.

From the Psychrometric chart, the humidity ratio at indoor condition

$W_1 = 0.0078\ kg/kg\ dry\ air$ at 21°c, 50% rh

$W_2 = 0.00198\ kg/kg\ dry\ air$ at -5°c 80% rh

The air density at indoor condition,

$$\rho_a = \frac{P_a}{R_a \cdot T} = \frac{101300 - 2486 \times 50\%}{287(273+21)} = 1.186\ kg/m^3$$

Therefore, the amount of the condensation

$$W = \rho_a \cdot V_a \cdot (W_1 - W_2) = 1.186 \times 0.5 \times (0.0078 - 0.00198) \times 10^{-3}$$
$$= 3.45 \times 10^{-3}\ g/s \cdot m^2$$

7. The air leakage rate under a certain pressure difference can be
1) calculated using the following equation:

$$Q = C \cdot A \cdot (\Delta p)^n$$

C — coefficient, A — leakage area, Δp — pressure difference (Pa)

n — normally 0.5 ~ 1.0 for buildings, n = 0.65 for window cracks.

From the above equation, it can be shown that the pressure distribution across building assembly is directly linked to the leakage areas.

$$\frac{A_1}{A_2} = \left(\frac{\Delta p_2}{\Delta p_1}\right)^n \quad \text{in case of window } n = 0.65$$

Therefore, in this question known that the inner sash is twice as tight as the outer sash, which can be denoted as $A_1 = 2A_2$ (Suppose A_2 is the leakage area of the inner sash, A_1 is the leakage area of the outer sash)

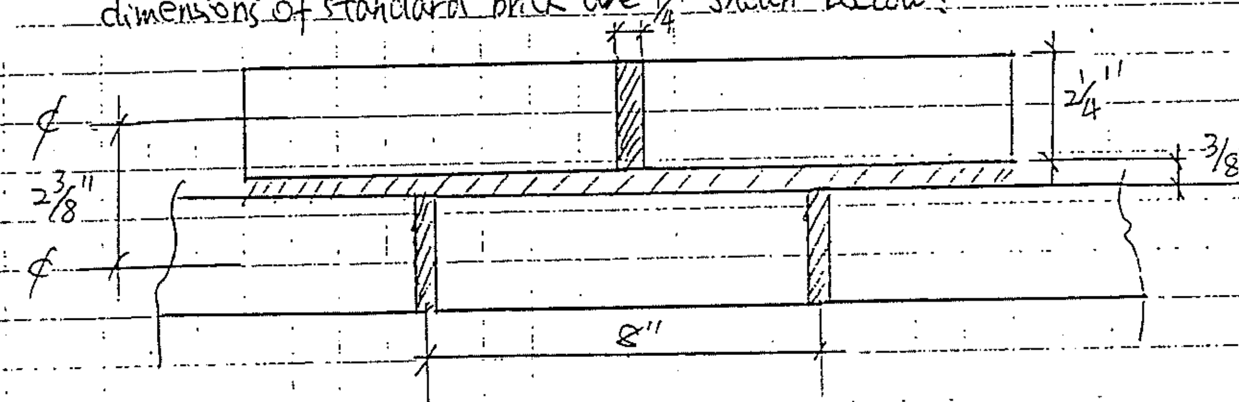
Then, $\left(\frac{\Delta p_2}{\Delta p_1}\right)^{0.65} = 2 \Rightarrow \frac{\Delta p_2}{\Delta p_1} = 2.9 \Rightarrow \frac{\Delta p_2}{\Delta p_2 + \Delta p_1} = 74.4\%$

$$\frac{\Delta p_1}{\Delta p_2 + \Delta p_1} = 25.6\%$$

Therefore, the inner sash would take 74.4% of the overall pressure difference exerted across the window, and the outer sash would take 25.5% of the overall pressure difference.

2) If the overall pressure is 500Pa, then the pressure difference across the outer sash would be: $500 \times 25.6\% = 128Pa$. Therefore, an upstand shall be at least 12.8mm high to prevent water from crossing the wall.

8. The same principle used in question 7 applies to this problem as well. 50mm head of water equals to 500 Pa. The overall pressure difference is 1000 Pa, if the maximum pressure difference across the outer wythe is 500 Pa, then the inner wythe will take 500 Pa, which is the same as the outer wythe. In this case, the inner wythe shall have at least the same air tightness as the outer wythe or tighter. So, the question is how to calculate the ventilation area in the exterior brick veneer. The dimensions of standard brick are shown below:



Suppose the height of one floor is 3m, then in every meter, there are $4/3 = 1.33$ open vertical mortar joint. The ventilation area is:

$$\frac{4/3 \times 1/4" \times 2\,3/8"}{3 \times 1} = 6.7\, mm^2/m^2.$$

Therefore, the leakage area per square meter shall be less than $6.7 mm^2/m^2$ in the inner wythe in order to limit the head of water in the drained space to 50mm.

9. The key to answer this question is to estimate the maximum temperature differential the wall experiences.

The 1% winter design temperature can be chosen as the minimum temp. of the aluminum curtain wall. From ASHRAE Fundamental, chapter 26, Table 2A. for Montreal, the 1% winter design temperature is $-24°C$.

The 1% summer design temperature can be chosen as the maximum temp but the solar radiation will raise the surface temperature to a much higher value. Therefore, the sol-air temp. shall be used. From Table 9.7 in "Building Science" book, for a dark-colored surface, the maximum sol-air temp in July for a west-facing wall can be as high as:

$$27.2 + 41 = 68.2°C$$

Therefore, the temp change from summer to winter could be as high as $(68.2 + 24) = 92.2°C$.

(1) The maximum movement, if the vertical mullions extend the full height of the building, would be:

$$3.3 \times 20 \times 23.5 \times 10^{-6} \times 92.2 = 0.143 \text{ (m)} = 143 \text{ mm}$$

(2) If the mullion is separately supported on each floor, the movement would be

$$3.3 \times 23.5 \times 10^{-6} \times 92.2 = 7.15 \text{ mm}.$$

7.15mm is the minimum allowance that must be left between panels installed at the midwinter low temperature to avoid contact under the peak conditions in summer. less allowance will be required if they were installed at some other time.

10. The maximum temperature change for a light-colored precast concrete panel will be:

$$27.2 + 41/2 + 24 = 71.7^\circ C$$

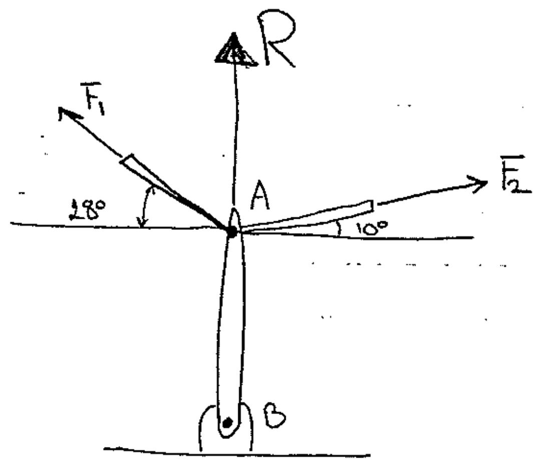
The maximum movement:

$$71.7 \times 11.7 \times 10^{-6} \times 10 = 8.4\,mm$$

If a sealant having a movement capability of plus or minus 25% was use, then the sealant can take a total movement of 50% of the joint width when the sealant is installed under temperature conditions midway between the maximum and the minimum. The minimum joint width allowable would be.

$$8.4/50\% = 16.8\,mm$$

2.5)



$F_1 = 120\,KN$

a)

$\Sigma F_x = 0 \quad -F_1 \cos 28^\circ + F_2 \cos 10^\circ$

$0 = -105.95 + F_2(0.98481)$

$\Longrightarrow F_2 = 107.58$

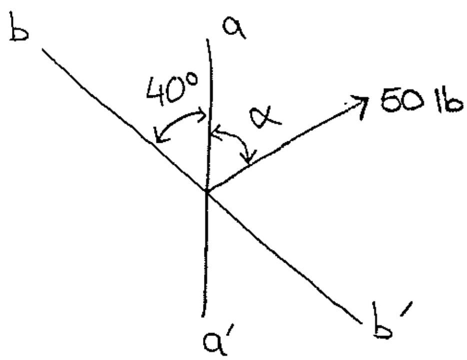
b) $R = F_1 + F_2$

$= \cancel{F_{1x}} + F_{1y} + \cancel{F_{2x}} + F_{2y}$

$= 120 \sin 28^\circ + 107.58 \sin 10^\circ$

$= \underline{75\,KN}$

2.7)



a-a' component = 35 lb

$F_{aa'} = 35$

$\frac{\sin\beta}{35} = \frac{\sin 40}{50}$

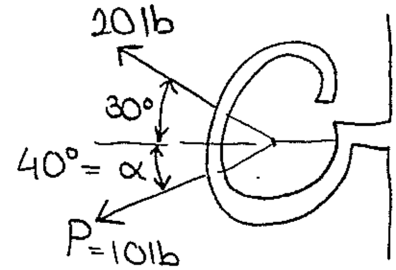
$\Longrightarrow \sin\beta = 0.45$

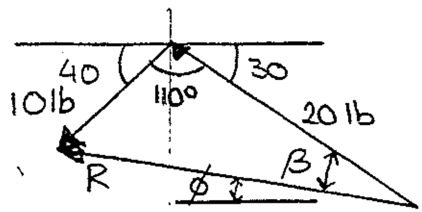
$\Longrightarrow \beta = 26.74^\circ$

$\frac{F_{bb'}}{\sin\alpha} = \frac{50}{\sin 40}$

$\Longrightarrow \underline{\alpha = 113.26^\circ} \Longrightarrow \underline{F_{bb'} = 71.5\,lb}$

2.15



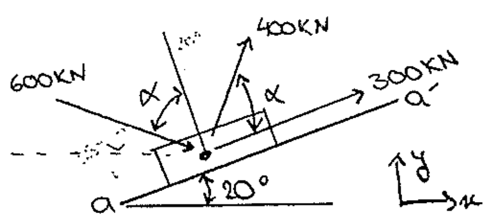


$$R^2 = (10)^2 + (20)^2 - 2(10)(20)\cos 110^\circ$$
$$R^2 = 636.81 \implies R = 25.24 \text{ lb}$$

$$\frac{10}{\sin\beta} = \frac{25.24}{\sin 110} \implies \sin\beta = 0.37 \implies \beta = 21.86^\circ$$

Two parallel lines : $\phi + \beta = 30^\circ \implies \phi = 8.14^\circ$

2.35



$\alpha = 35^\circ$

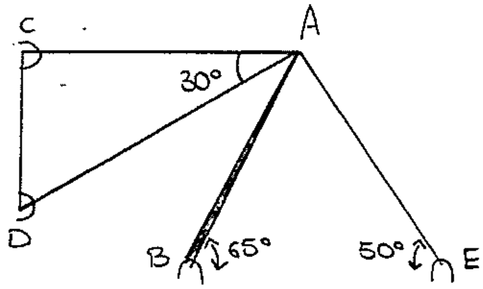
$$\vec{R} = \vec{R_x} + \vec{R_y}$$

$$R_x = 300\cos 20^\circ + 400\cos 55^\circ + 600\cos 35^\circ = 1002.83 \text{ KN}$$
$$R_y = 300\sin 20^\circ + 400\sin 55^\circ - 600\sin 35^\circ = 86.12 \text{ KN}$$

$$\implies R = \sqrt{R_x^2 + R_y^2} = 1006.52$$

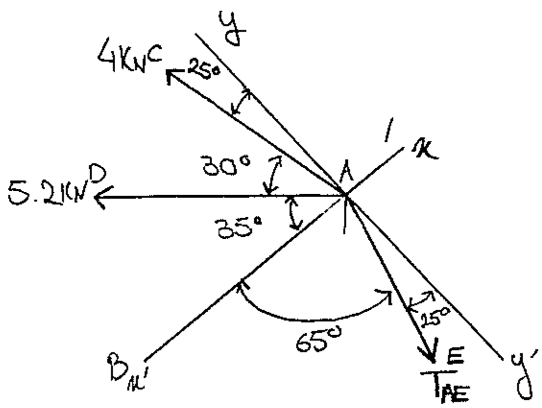
$$\tan\alpha = \frac{86.12}{1002.83} \implies \alpha = 4.91^\circ$$

2.41



$T_{AC} = 4$ KN
$T_{AD} = 5.2$ KN

$T_{AE} = ?$



$R_y = 0$

$$4 \cos 25 + 5.2 \cos 55 = T_{AE} \cos 25°$$

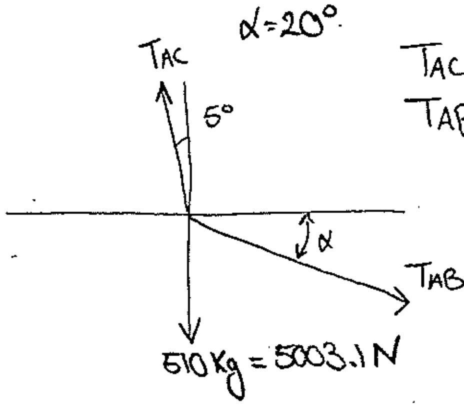
$$\Rightarrow T_{AE} = 7.29 \text{ KN}$$

$$R_x = 4 \sin 25 + 5.2 \sin 55 + 7.29 \sin 25$$

$$= 9.03 \text{ KN}$$

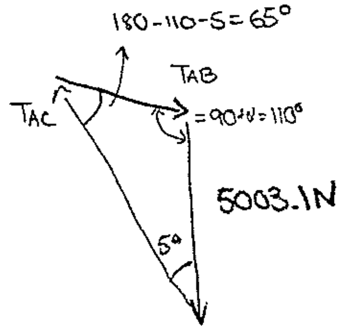
see ya next week!

2.45)



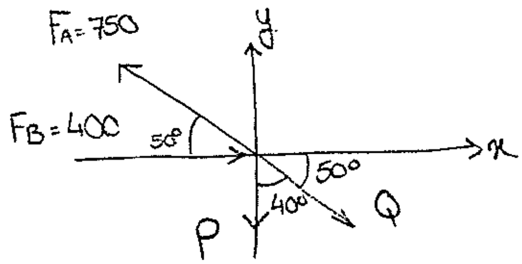
$T_{AC} = ?$

$T_{AB} = ?$



$$\frac{T_{AC}}{\sin 110°} = \frac{T_{AB}}{\sin 5°} = \frac{5003.1}{\sin 65°} \Longrightarrow \begin{cases} T_{AC} = 5187.39\,N \\ T_{AB} = 481.13\,N \end{cases}$$

2.52)



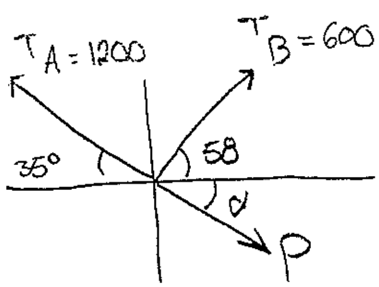
$$\Sigma F_x = 0 : -750\cos 50° + 400 + Q\cos 50° = 0$$

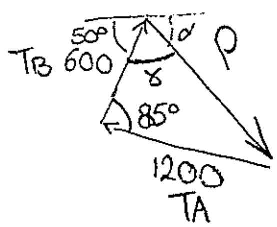
$$\Rightarrow Q = 127.71$$

$$\Sigma F_y = 0 : \; 750 \sin 50 - Q \sin 50 - P = 0$$

$$\Rightarrow P = 476.70$$

2.58)



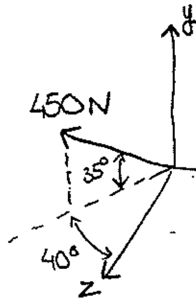


$$P^2 = 600^2 + 1200^2 - 2(600)(1200)\cos 85° \Longrightarrow P = 1294$$

$$\frac{\sin\gamma}{1200} = \frac{\sin 85}{1294} \rightarrow \gamma = 67.50°$$

$$\gamma + 50 + \alpha = 180° \Longrightarrow \alpha = 62.5°$$

2.72)



$F_x = -450\ \cos 35° \sin 40° = -236.94\ N$

$F_y = +450\ \sin 35° = 258.11\ N$

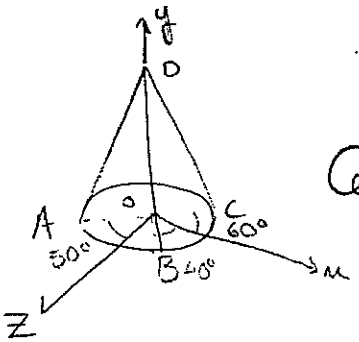
$F_z = +450\ \cos 35° \cos 40° = 282.38\ N$

$\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-236.44}{450} = -0.5254 \longrightarrow \theta_x = 121.69°$

$\cos\theta_y = \dfrac{F_y}{F} = \dfrac{258.11}{450} = 0.5736 \longrightarrow \theta_y = 54.99°$

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{282.38}{450} = 0.6275 \longrightarrow \theta_z = 51.13°$

2.76)



$F_z\!: -32.14 = T_{BD} \sin 30 \sin 40 \Rightarrow T_{BD} = 100\ N$

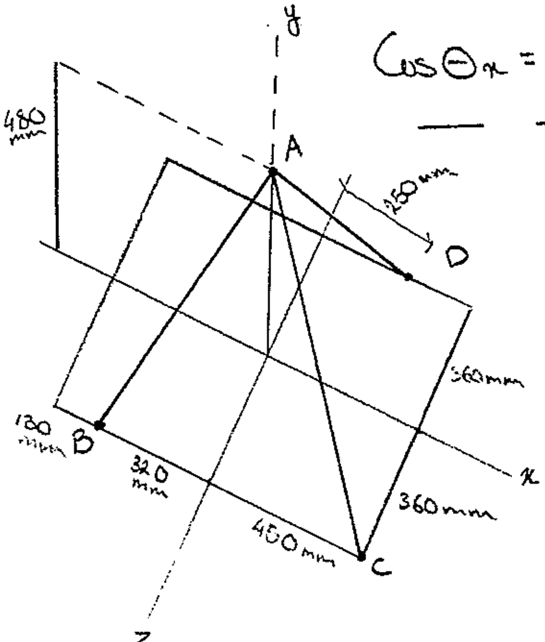
$\cos\theta_z = \dfrac{-32.14}{100} \rightarrow \theta_z = 108.74$

$\theta_y = 30°$

$F_x = -100 \sin 30 \cos 40 = -38.30$

$\cos\theta_x = \dfrac{-38.30}{100} \longrightarrow \theta_x = 112.52°$

2.86)



$T_{AD} = 429\ N$

$\vec{AD} = +250\vec{i} - 480\vec{j} - 360\vec{k}$

$\vec{DA} = -250\vec{i} + 480\vec{j} + 360\vec{k}$

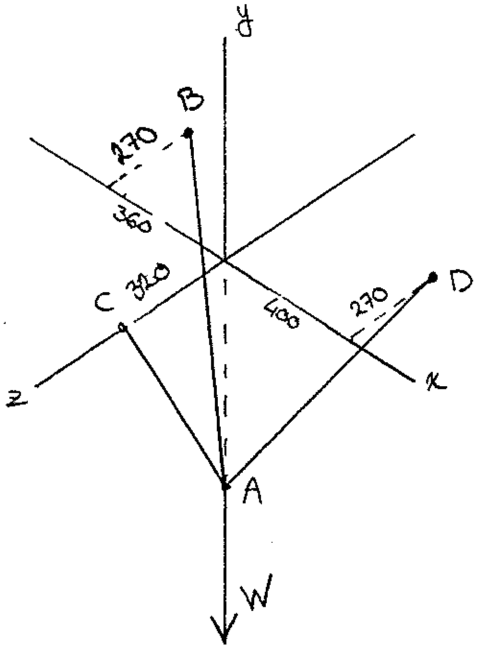
$|DA| = \sqrt{(250)^2 + (480)^2 + (360)^2} = 650$

$F = F\lambda_{DA} = \dfrac{429}{650}\left(-250\vec{i} + 480\vec{j} + 360\vec{k}\right)$

$= -165\vec{i} + 316.8\vec{j} + 237.6\vec{k}$

$F_x = -165\ N \quad F_y = +316.8\ N \quad F_z = +237.6\ N$

2.104)



$\vec{AB} = -360\vec{i} + 600\vec{j} - 270\vec{K}$

$\vec{AC} = 600\vec{j} + 320\vec{K}$

$\vec{AD} = 400\vec{i} + 600\vec{j} - 270\vec{K}$

$|AB| = 750$

$|AC| = 680$

$|AD| = 770$

$T_{AD} = 616\,N$

$\vec{T_{AB}} + \vec{T_{AC}} + \vec{T_{AD}} - \vec{W} = 0$ (I)

$\vec{T_{AB}} = T_{AB}\lambda_{AB} = T_{AB}\dfrac{\vec{AB}}{|AB|} = (-0.48\vec{i} + 0.8\vec{j} - 0.36\vec{K})\,T_{AB}$

$\vec{T_{AC}} = T_{AC}\lambda_{\vec{AC}} = T_{AC}\dfrac{\vec{AC}}{|AB|} = (0.88\vec{j} + 0.47\vec{K})\,T_{AC}$

$\vec{T_{AD}} = T_{AD}\lambda_{\vec{AD}} = T_{AD}\dfrac{\vec{AD}}{|AD|} = (0.51\vec{i} + 0.78\vec{j} - 0.35\vec{K})\,T_{AD}$

(I) : $(-0.48\vec{i} + 0.8\vec{j} - 0.36\vec{K})T_{AB} + (0.88\vec{j} + 0.47\vec{K})T_{AC} + (0.51\vec{i} + 0.78\vec{j} - 0.35\vec{K})T_{AD}$

$- W\vec{j} = 0$

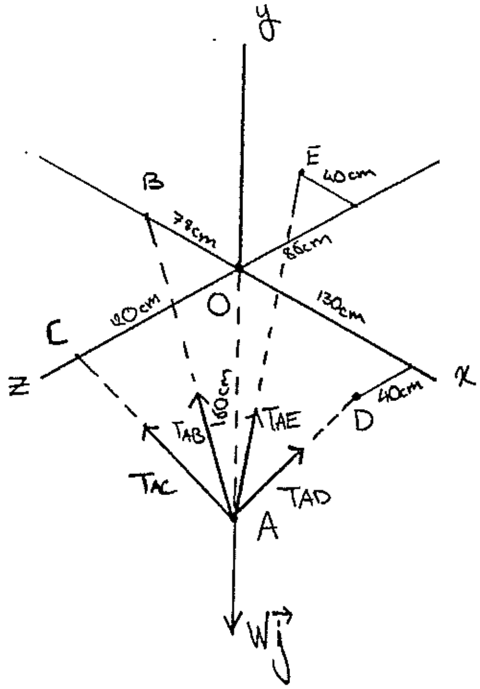
$\Rightarrow -0.48\,T_{AB} + 0.51\,T_{AD} = 0 \longrightarrow T_{AB} = 654.5\,N$

$0.8\,T_{AB} + 0.88\,T_{AC} + 0.78\,T_{AD} - W = 0$

$-0.36\,T_{AB} + 0.47\,T_{AC} - 0.35\,T_{AD} = 0 \longrightarrow T_{AC} = 960N$

$\Rightarrow W = 1848.88\,N$

2.121)



$W = 1000N$

$T_{AB} = T_{AD} = P$

EQUILIBRIUM: $\Sigma F = 0 \longrightarrow \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{T}_{AE} = W\vec{j}$

$\vec{AB} = -0.78\vec{i} + 1.6\vec{j}$ $\quad |AB| = 1.78\,m$

$\vec{AC} = 1.6\vec{j} + 1.2\vec{K}$ $\quad |AC| = 2.00\,m$

$\vec{AD} = 1.3\vec{i} + 1.6\vec{j} + 0.4\vec{K}$ $\quad |AD| = 2.10\,m$

$\vec{AE} = -0.4\vec{i} + 1.6\vec{j} - 0.86\vec{K}$ $\quad |AE| = 1.86\,m$

$$\vec{T}_{AB} = T_{AB}\vec{\lambda}_{AB} = P\frac{\vec{AB}}{|AB|} = P(-0.44\vec{i} + 0.90\vec{j})$$

$$\vec{T}_{AC} = T_{AC}\vec{\lambda}_{AC} = T_{AC}\frac{\vec{AC}}{|AC|} = T_{AC}(0.8\vec{j} + 0.6\vec{K})$$

$$\vec{T}_{AD} = T_{AD}\vec{\lambda}_{AD} = P\frac{\vec{AD}}{|AD|} = P(0.62\vec{i} + 0.76\vec{j} + 0.19\vec{K})$$

$$\vec{T}_{AE} = T_{AE}\vec{\lambda}_{AE} = T_{AE}\frac{\vec{AE}}{|AE|} = T_{AE}(-0.21\vec{i} + 0.86\vec{j} - 0.46\vec{K})$$

WRITE THE EQUILIBRIUM EQUATION:

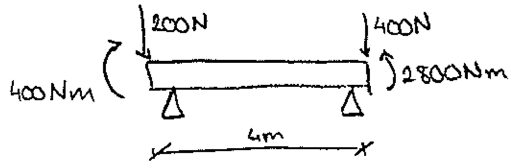
$\vec{i}$: $-0.44P + 0.62P - 0.21T_{AE} = 0 \Rightarrow 0.18P = 0.21T_{AE} \Rightarrow T_{AE} = 0.86P$ (I)

$\vec{j}$: $0.9P + 0.8T_{AC} + 0.76P + 0.86T_{AE} = W$ (I) and (II) $\Rightarrow W = 2.68P \rightarrow P = \frac{W}{2.68}$ (III)

$\vec{K}$: $0.6T_{AC} + 0.19P - 0.46T_{AE} = 0$ (I) $\rightarrow 0.6T_{AC} = 0.21P \rightarrow T_{AC} = 0.35P$ (II)

(III) and $W = 1000N \Rightarrow$ $\boxed{P = 373.13N}$

3.102)

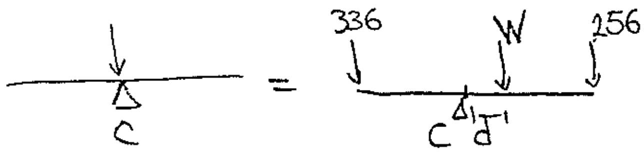


$R = \sum F = -200 - 400 = -600\,N$

$M = \sum M_A = 400\,Nm + 2800\,N.m - \underset{m}{(4)}\underset{N}{(400)} = 800\,Nm$

$\longrightarrow$ EQUIVALENT TO LOADING (f)

3.105)

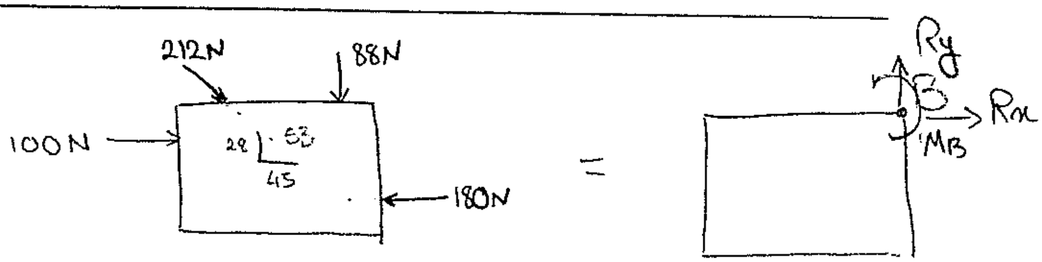


$\overset{+}{\circlearrowleft} \sum M_C = 0 \qquad (2)(336) - Wd - (2)(256) = 0$

a) $W = 240\,N \longrightarrow d = 0.66\,m$

b) $W = 208\,N \longrightarrow d = 0.77\,m$

3.109)



$\sum F_x = 100 + 212\frac{45}{53} - 180 = 100\,N$

$\sum F_y = -212\frac{28}{53} - 88 = -200\,N$

$\vec{R} = 100\,\vec{i} - 200\,\vec{j}$ OR $R = 223.61\,N \quad \measuredangle\, 63.43^\circ$

$R = \sqrt{R_x^2 + R_y^2}$ $\qquad \tan\theta = \frac{R_y}{R_x}$

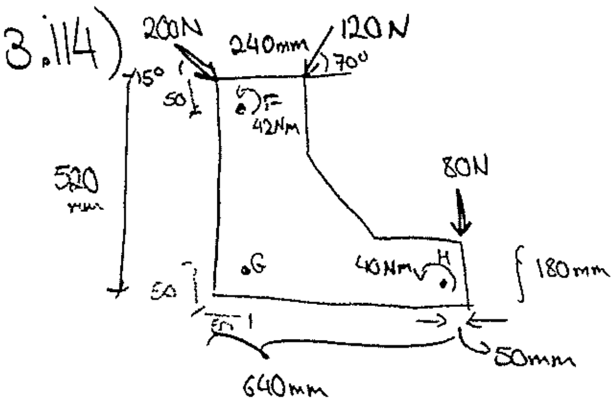
$+\circlearrowleft M_B = (0.1)(100) - (0.28)(180) + (0.08)(88) + (0.53)\left(\frac{28}{53}\right)(212) = 26\,N.m$

b) We should find the two places where if we put R there would be zero moment:

$26 = x(200) \longrightarrow x = 0.13\,m$ (left of B)

$26 = y(100) \longrightarrow y = 0.26\,m$ (below B)

3.114)



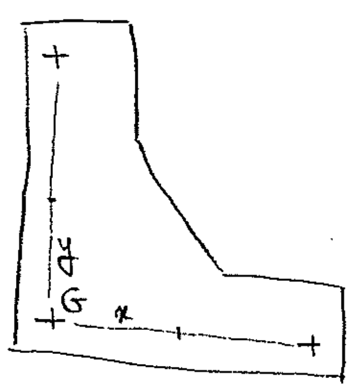
STEP ONE: FIND THE EQUIVALENT @ G!

ΣF_x : $200 \cos 15 - 120 \cos 70 = 152.143$

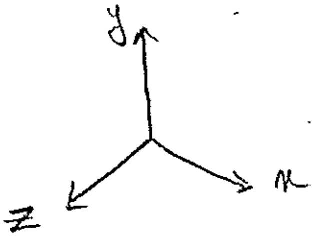
ΣF_y : $-(200 \sin 15 + 120 \sin 70 + 80) = -244.527$

ΣM_G: $-(0.47)(200 \cos 15) + (0.05)(200 \sin 15) + (0.47)(120 \cos 70) - (0.19)(120 \sin 70)$
$-(0.59)(80) + 42 + 40 = -55.544$ N.m

STEP TWO: x : $55.544 = 244.527(x) \rightarrow x = 0.227$ m
y : $55.544 = 152.143\ (y) \rightarrow y = 0.365$ m



3.133) STEP ONE: EQUIVALENT ON $\underline{O}$



WE HAVE:

$$\Sigma \vec{F} = -P\vec{j} + P\vec{K} + P\vec{j} = P\vec{K} = \vec{R}$$

$$\Sigma M_O = -Pa\vec{j} + \left[-(aP)\vec{i} + (\tfrac{5}{2}aP)\vec{K}\right] = aP(-\vec{i} - \vec{j} + \tfrac{5}{2}\vec{K})$$

(a) THEN FOR THE WRENCH:

$R = P$

AND $\vec{\lambda}$ AXIS $= \dfrac{\vec{R}}{R} = \vec{K}$

$\rightarrow \cos\theta_x = 0 \quad \cos\theta_y = 0 \quad \cos\theta_z = 1$

$\rightarrow \theta_x = 90° \quad \theta_y = 90° \quad \theta_z = 0°$

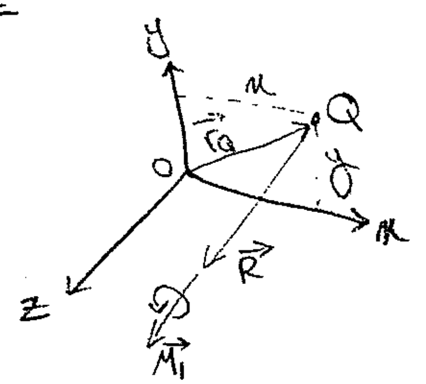
(b) NOW $\quad M_1 = \vec{\lambda}_{AXIS} \cdot \vec{M}_O^R$

$= \vec{K} \cdot aP(-\vec{i} - \vec{j} + \tfrac{5}{2}\vec{K}) = \tfrac{5}{2}aP$

THEN... $\quad p = \dfrac{M}{R} = \dfrac{\frac{5}{2}a\cancel{P}}{\cancel{P}} \Rightarrow p = \tfrac{5}{2}a$

(c) THE COMPONENTS OF THE WRENCH ARE $(\vec{R}, \vec{M}_1)$ WHERE $\vec{M}_1 = M_1 \vec{\lambda}_{AXIS}$ AND THE AXIS OF THE WRENCH IS ASSUMED TO INTERSECT THE XY PLANE AT POINT Q WHOSE COORDINATES ARE $(X, Y, 0)$ THUS REQUIRE

$\vec{M}_2 = \vec{r}_Q \times \vec{R} \quad$ WHERE $\quad \vec{M}_2 = \vec{M}_O^R - \vec{M}_1$



THEN...

$$aP(-\vec{i} - \vec{j} + \tfrac{5}{2}\vec{K}) - \tfrac{5}{2}aP\vec{K} = (x\vec{i} + y\vec{j})P\vec{K}$$

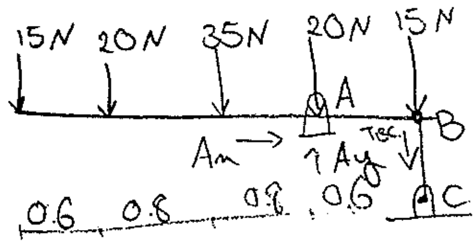
EQUATING COEFFICIENTS:

$i: \ -aP = yP \longrightarrow y = -a$

$j: \ -aP = -xP \longrightarrow x = a$

$\therefore$ THE AXIS OF THE WRENCH IS PARALLEL TO THE Z AXIS AND INTERSECTS THE XY PLANE @ $\begin{cases} X = a \\ Y = -a \end{cases}$

4.4)



$\Sigma F_x = 0 \longrightarrow A_x = 0$

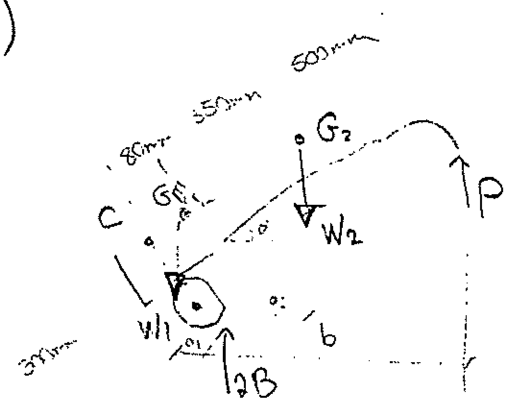
$+\circlearrowleft \Sigma M_B = 0 \quad (15)(2.8) + (20)(2.2) + (35)(1.4) + (20)(0.6) - (0.6)A_y = 0$

$\Longrightarrow A_y = 245\,N$

$\Sigma F_y = 0 : \quad 15 + 20 + 35 + 20 + 15 - 245 + T_{BC} = 0$

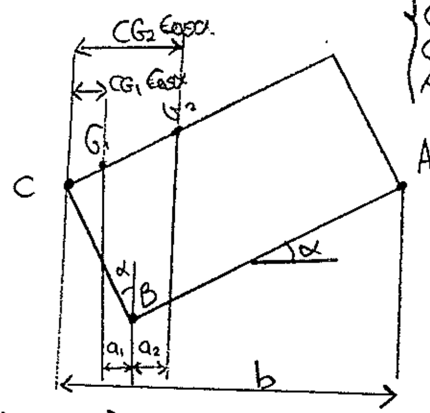
$\Longrightarrow T_{BC} = 140\,N$

4.7)



$W = mg = 40(9.81) = 392.4\,N$

$BC = 300\,mm$
$CG_1 = 80\,mm$
$CG_2 = 430\,mm$
$AB = 930\,mm$



$+\circlearrowright \Sigma M_B = 0 : \quad W_1 a_1 - W_2 a_2 + Pb = 0 \Rightarrow P = \frac{W}{b}(a_2 - a_1) \quad (I)$

$\Sigma F_y = 0 : \quad -W - W + P + 2B = 0 \rightarrow B = W - \frac{P}{2} \quad (II)$

$a_1 = BC \sin\alpha - CG_1 \cos\alpha =$

$a_2 = CG_2 \cos\alpha - BC \sin\alpha =$

$b = AB \cos\alpha =$

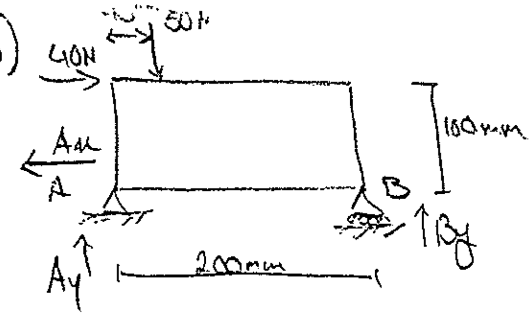
$(I) \longrightarrow P = \frac{W(a_2 - a_1)}{b} = W[(430\cos\alpha - 300\sin\alpha) - (300\sin\alpha - 80\cos\alpha)] \div 930\cos\alpha$

$\alpha = 35° \rightarrow P = 37.9\,N \uparrow$

$(II) \rightarrow B = 374\,N \uparrow$

4.23)

a)



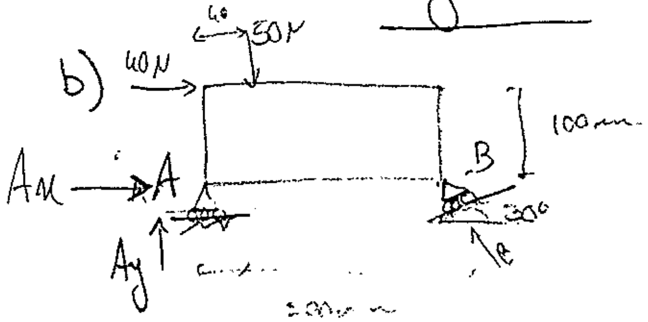
$\Sigma F_x = 0: \; A_x - 40 = 0 \Rightarrow \underline{A_x = 40N}$

$\Sigma F_y = 0 \quad A_y + B_y - 50 = 0$

$+\circlearrowleft \Sigma M_A = 0 \quad 200 B_y - 50 \times 40 - 40 \times 100 = 0$

$\Rightarrow \underline{B_y = 30N} \Rightarrow A_y = \underline{20N}$

b)

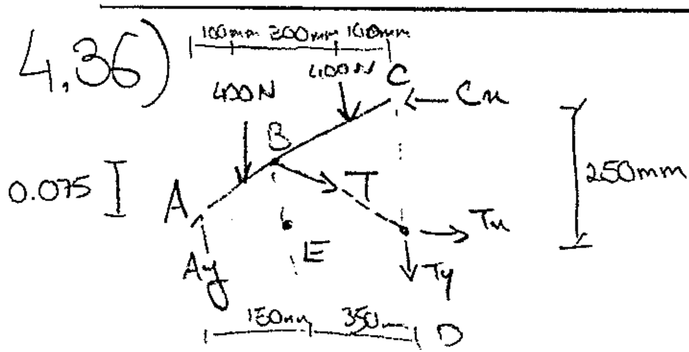


$+\circlearrowleft \Sigma M_B = 0 \; -200 A_y - 40 \times 100 + 50 \times 160 = 0 \Rightarrow \underline{A_y = 20N \uparrow}$

$\Sigma F_y = 0 \quad A_y - 50 + B\cos 30^\circ = 0 \Rightarrow \underline{B = 34.64\,N}$ 60°

$\Sigma F_x = 0 \quad A_x + 40 - B\sin 30 = 0 \Rightarrow \underline{A_x = 22.68\,N \leftarrow}$

4.36)



$\Delta ABE \sim \Delta ACD: \; \frac{AE}{BE} = \frac{AD}{CD} \Rightarrow BE = 0.075\,m$

$\Sigma F_x = 0 : \; C_x = T_x$

$|T_y| = \left| \frac{0.075}{0.35} T_x \right|$

a) $+\circlearrowleft \Sigma M_A = 0 \quad T_x(0.25) - \left(\frac{0.075}{0.35}\right) T_x(0.5) - (400)(0.1) - (400)(0.4) = 0$

$\Rightarrow T_x = 1400\,N \quad \Rightarrow T = 1432\,N$

$\Rightarrow T_y = 300N$

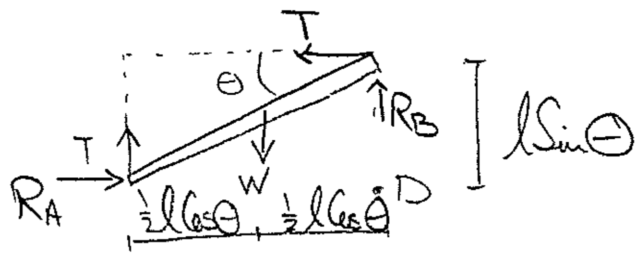
b) $\Sigma F_y = 0:$

$A - 300 - 400 - 400 = 0 \Rightarrow A = 1100N$

c) $\Sigma F_x = 0: \quad -C_x + 1400 = 0 \quad$ (OR $C_x = T_x$) $\Rightarrow$

$\Rightarrow C_x = 1400$

4.51)



a) $+\circlearrowleft \Sigma M_D = 0$:

$$T(\cancel{l}\sin\theta) - T(\cancel{l}\cos\theta) + W(\tfrac{1}{2}\cancel{l}\cos\theta) = 0$$

$$\Rightarrow T(\sin\theta - \cos\theta) = -\frac{W\cos\theta}{2}$$

$$\Rightarrow T = \frac{\frac{1}{2}W\cos\theta}{\cos\theta - \sin\theta} \Rightarrow \frac{1}{T} = \frac{2(\cos\theta - \sin\theta)}{W\cos\theta} = \frac{2(1-\tan\theta)}{W}$$

$$\Rightarrow T = \frac{W}{2} \times \frac{1}{(1-\tan\theta)}$$

b) $T = 3W$

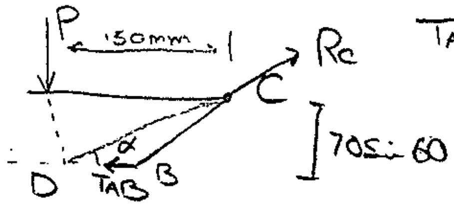
$$\Rightarrow 3\cancel{W} = \frac{\cancel{W}}{2(1-\tan\theta)} \Rightarrow 6(1-\tan\theta) = 1 \Rightarrow 6\tan\theta = 5$$

$$\Rightarrow \theta = 39.8^\circ$$

See y'all next week! :)

—ARASH

4.63)



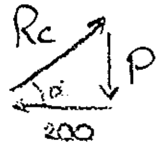
$T_{AB} = 200N$

Three force member:

$\Longrightarrow$ Reaction at C <u>MUST</u> pass through <u>D</u>

$$\tan\alpha = \frac{70\sin 60}{150} \longrightarrow \alpha = 22^\circ$$

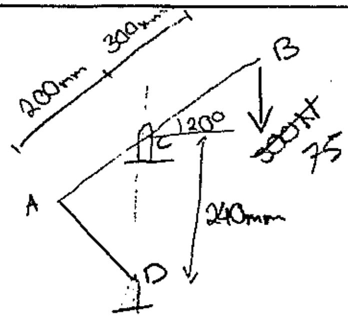
FORCE TRIANGLE :

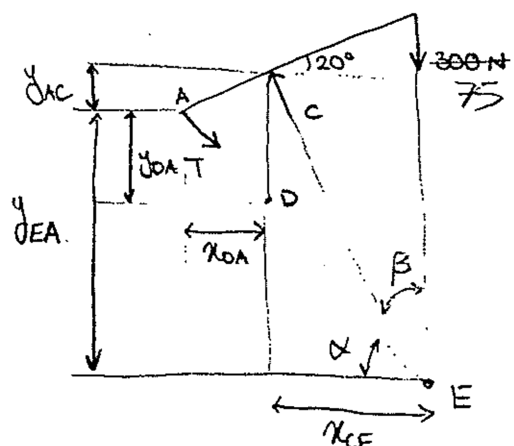


$P = 200 \tan\overset{22}{\alpha} = 80.8\ N$

$R_C = \dfrac{200}{\cos 22} = 215.7\ N \quad \angle 22^\circ$

4.80)





$$\alpha = \tan^{-1}\left[\frac{y_{DA}}{x_{DA}}\right]$$

$y_{DA} = 0.24 - y_{AC} = 0.24 - (0.2)\sin 20 = 0.171596$

$x_{DA} = 0.2\cos 20 = 0.187939\,m$

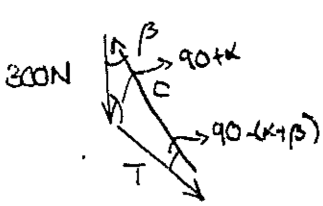
$\Longrightarrow \alpha = 42.40^\circ$

$$\beta = 90^\circ - \tan^{-1}\left(\frac{y_{AC} + y_{EA}}{x_{CE}}\right)$$

$\Longrightarrow \beta = 29.54^\circ$

$\left\{\begin{array}{l} x_{CE} = 0.3\cos 20 = 0.28191\,m \\ y_{AC} = 0.2\sin 20 = 0.068404\,m \\ y_{EA} = (x_{DA} + x_{CE})\tan\alpha = (0.187939 + 0.28191)\tan 42.40 \\ \quad = 0.42898\,m \end{array}\right.$

FORCE TRIANGLE



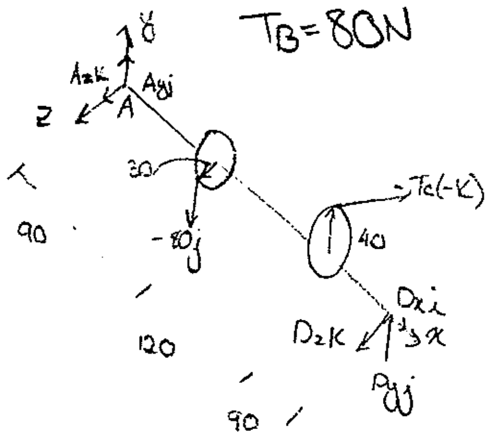
$$\frac{\overset{75}{\cancel{300}}}{\sin 18.06} = \frac{T}{\sin 29.54} = \frac{C}{\sin 132.40}$$

$90 - (\alpha+\beta) = 18.06^\circ$

$90 + \alpha = 132.40^\circ$

$\Longrightarrow \left\{\begin{array}{l} T = \cancel{477N} \\ C = \cancel{715N} \quad \angle 60.46^\circ \end{array}\right.$

4.91) $T_B = 80N$



$\Sigma M_A = 0: (90i + 30K) \times (-80j) + (210i + 40j) \times (-T_C K) + (300i) \times (D_x i + D_y j + D_z K) = 0$

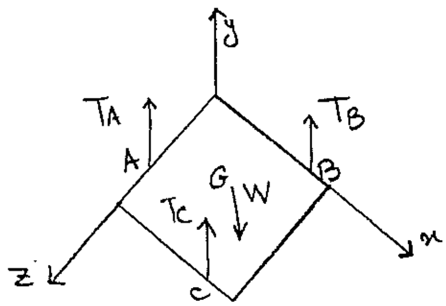
$-7200K + 2400i + 210T_C j - 40T_C i + 300D_y K - 300D_z j = 0$

$(2400 - 40T_C)i + (210T_C - 300D_z)j + (-7200 + 300D_y) = 0$

$\begin{cases} 2400 - T_C \times 40 = 0 \rightarrow T_C = 60N \\ 210T_C - 300D_z = 0 \rightarrow \underline{D_z = 42N} \\ -7200 + 300D_y = 0 \rightarrow \underline{\underline{D_y = 24N}} \end{cases}$

$\begin{cases} \Sigma F_x = 0 \rightarrow \underline{D_x = 0} \\ \Sigma F_y = 0 \rightarrow A_y + D_y - 80 = 0 \rightarrow \underline{\underline{A_y}} = 56N \\ \Sigma F_z = 0 \rightarrow A_z + D_z - 60 = 0 \rightarrow \underline{\underline{A_z}} = 18N \end{cases}$

4.99)



$W = mg = (25)(9.81) = 245.25N$

$\Sigma M_x = 0: (245.25)(100) - T_A(100) - T_C(200) = 0 \rightarrow T_A + 2T_C = 245.25N \quad (I)$

$\Sigma M_z = 0: T_B(160) + T_C(160) - (245.25)(100) = 0 \rightarrow T_B + T_C = 153.281N \quad (II)$

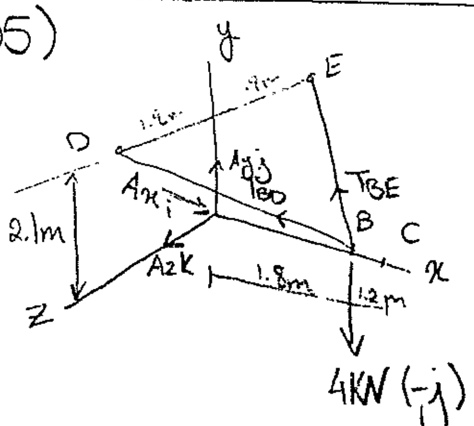
$\Sigma F_y = 0: T_A + T_B + T_C - 245.25 = 0 \rightarrow T_B + T_C = 245.25 - T_A \quad (III)$

$(II) \& (III) \Rightarrow T_A = 92N$

$(I) \Rightarrow T_C = 76.641N \simeq 76.6N$

$(II) \Rightarrow T_B = 76.6N$

4.105)



$\overrightarrow{BD} = (-1.8)i + (2.1)j + (1.8)K \quad BD = 3.3m$

$\overrightarrow{BE} = (-1.8)i + (2.1)j + (-1.8)K \quad BE = 3.3m$

$\overrightarrow{T_{BD}} = T_{BD} \frac{\overrightarrow{BD}}{|BD|} = T_{BD} \frac{-1.8i + 2.1j + 1.8K}{3.3}$

$\overrightarrow{T_{BE}} = T_{BE} \frac{\overrightarrow{BE}}{|BE|} = T_{BE} \frac{-1.8i + 2.1j - 1.8K}{3.3}$

$\sum M_A = 0 \quad \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{BE} + \vec{r}_C \times (-4\vec{j}) = 0$

$$\underbrace{\left(\frac{-3.24}{3.3}T_{BD} + \frac{3.24}{3.3}T_{BE}\right)}_{0}\vec{j} + \underbrace{\left(\frac{3.78}{3.3}T_{BD} + \frac{3.78}{3.3}T_{BE} - 12\right)}_{0}\vec{K} = 0$$

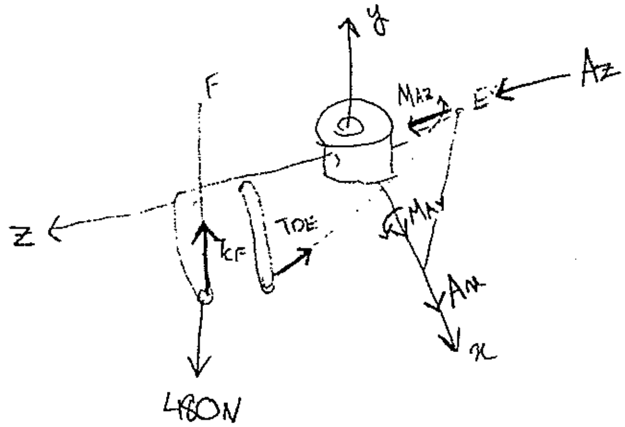
$$\Longrightarrow \begin{cases} T_{BE} = 5.238\ KN \\ T_{BD} = 5.238\ KN \end{cases}$$

$\sum F_x = 0: A_x - \frac{1.8}{3.3}(5.238) - \frac{1.8}{3.3}(5.238) = 0 \longrightarrow A_x = 5.714\ KN$

$\sum F_y = 0: A_y + \frac{2.1}{3.3}(5.238) + \frac{2.1}{3.3}(5.238) - 4 = 0 \longrightarrow A_y = -2.667\ KN$

$\sum F_z = 0: A_z + \frac{1.8}{3.3}(5.238) - \frac{1.8}{3.3}(5.238) = 0 \longrightarrow A_z = 0\ KN$

4.122)



$\vec{T}_{CF} = \vec{\lambda}_{CF} T_{CF} = \frac{-(0.8)\vec{i} + (0.06)\vec{j}}{\sqrt{0.8^2 + 0.06^2}} T_{CF}$

$\vec{T}_{CF} = T_{CF}(-0.8\vec{i} + 0.06\vec{j})$

$\vec{T}_{DE} = \vec{\lambda}_{DE} T_{DE} = \frac{(0.12)\vec{j} - (0.09)\vec{K}}{\sqrt{0.12^2 + 0.09^2}}$

$\vec{T}_{DE} = T_{DE}(0.8\vec{j} - 0.6\vec{K})$

$\sum F_y = 0: 0.6T_{CF} + 0.8T_{DE} - 480 = 0 \implies 0.6T_{CF} + 0.8T_{DE} = 480 \quad (I)$

$\sum M_y = 0: -(0.8T_{CF})(0.135) + (0.6T_{DE})(0.08) = 0 \Rightarrow T_{DE} = 2.25T_{CF} \quad (II)$

(I) and $(II) \implies T_{CF} = 200N \quad (II) \Longrightarrow T_{DE} = 450N$

$$\left.\begin{array}{l} \sum F_x = 0 : A_x - (0.8)(200) = 0 \longrightarrow A_x = 160\ N \\ \sum F_z = 0 : A_z - (0.6)(450) = 0 \longrightarrow A_z = 270\ N \end{array}\right\} \Rightarrow \vec{A} = 160\vec{i} + 270\vec{K}$$

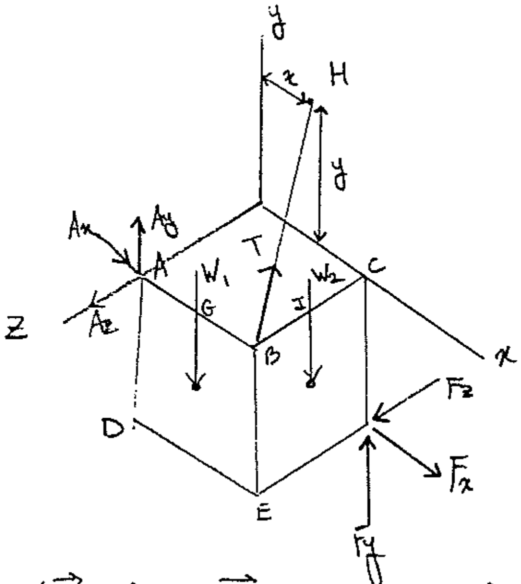
$\sum M_x = 0: M_{A_x} + \underbrace{(480)(0.135)}_{64.8} - \underbrace{[(200)(0.6)](0.135)}_{16.2} - \underbrace{[(450)(0.8)](0.09)}_{32.4} = 0 \implies M_{A_x} = -16.2\,Nm$

$\sum M_z = 0 \quad M_{A_z} - (480)(0.08) + [(200)(0.6)](0.08) + [(450)(0.8)](0.08) = 0$

$\implies M_{A_z} = 0$

$\implies \vec{M}_A = -16.2\,\vec{i}\ Nm$

4.140)



60N

$$W_1=W_2=-(mg)\vec{j}=-(15)(9.81)=\cancel{-147.15N}$$

$$\begin{cases} \vec{r}_{G/A}=\vec{i} \\ \vec{r}_{B/A}=2\vec{i} \\ \vec{r}_{I/A}=2\vec{i}-\vec{k} \end{cases}$$

$$\Sigma \vec{M}_{AF}=0:\ \vec{\lambda}_{AF}\cdot(\vec{r}_{G/A}\times\vec{W}_1)+\vec{\lambda}_{AF}\cdot(\vec{r}_{B/A}\times\vec{T})+\vec{\lambda}_{AF}\cdot(\vec{r}_{I/A}\times\vec{W}_2)=0$$

$$\vec{\lambda}_{AF}=\frac{2\vec{i}-\vec{j}-2\vec{k}}{\sqrt{4+1+4}}=\frac{1}{3}(2\vec{i}-\vec{j}-2\vec{k})$$

$$\vec{\lambda}_{BH}=\frac{(x-2)i+yj-2k}{\sqrt{(x-2)^2+y^2+4}}\quad:\quad \vec{T}=\lambda_{BH}T=\frac{T[(x-2)i+yj-2k]}{\sqrt{(x-2)^2+y^2+4}}$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right)+\begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ x-2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{(x-2)^2+y^2+4}}\right)+\begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right)=0$$

$$\frac{2}{3}(147.15)-(4+4y)\frac{T}{3\sqrt{(x-2)^2+y^2+4}}+(-2+4)\left(\frac{147.15}{3}\right)=0$$

$$T=\frac{147.15}{1+y}\sqrt{(x-2)^2+y^2+4}\qquad x=2\,m \Longrightarrow T=T_{min}$$

$$\frac{dT}{dy}=0 \Longrightarrow \frac{(1+y)\frac{1}{2}(y^2+4)^{-\frac{1}{2}}-(y^2+4)^{\frac{1}{2}}(1)}{(1+y)^2}=0$$

$$(1+y)y=y^2+4 \Longrightarrow y=4\,m$$

$$\begin{cases} x=2m \\ y=4m \end{cases} \Longrightarrow T_{min}=131.615\,N \simeq 131.6\,N$$

29/11/04

ENGR 242

ENGINEERING STATICS

DR HAMMAD

CHAPTER 8

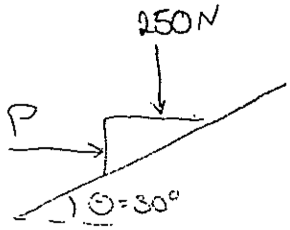
ARASH RANJBARAN

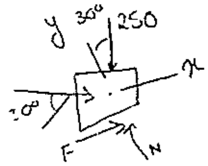
20c

2 pages

8.1)

$\mu_s = 0.30$
$\mu_k = 0.20$





$\Sigma F_y = 0: \; N - (250)\cos 30 - (50)\sin 30 = 0$

$N = 241.5N$

$\Sigma F_x = 0: \; F - (250)\sin 30 + (50)\cos 30 = 0$

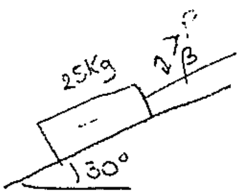
$F = 81.7N$

Maximum Friction Force

$F_f = \mu_s N = 0.3(241.5) = 72.5N \quad F > F_f$: Block moves down

Friction Force : $F = \mu_k N = (0.20)(241.5) = 48.3N$

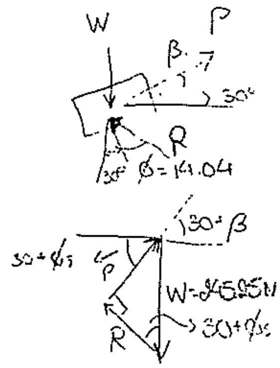
8.7)



$\mu_s = 0.25$

$W = 25(9.81) = 245.25N$

$\phi_s = \tan^{-1} 0.25 = 14.04°$



For smallest P

We choose $P \perp R$

$30 + \phi_s = 30 + \beta$

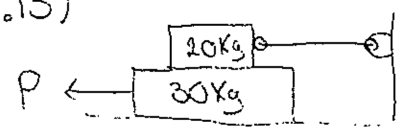
$\therefore \beta = \phi_s = 14.04$

$P = W(\sin(30+\phi_s))$

$= 245.25 \sin 44.04$

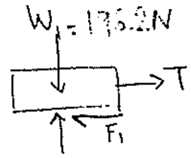
$= 170.49N$

8.13)

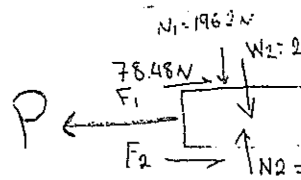


$\mu_s = 0.40$
$\mu_k = 0.30$

$W_1 = 20 \times 9.81 = 196.2N$
$W_2 = 30 \times 9.81 = 294.3N$



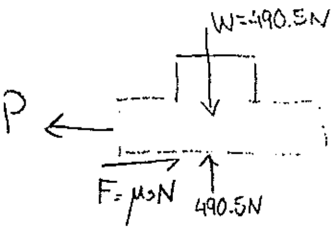
$F_1 = \mu_s N_1 = 0.4(196.2) = 78.48N$



$F_2 = \mu_s N_s = 0.4(490.5) = 196.2N$

$\Sigma F_x = 0: \; P - F_1 - F_2 = 0$

$\rightarrow P = 274.7 \; N \cong 275 \; N$

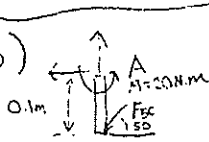


$W = (50)(9.81) = 490.5N$

$\Sigma F_x = 0 \quad P - F = 0 \; ; \; P = \mu_s N = 0.4(490.5) = 196.2N$

$P = 196.2N \leftarrow$

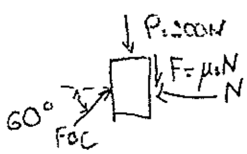
8.45)



BC is a two force member

$+\circlearrowleft \Sigma M_A = 0 \quad M - F_{BC}\cos 60(0.1) = 0 \quad M = 0.05 F_{BC} \; (I)$

C impending upward

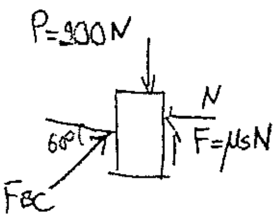


$\Sigma F_x = 0: \; F_{BC}\cos 60 - N = 0 \quad N = 0.5 F_{BC}$

$\Sigma F_y = 0: \; F_{BC}\sin 60 - 200 - (0.4)(0.5F_{BC}) = 0$

$F_{BC} = 300.29$

$(I): \; M = 15.014 \; N.m$



C impending downwards

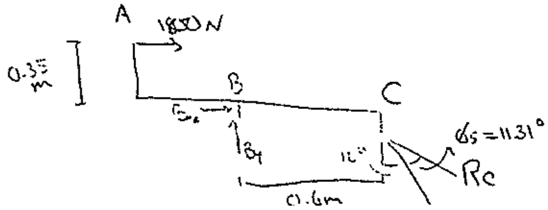
$\Sigma F_x = 0: \; F_{BC}\cos 60 - N = 0 \quad N = 0.5 F_{BC}$

$\Sigma F_y = 0: \; F_{BC}\sin 60 - 200 + (0.40)(0.5F_{BC}) = 0$

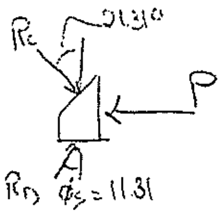
$F_{BC} = 187.613 N$

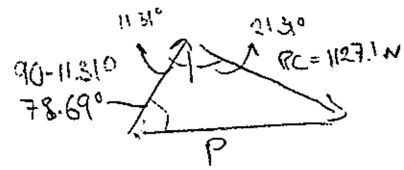
$(I) \; : \; M = 0.05(187.613) = 9.381 N.m$

8.62) $\phi_s = \tan^{-1} 0.20 = 11.31$



$\sum M_B = 0$: $(1800)(0.35) - R_C \cos 21.31 (0.6) = 0$ ∴ $R_C = 1127.1\ N$





$$\frac{P}{\sin(32.62°)} = \frac{1127.1}{\sin(78.69°)}$$

$\Longrightarrow P = 620\ N \leftarrow$

$\xrightarrow{+} \sum F_x = 0 \quad B_x + 1800 - R_C \sin 21.31 = 0$

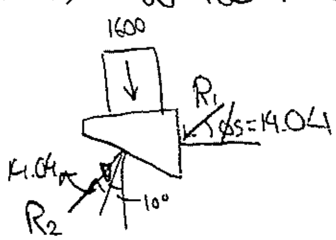
$\Longrightarrow B_x = 1390\ N \leftarrow$

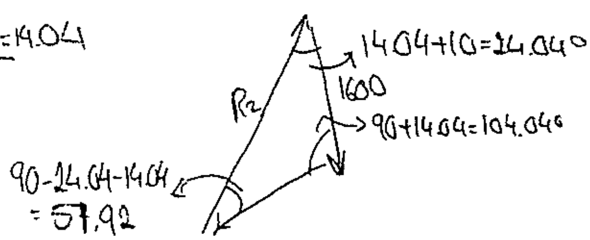
$+\uparrow \sum F_y = 0 \quad B_y + R_C \cos 21.31 = 0$

$B_y = 1050\ N \downarrow$

8.64) $W = 163 \times 9.81 = 1600\ N$ $\mu_s = 0.25$

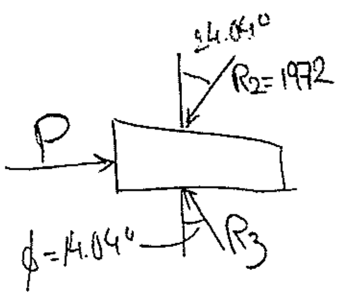
$\phi_s = \tan^{-1} \mu_s = 14.04°$

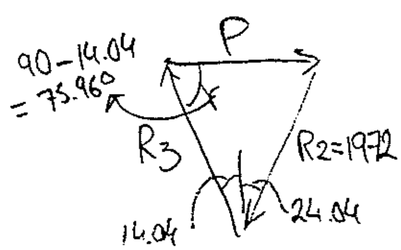




$$\frac{R_2}{\sin 104.04} = \frac{1600}{\sin 51.92}$$

$\Longrightarrow R_2 = 1972\ N$

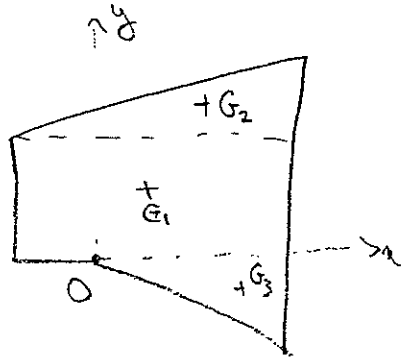




$$\frac{P}{\sin 38.08} = \frac{1972}{\sin 75.96}$$

$\Longrightarrow P = 1254\ N$

5.6)

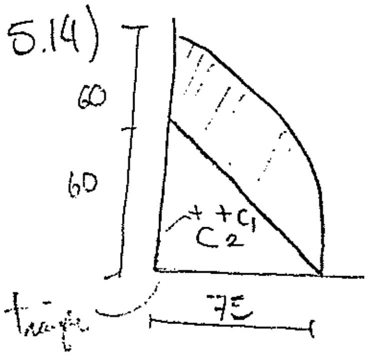


	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$\bar{x}A$	$\bar{y}A$
1	126x54	9	27	61236	18370
2	½ x 126x30	30	64	56700	120960
3	½ x 72x48	40	−16	82944	−2764
Σ	10422			200880	27702

$\bar{x}\Sigma A = \Sigma \bar{x} A \Rightarrow \bar{x} = 19.27\,mm$

$\bar{y}\Sigma A = \Sigma \bar{y} A \Rightarrow \bar{y} = 26.6\,mm$

5.14)

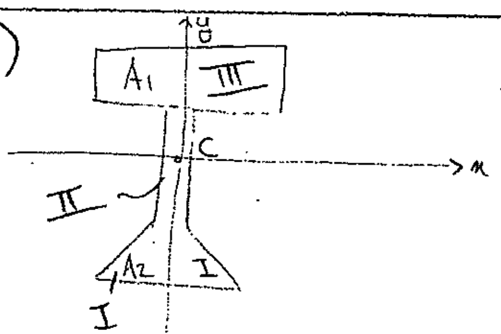


	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$\bar{x}A$ mm^3	$\bar{y}A$ mm^3
1	2/3x 75x60 = 6000	28.125	48	168750	288000
2	−½x 75x60 = 2250	25	20	−56250	−45000
Σ	3750			112500	243000

$\bar{X}\Sigma A = \Sigma \bar{x} A \Rightarrow \bar{x}(3750) = 112500 \rightarrow \bar{x} = 30\,mm$

$\bar{y}\Sigma A = \Sigma \bar{y} A \Rightarrow \bar{y}(3750) = 243000 \rightarrow \bar{y} = 64.8\,mm$

5.25)



	A	$\bar{y}$	$\bar{y}A$
I	2(½x2x15) = 3	0.5	1.5
II	1.5x 5.5 = 8.25	2.75	22.69
III	4.5x2 = 9	6.5	58.5
Σ	20.25		82.6875

$\bar{y}\Sigma A = \Sigma \bar{y} A \rightarrow \bar{y}\Sigma A = \Sigma \bar{y} A \Rightarrow \bar{y}(20.25) = 82.6875 \Rightarrow \bar{y} = 4.083$

$Q_x = \Sigma \bar{y} A$

$(Q_{x_1}) = \left[\frac{1}{2}(5.5-4.0833)\right]\left[(1.5)(5.5-4.0833)\right] + \left[(6.5-4.0833)\right]\left[(4.5)(2)\right] = 23.3$

$(Q_{x_2}) = -\left[\left(\frac{1}{2}\right)(4.0833)\right]\left[(1.5)(4.0833)\right] - \left[4.0833-0.5\right] \times 2\left[\left(\frac{1}{2}\right)(2\times15)\right] = -23.3$

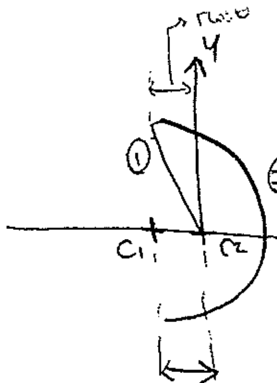
$Q_x = (Q_x)_1 + (Q_x)_2 = 0$

explanation: $\underline{x}$ is a centroid axis $\Rightarrow \bar{Y} = 0$

$Q_x = \Sigma \bar{y} A = \bar{y}\Sigma A \quad \bar{y}=0 \Rightarrow Q_x = 0$

1

5.32)



$\frac{2r}{\pi}$ (π for semicircular arc)

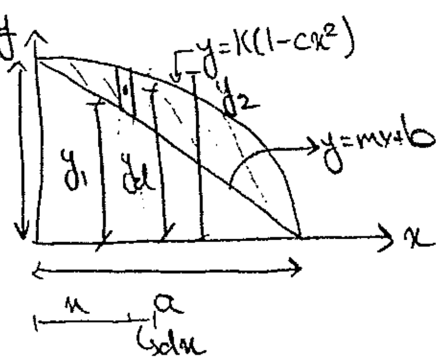
For equilibrium the center of gravity of the wire must lie on a vertical line through A. Further, because the wire is homogeneous its center of gravity will coincide with the centroid of the corresponding line. Thus:

$$\bar{x}=0$$
$$\Sigma \bar{x}L=0$$

Then $\left(-\frac{1}{2}r\cos\theta\right)(r)+\left(\frac{2r}{\pi}-r\cos\theta\right)(\pi r)=0$

$$\cos\theta=\frac{4}{1+2\pi}=0.5491 \longrightarrow \theta=56.7°$$

5.43)



x	y_1
0	h
a	0

$$\frac{x-0}{a-0}=\frac{y_1-h}{-h}$$

$\Rightarrow ay_1-ah=-xh$

$\Rightarrow y_1=-\frac{xh}{a}+h=h\left(1-\frac{x}{a}\right)$

*

$\bar{x}A=\int \bar{x}_{el}\,dA \rightarrow x=\frac{1}{2}a$

$\bar{y}A=\int \bar{y}_{el}\,dA \rightarrow y=\frac{3}{5}h$

y_2 : $\begin{cases} x=0 \quad y_2=h \Rightarrow \underline{h=K} \\ x=a \quad y_2=0 \Rightarrow 0=K(1-ca^2) \Rightarrow \underline{c=1/a^2} \end{cases}$

$$\Rightarrow y_2=h\left(1-\frac{x^2}{a^2}\right)$$

$$dA=(y_2-y_1)dx=h\left[\left(1-\frac{x^2}{a^2}\right)-\left(1-\frac{x}{a}\right)\right]dx=h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)dx$$

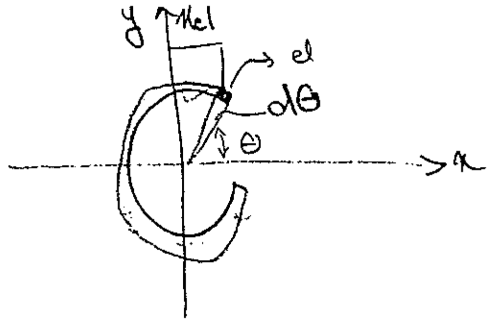
$$\bar{x}_{el}=x \Rightarrow \bar{y}_{el}=\frac{1}{2}(y_1+y_2)=\frac{h}{2}\left[\overbrace{\left(1-\frac{x^2}{a^2}\right)}^{y_2}+\overbrace{\left(1-\frac{x}{a}\right)}^{y_1}\right]=\frac{h}{2}\left[2-\frac{x}{a}-\frac{x^2}{a^2}\right]$$

$$A=\int dA=\int_0^a h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)dx=h\left[\frac{x^2}{2a}-\frac{x^3}{3a^2}\right]_0^a=\frac{1}{6}ah$$

$$\int \bar{x}_{el}\,dA=\int_0^a x\left[h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\right]dx=h\left[\frac{x^3}{3a}-\frac{x^4}{4a^2}\right]_0^a=\frac{1}{12}a^2h$$

$$\int \bar{y}_{el}\,dA=\int_0^a \frac{h}{2}\left(2-\frac{x}{a}-\frac{x^2}{a^2}\right)\left[h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\right]dx=\frac{h^2}{2}\left[\frac{x^2}{a}-\frac{x^3}{a^2}+\frac{x^5}{5a^4}\right]_0^a=\frac{1}{10}ah^2 \ *$$

5.52)



Note: wire is homogeneous
center of gravity coincides with centroid of corresponding line

$$\bar{x}_{el} = r\cos\theta \qquad dL = r\,d\theta$$

$$L = \int dL = \int_{\pi/4}^{5\pi/4} r\,d\theta = r\left[\theta\right]_{\pi/4}^{3\pi/4} = \tfrac{3}{2}\pi r$$

$$\int \bar{x}_{el}\,dL = \int_{\pi/4}^{3\pi/4} r\cos\theta\,(r\,d\theta) = r^2\left[\sin\theta\right]_{\pi/4}^{7\pi/4} = r^2\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right] = -r^2\sqrt{2}$$

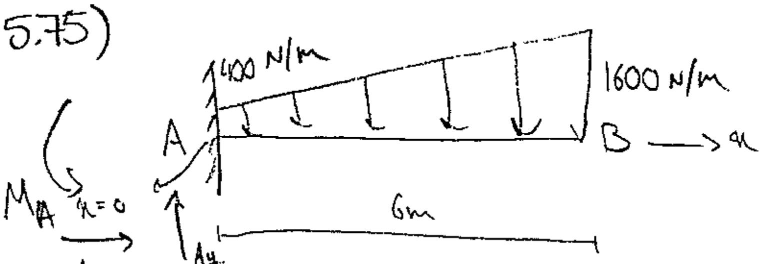
$$\bar{x}L = \int_{\pi/4} \bar{x}\,dL \;:\; \bar{x}\left[\frac{3}{2}\pi r\right] = -r^2\sqrt{2} \Longrightarrow \bar{x} = \frac{2\sqrt{2}}{3\pi}r$$

5.63) SECTION: $A = \frac{\pi}{4}d^2 \quad C = \pi d$

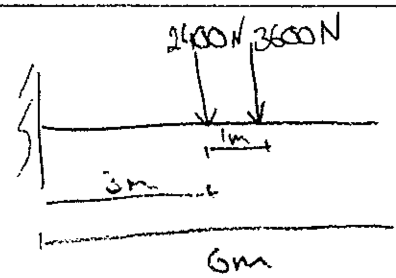
$$\text{Volume}: 2(V_{side}) + 2(V_{end}) = 2AL + 2(\pi R A) = 2A(L + \pi R)$$
$$= 2\left[\frac{\pi}{4}(6)^2\right]\left[30 + \pi(10)\right] = 3470\text{ mm}^3$$

$$\text{Area} = 2(A_{side}) + 2(A_{end}) = 2CL + 2\pi R C = 2(L + \pi R)C$$
$$= 2\left[30 + \pi(10)\right]\left[\pi(6)\right] = 2320\text{ mm}^2$$

5.75)



$\equiv$



$400 \times 6 = 2400\text{ N} \longrightarrow x = 3\text{ m}$

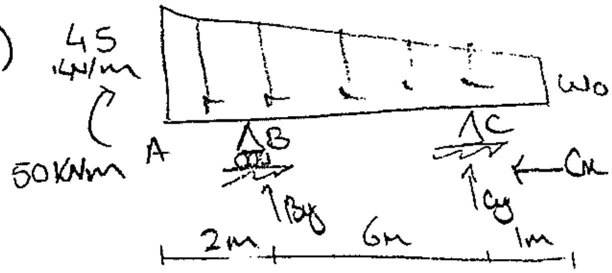
$(1600 - 400) \times \frac{6}{2} = 3600\text{ N} \longrightarrow x = 4\text{ m}$

$\Sigma F_y = 0 \quad A_y - 2400 - 3600 = 0 \Longrightarrow A_y = 6000\text{ N}$

$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma M_A = 0 \quad M_A - 2400 \times 3 - 3600 \times 4 = 0 \Longrightarrow M_A = 21{,}600\text{ Nm}$

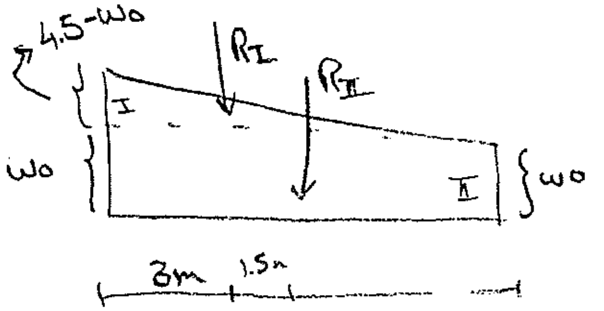
5.84)



$B_y = 0$

$\sum F_y = 0$

$C_y = \frac{4.5 + w_o}{2} \times 9$ (I)



$R_1 = \frac{1}{2}(4.5 - w_o) \times 9 = 4.5(4.5 - w_o)$ KN

$R_2 = 9w_o$ KN

$+\circlearrowleft \sum M_C = 0 : -50 + 5 \times [4.5(4.5 - w_o)] + 3.5 \times (9w_o) = 0$

$-50 + 101.25 - 22.5w_o + 31.5w_o = 0$

$9w_o = 51.25$

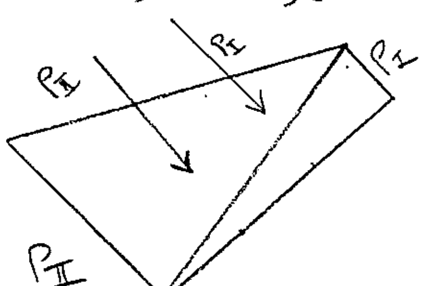
$\Rightarrow w_o = 5.69$

(I) $\Rightarrow C_y = \frac{4.5 + 5.69}{2} \times 9 = 45.86$ KN

5.97) Calculate P for A & B

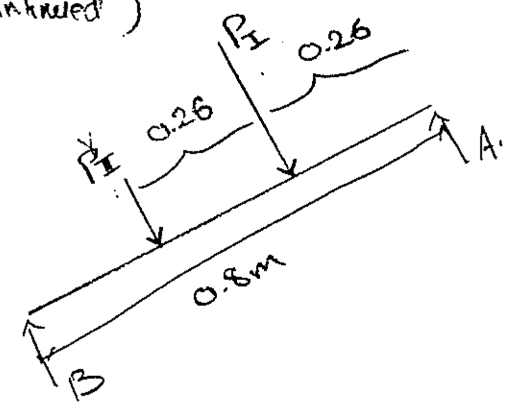
$P_I = \underbrace{0.5(0.5 \times 0.8)}_{A} \underbrace{(1000)}_{\rho H_2O} \underbrace{(9.81)}_{g} \underbrace{(0.45)}_{h} = 882.9N$

$P_{II} = 0.5(0.5 \times 0.8)(1000)(9.81)(0.93) = 1824.66N$



T=0 given in question

5.97 Continued)



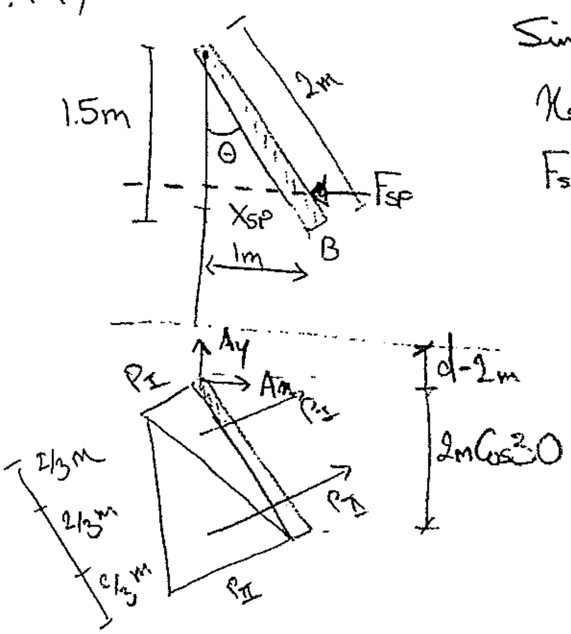
$+\circlearrowleft \sum M_A = 0 : 0.26P_I + 0.52P_{II} = 0.8B$

$\Rightarrow B = 1510.74\,N$ 53.1°

$\sum F_y = 0 \quad A + B = P_I + P_{II}$

$\longrightarrow A = 1197\,N$ 53.1°

5.99)



$\sin\theta = \frac{1}{2} \rightarrow \theta = 30°$

$x_{sp} = (1.5)\tan 30$

$F_{sp} = Kx_{sp} = \underbrace{(12)}_{K}(1.5)\tan 30 = 10.39\,KN$

$P_I = \frac{1}{2}\overbrace{[2\times3]}^{A}\overbrace{[1000]}^{\rho}\overbrace{[9.81]}^{g}\overbrace{[(d-2)]}^{h} = 29.43(d-2)$ KN

$P_{II} = \frac{1}{2}[2\times3][1000][9.81][d-2+2\cos30] =$

$= 29.43(d-0.2679)$ KN

$+\circlearrowleft \sum M_A = 0 : (\frac{2}{3})(29.43)(d-2) + (\frac{4}{3})(29.43)(d-0.2679) - (1.5)(10.39) = 0$

$\Rightarrow 58.86d = 65.3374$

$d = 1.1105$ m ✓

see y'all next week! :)

5.105)

	V	$\bar{x}$	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L+\frac{1}{4}h$	$\frac{1}{6}abh\left[L+\frac{1}{4}h\right]$

$$\Sigma V = ab\left[L+\frac{1}{6}h\right] \qquad \Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2+h\left(L+\frac{1}{4}\right)\right]$$

$$\bar{X}\Sigma V = \Sigma\bar{x}V$$

$$\bar{X}\left[\cancel{ab}\left(L+\frac{1}{6}h\right)\right] = \frac{1}{6}\cancel{ab}\left[3L^2+hL+\frac{1}{4}h^2\right]$$

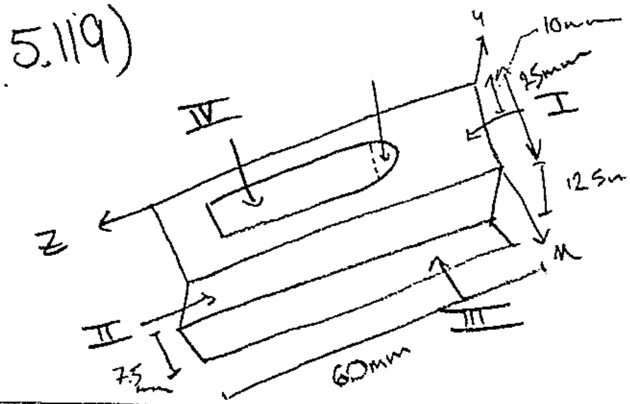
(a) $h=\frac{L}{2} \implies \boxed{\bar{X} = \frac{57}{104}L}$

(b) $$\bar{X}\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left[3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right]$$

$$\bar{X}=L \implies \frac{h^2}{L^2}=12$$

$$\boxed{\frac{h}{L}=2\sqrt{3}}$$

5.119)



$$A_{V} = -\frac{\pi}{2}(6.25)^2 = -61.36\,mm^2$$

$$\bar{z}_{V} = 22.5 - \frac{4(6.25)}{3\pi} = 19.85\,mm$$

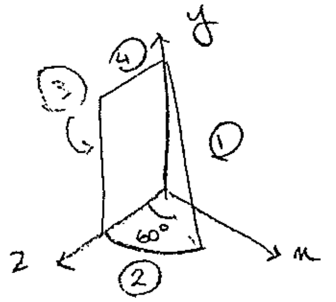
	A	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$ mm³	$\bar{y}A$ mm³	$\bar{z}A$ mm³
I	(25)(60) = 1500	12.5	0	30	18 750	0	45000
II	(12.5)(60) = 750	25	-6.25	30	18750	-4687.5	22500
III	(7.5)(60) = 450	28.75	-12.5	30	12 937.5	-5625	13 500
IV	-(12.5)(30) = -375	10	0	37.5	-3750	0	-14062.5
V	-61.36	10	0	19.85	-613.6	0	-1218.0
Σ	2263.64				46074	-10313	65720

$\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(2263.64) = 46074 \longrightarrow \bar{X} = 20.4\,mm$

$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(2263.64) = -10313 \longrightarrow \bar{Y} = -4.55\,mm$

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\bar{Z}(2263.64) = 65720 \longrightarrow \bar{Z} = 29.0\,mm$

5.125)



wire homogenous → center of gravity coincides with centroid of the corresponding line

$\bar{x}_1 = 0.3\sin 60 = 0.15\sqrt{3}\,m$

$\bar{z}_1 = 0.3\cos 60 = 0.15\,m$

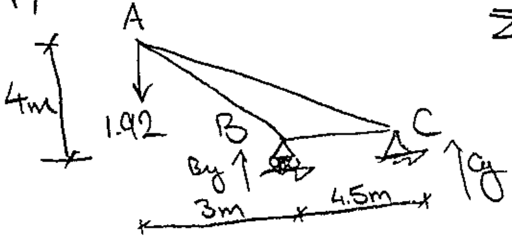
$\bar{x}_2 = \left(\frac{0.6\sin 30}{\pi/6}\right)\sin 30 = \frac{0.9}{\pi}$ $\quad \bar{z} = \left(\frac{0.6\sin 30}{\pi/6}\right)\cos 30 = \frac{0.9}{\pi}\sqrt{3}$

$L = \left(\frac{\pi}{3}\right)(0.6) = 0.2\pi\,m$

	L m	$\bar{x}$ m	$\bar{y}$ m	$\bar{z}$ m	$\bar{x}L$ m	$\bar{y}L$ m	$\bar{z}L$ m
1	1.0	$0.15\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2π	$0.4/\pi$	0	$0.9\sqrt{3}/\pi$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.40	0.18
Σ	3.0283				0.43981	1.20	1.12177

$\bar{X} = 0.1452\,m$ $\bar{Y} = 0.396\,m$ $\bar{Z} = 0.370\,m$

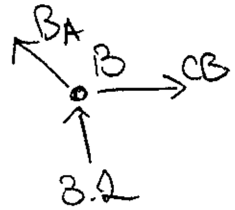
6.1)



$\Sigma M_B = 0$

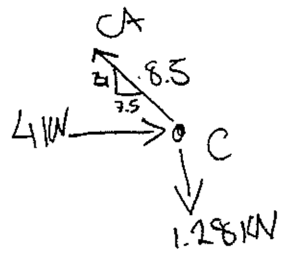
$1.92 \times 3 + 4.5 \times C_y = 0 \rightarrow C_y = -1.28\text{ KN}$

$\Sigma F_y = 0 \quad B_y + C_y = 1.92 \rightarrow B_y = 3.2\text{ KN}$



$\Sigma F_y = 0 \quad 3.2 + BA\frac{4}{5} = 0 \rightarrow \underline{BA = 4\text{KN (C)}}$

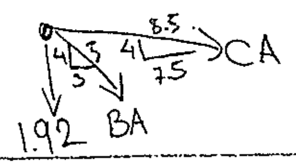
$\Sigma F_x = 0 \quad CB + BA\frac{3}{5} = 0 \rightarrow \underline{CB = 2.4\text{ KN (C)}}$



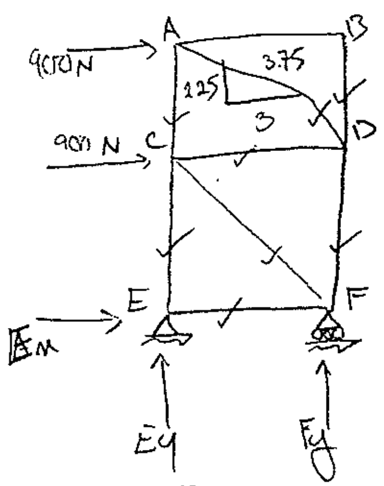
$1.28 = \frac{4}{8.5} CA \Rightarrow \underline{CA = 3.44}$

$1.92 + CA\frac{4}{8.5} - \frac{4}{5}BA = 0$ ✓

CHECK:



6.6)



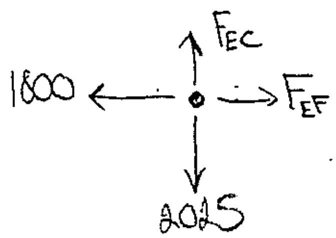
$\Sigma M_E = 0$

$900(2.25 + 4.50) = 3F_y$

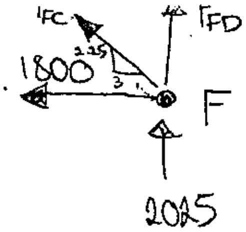
$\rightarrow F_y = 2025 \uparrow$

$\Sigma F_y = 0 \quad E_y = 2025 \downarrow$

$\Sigma F_x = 0 \quad E_x = 1800 \leftarrow$



$\begin{cases} F_{EF} = 1800 \ (T) \\ F_{EC} = 2025 \ (T) \end{cases}$

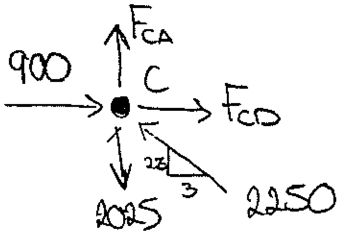


$$\sum F_x = 0$$

$$1800 + \frac{3}{3.75}F_{FC} = 0 \Rightarrow F_{FC} = 2250\,N \searrow (C)$$

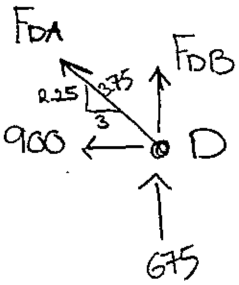
$$\sum F_y = 0$$

$$2025 - 2250\frac{2.25}{3.75} + F_{FD} = 0 \Rightarrow F_{FD} = 675\,N\ (C)$$



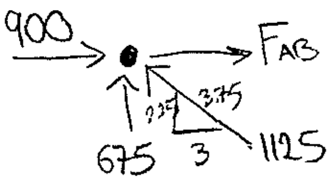
$$\sum F_x = 0 \quad 900 + F_{CD} - 2250 \times \frac{3}{3.75} = 0 \quad : F_{CD} = 900\,N\ (T)$$

$$\sum F_y = 0 \quad -2025 + 2250 \times \frac{2.25}{3.75} + F_{CA} = 0 \quad : F_{CA} = 675\ (T)$$



$$\sum F_y = 0: \quad 675 + F_{DB} + \frac{2.25}{3.75}F_{DA} = 0 \longrightarrow F_{DB} = 0\,N$$

$$\sum F_x = 0: \quad 900 + \frac{3}{3.75}F_{DA} = 0 \longrightarrow F_{DA} = 1125\,N\ (C)$$



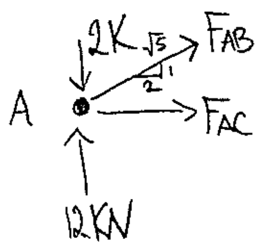
$$\sum F_x = 0$$

$$900 + F_{AB} - 1125\frac{3}{3.75} = 0 \Rightarrow F_{AB} = 0$$

6.21) $\sum F_x = 0 \longrightarrow A_x = 0$

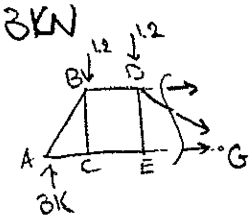
$$\sum M_L = 0 \qquad 2(36) + 4\left(\overbrace{30+24+18+12+6}^{90}\right) = 36A_y$$

$$\Rightarrow A_y = 12\,KN$$



$$\sum F_y = 0: \quad 12 - 2 + F_{AB}\frac{1}{\sqrt{5}} = 0 \quad : F_{AB} = -10\sqrt{5} = 22.36N\ (C)$$

6.43) $A_y = L_y = 3\,KN$



$\Sigma M_G = 0$:

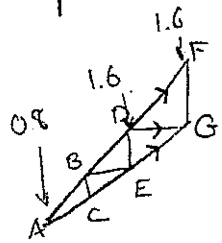
$(1.2)(8) + (1.2)(4) - (3)(10.25) - F_{DF}(3) = 0$

$F_{DF} = 5.45\,KN\,(C)$

$\Sigma F_y = 0$: $3 - 1.2 - 1.2 - \frac{3}{5} F_{DG} = 0 \rightarrow F_{DG} = 1.00\,KN\,(T)$

$+\circlearrowleft \Sigma M_D = 0 \quad (1.2)(4) - (3)(6.25) + F_{EG}(3) = 0 \rightarrow F_{EG} = 4.65\,KN\,(T$

6.49) $A_y = L_y = 4.8\,KN$



$+\circlearrowleft \Sigma M_G = 0 : (0.8)(24) + (1.6)(8) - (4.8)(24) - \frac{8 F_{DF}}{\sqrt{8^2 + 3.5^2}}(6) = 0$

$\rightarrow F_{DF} = 10.48\,KN\,(C)$

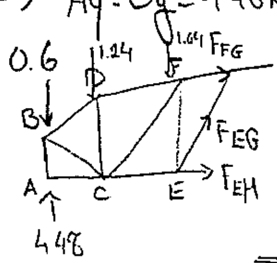
$+\circlearrowleft \Sigma M_A = 0 : -(1.6)(8) - (1.6)(16) - \frac{2.5 F_{DG}}{\sqrt{8^2 + 2.5^2}}(16) = 0$

$\rightarrow F_{DG} = 3.35\,KN\,(C)$

$+\circlearrowleft \Sigma M_D = 0 : (0.8)(16) + (1.6)(8) - (4.8)(16) + \frac{8 F_{EG}}{\sqrt{8^2 + 1.5^2}}(4) = 0$

$\rightarrow F_{EG} = 13.02\,KN\,(T)$

6.53) $A_y = O_y = 4.48\,KN$



$+\circlearrowleft \Sigma M_E = 0 : (0.6)(7.44) + (1.24)(3.84) - (4.48)(7.44) - \frac{6 F_{FG}}{\sqrt{6^2 + 1.75^2}}(4.8) = 0$

$\rightarrow F_{FG} = 5.23\,KN\,(C)$

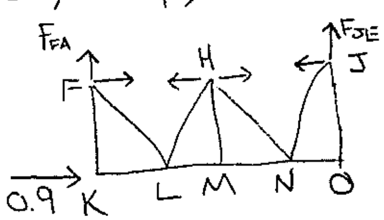
$+\circlearrowleft \Sigma M_G = 0 : F_{EH}(5.5) + (0.6)(9.84) + (1.24)(6.24) + (1.04)(2.4)$

$-(4.48)(9.84) = 0 \Rightarrow F_{EH} = 5.08\,KN\,(T)$

$\Sigma F_y = 0 : \frac{5.50}{\sqrt{5.5^2 + 2.4^2}} F_{EG} + \frac{1.75}{\sqrt{6^2 + 1.25^2}}(-5.23) + 4.48 - 0.6 - 1.24 - 1.04 = 0$

$\Rightarrow F_{EG} = 0.14\,KN\,(C)$

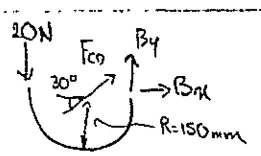
6.62) $+\circlearrowleft \Sigma M_A = 0 : \quad K_x(8) - (1.2)(6) = 0 \rightarrow K_x = 0.9\,KN$



$+\circlearrowleft \Sigma M_F = 0 : \quad F_{JE}(12) + 4(0.9) - (1.2)(6) = 0 \rightarrow F_{JE} = 0.3\,KN\,(T)$

$\Sigma F_y = 0 : \quad F_{FA} + 0.3 - 1.2 = 0 \rightarrow F_{AF} = 0.9\,KN\,(T)$

6.77)

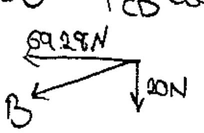


DC: 2 FORCE MEMBER

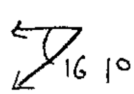
$+\circlearrowleft \Sigma M_B = 0 : (20)(2 \times 0.15) - (F_{CD} \sin 30) \overset{=0.15}{R} = 0 \rightarrow F_{CD} = 80\,N\,(T)$

$+\circlearrowleft \Sigma M_C = 0 \quad (20)(0.15) + (B_y)(0.15) \rightarrow B_y = 20\,N \downarrow$

$\Sigma F_x = 0 \quad F_{CD} \cos 30 + B_x = 0 \Rightarrow B_x = 69.28\,N \leftarrow$

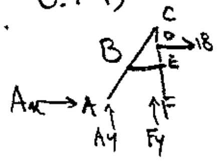


$\Rightarrow B = 72.11\,N$



(2)

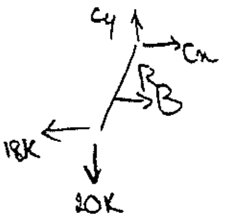
6.79) $\Sigma F_x = 0 : A_x + 18 = 0 \Rightarrow A_x = 18\,kN \leftarrow$



$+\circlearrowleft \Sigma M_F = 0 \quad (-18)(4) - A_y(3.6) = 0 \Rightarrow A_y = 20\,kN \downarrow$

$\Sigma F_y = 0 \; -20 + E_y = 0 \Rightarrow F_y = 20\,kN \uparrow$

BE : 2 FORCE MEMBER $\Rightarrow R_B \rightarrow$ along BE

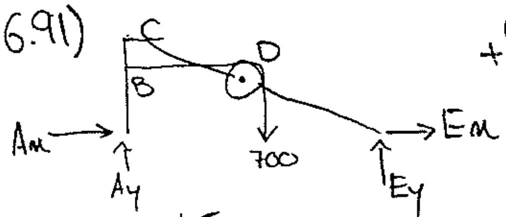


$+\circlearrowleft \Sigma M_C = 0 : 4R_B - (18)(6) + (20)(3.6) = 0 \Rightarrow R_B = 9\,kN \longrightarrow$

$\Sigma F_x = 0 \quad C_x - 18 + 9 = 0 \Rightarrow C_x = 9\,kN \longrightarrow$

$\Sigma F_y = 0 \quad C_y - 20 = 0 \Rightarrow C_y = 20\,kN \uparrow$

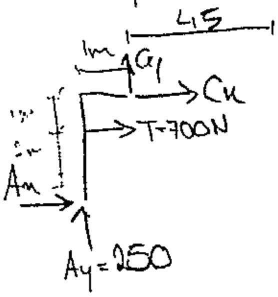
6.91)



$+\circlearrowleft \Sigma M_A = 0 : 7E_y - (700)(4.5) = 0 \quad E_y = 450\,N \uparrow$

$\Sigma F_y = 0 : A_y + 450 - 700 = 0 \quad A_y = 250\,N \uparrow$

$\Sigma F_x = 0 : A_x + E_x = 0$

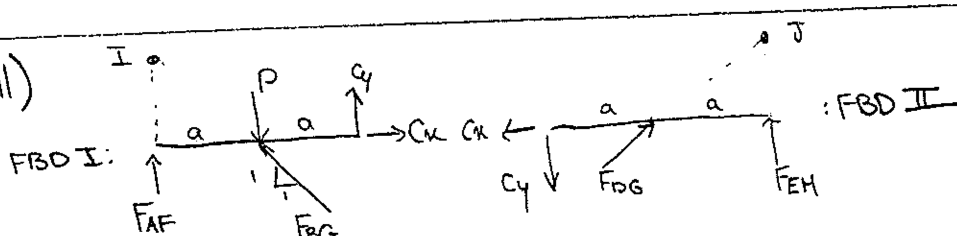


$+\circlearrowleft \Sigma M_C = 0 \quad (700)(1) + 3A_x - (250)(1) = 0$

$\Rightarrow A_x = 150\,N \longleftarrow$

$\Rightarrow E_x = 150\,N \longrightarrow$

6.111)



FBD I: $\Sigma M_I = 0 \quad 2aC_y + aC_x - aP = 0 \Rightarrow 2C_y + C_x = P$

FBD II: $\Sigma M_J = 0 \quad 2aC_y - aC_x = 0 \Rightarrow 2C_y - C_x = 0$

$\Rightarrow C_y = P/4 \Rightarrow C_x = \frac{P}{2}$

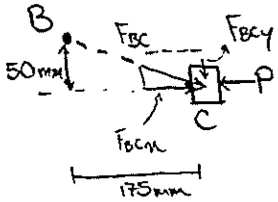
FBD I: $\Sigma F_x = 0 : \frac{-1}{\sqrt{2}} F_{BG} + C_x = 0 \quad F_{BG} = C_x\sqrt{2} \Rightarrow F_{BG} = \frac{\sqrt{2}}{2} P \; (C)$

$\Sigma F_y = 0 \quad F_{AF} - P + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}(P)\right) + P/4 = 0 \Rightarrow F_{AF} = P/4 \; (C)$

FBD II: $\Sigma F_x = 0 : -C_x + \frac{1}{\sqrt{2}} F_{DG} = 0 \Rightarrow F_{DG} = C_x\sqrt{2} \Rightarrow F_{DG} = \frac{\sqrt{2}}{2} P \; (C)$

$\Sigma F_y = 0 : -C_y + \frac{1}{\sqrt{2}}\left[\frac{\sqrt{2}}{2}(P)\right] + F_{EH} = 0 \Rightarrow F_{EH} = P/4 \; (T)$

6.129)



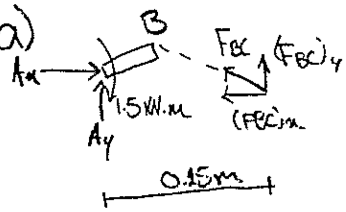
$\Sigma F_x = 0 \quad (F_{BC})_x - P = 0$

$P = (F_{BC})_x \quad (1)$

BC: 2 FORCE MEMBER $\quad \dfrac{(F_{BC})_y}{50} = \dfrac{(F_{BC})_x}{175} \; ; \; (F_{BC})_y = \dfrac{50}{175}(F_{BC})_x$

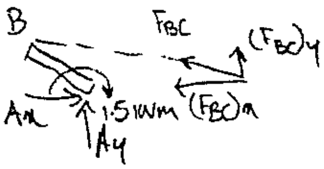
$(1) \rightarrow (F_{BC})_y = \dfrac{50}{175}P \; ; \; (F_{BC})_y = \dfrac{2}{7}P$

(a)



$+\circlearrowleft \Sigma M_A = 0 : (F_{BC})_y(0.25) - 1.5 = 0 : \frac{2}{7}P(0.25) = 1.5 \Rightarrow P = 21\text{ KN} \leftarrow$

(b)

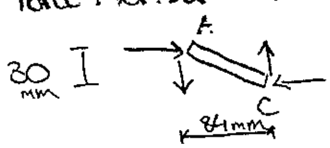


$+\circlearrowleft \Sigma M_A = 0 \quad (F_{BC})_y(0.1) - 1.5 = 0$

$\frac{2}{7}P(0.1) = 1.5$

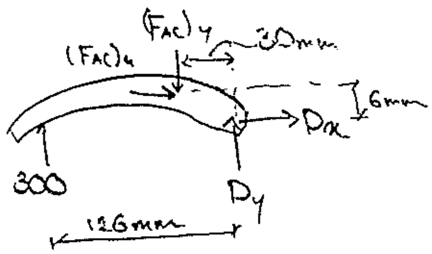
$\rightarrow P = 52.5\text{ KN} \leftarrow$

6.148) AC: Two Force Member :



$\dfrac{(F_{AC})_x}{84} = \dfrac{(F_{AC})_y}{30} \Rightarrow (F_{AC})_x = 2.8\,(F_{AC})_y$

Lower Handle FBD:

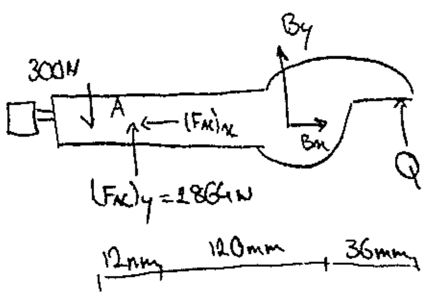


$+\circlearrowleft \Sigma M_D = 0$

$(F_{AC})_y(30) - \underbrace{(F_{AC})_x}_{2.8(F_{AC})_y}(6) - (300)(126) = 0$

$\Rightarrow (F_{AC})_x = 2864\text{ N}$

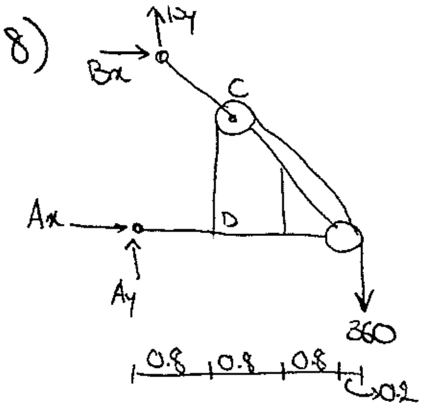
Upper Handle FBD:



$+\circlearrowleft \Sigma M_G = 0 \quad : (300)(132) - (2864)(120) + Q(36) = 0$

$\Longrightarrow Q = 8.46\text{ KN}$

7.18)



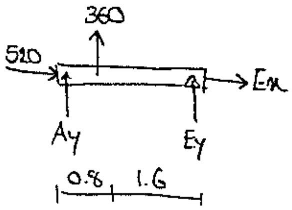
$+\circlearrowleft \sum M_A = 0 : -B_x(1.8) - (360)(2.6) = 0$

$B_x = 520\,N \leftarrow$

$\sum F_x = 0 \quad A_x - 520 = 0 \quad A_x = 520\,N \rightarrow$

$\sum F_y = 0 \quad A_y + B_y = 360\,N \quad (I)$

AE FBD:

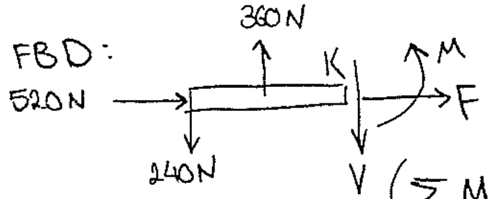


$+\circlearrowleft \sum M_E = 0 :$

$-A_y(2.4) - (360)(1.6) = 0 \Rightarrow A_y = 240 \downarrow$

$(I) \Rightarrow B_y = 600\,N \uparrow$

AK FBD:



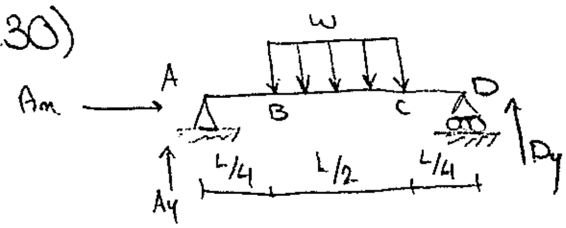
$\sum F_x = 0 \quad 520 + F = 0 \Rightarrow F = 520\,N \leftarrow$

$\sum F_y = 0 \quad 360 - 240 - V = 0 \Rightarrow V = 120\,N \downarrow$

$+\circlearrowleft \sum M_K = 0 \quad M + (240)(1.6) - (360)(0.8) = 0$

$\Rightarrow M = 96\,Nm \circlearrowright$

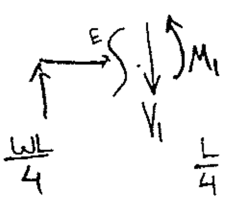
7.30)



$A_y + D_y = \frac{wL}{2}$

$\sum M_A = 0 \Rightarrow D_y = \frac{wL}{4} \Rightarrow A_y = \frac{wL}{4}$

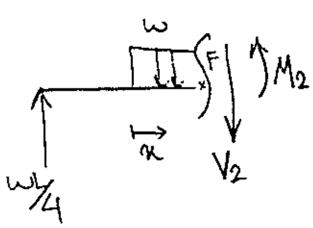
$A_x = 0$



$\frac{L}{4} \geq x \geq 0$

$V_1 = \frac{wL}{4}$

$\sum M_E = 0 \quad M_1 - \frac{wL}{4}x = 0 \Rightarrow M_1 = \frac{wL}{4}x \quad \begin{cases} x = 0 & M_1 = 0 \\ x = L/4 & M_1 = \frac{wL^2}{16} \end{cases}$



$\frac{L}{2} \geq x \geq 0$

$\sum F_y = 0 \quad \frac{wL}{4} - wx - V_2 = 0 \quad V_2 = w\left(\frac{L}{4} - x\right)$

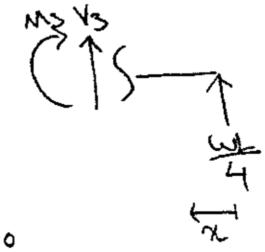
$\sum M_F = 0 \Rightarrow M_2 - \frac{wL}{4}\left(\frac{L}{4} + x\right) + \frac{wx^2}{2} = 0$

$M_2 = w\left(\frac{L^2}{16} + \frac{Lx}{4} - \frac{x^2}{2}\right) \quad \begin{cases} x = 0 & M_2 = \frac{wL^2}{16} \\ x = \frac{L}{2} & M_2 = \frac{wL^2}{16} \end{cases}$

$M_2 = max \longrightarrow \quad \frac{L}{4} - x = 0 \quad x = \frac{1}{4}L$

$\Rightarrow M_{max} = \frac{3wL^2}{16}$

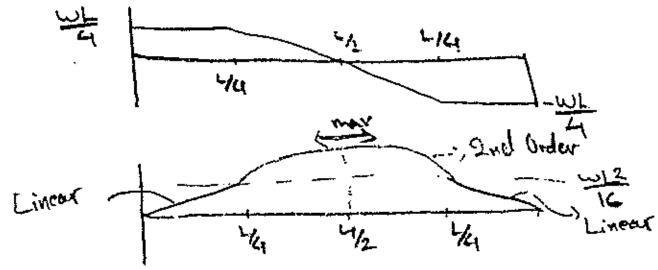
7.30 continued)



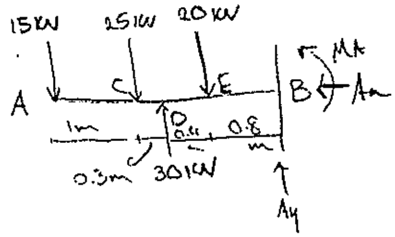
$V_3 = -\frac{wL}{4}$

$M_3 = \frac{wL}{4}x \longrightarrow \begin{cases} x=0 & M_3 = 0 \\ x = L/4 & M_3 = \frac{wL^2}{16} \end{cases}$

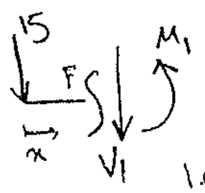
$\frac{L}{4} \geq x \geq 0$



7.35)



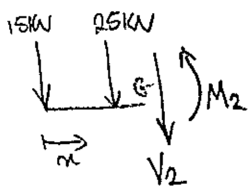
$A_x = 0$ KN
$A_y = 30$ KN
$M_A = 55$ KNm



$V_1 = -15$ KN

$M_1 = -15x \quad \begin{cases} x=0\text{ m} \rightarrow M_F = 0 \text{ KNm} \\ x = 1\text{m} \rightarrow M = -15 \text{ KNm} \end{cases}$

$1.0 \geq x \geq 0$

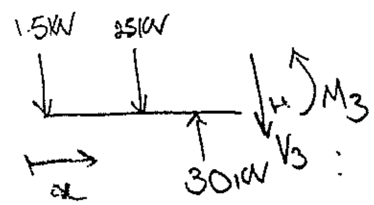


$V_2 = -40$ KN

$\sum M_G = 0 : M_2 + 15x + 25(x-1) = 0$

$M_2 = 25 - 40x \quad \begin{cases} x = 1\text{m} \longrightarrow M_2 = -15 \text{ KNm} \\ x = 1.3\text{m} \longrightarrow M_2 = -27 \text{ KNm} \end{cases}$

$1.3 \geq x \geq 1$



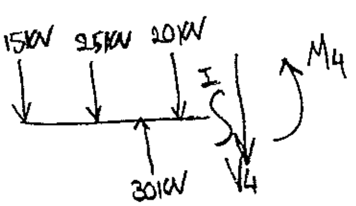
$V_3 + 15 + 25 - 30 = 0 : V_3 = -10$ KN

$\sum M_H = 0 \quad M_3 - 30(x-1.3) + 25(x-1) + 15x = 0$

$M_3 - 30x + 39 + 25x - 25 + 15x = 0$

$M_3 = -10x - 14 \quad \begin{cases} x = 1.3 & M_3 = -27 \text{ KNm} \\ x = 1.7 & M_3 = -31 \text{ KNm} \end{cases}$

$1.7 \geq x \geq 1.3$



$V_4 + 60 - 30 = 0 \; ; V_4 = -30$ KN

$M_4 + 15x + 25(x-1) - 30(x-1.3) + 20(x-1.7) = 0$

$M_4 + 15x + 25x - 25 - 30x + 39 + 20x - 34 = 0$

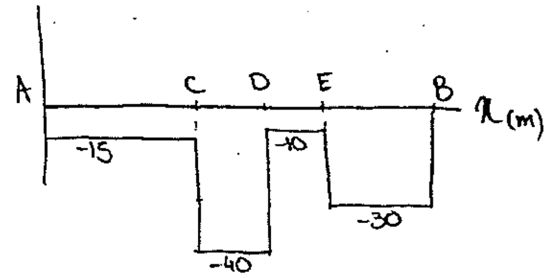
$M_4 + 15x + 25x - 25 - 30x \ldots$

$M_4 = -30x + 20 \quad \begin{cases} x = 1.7 \longrightarrow M_4 = -31 \\ x = 2.5 \rightarrow M_4 = -55 \end{cases}$

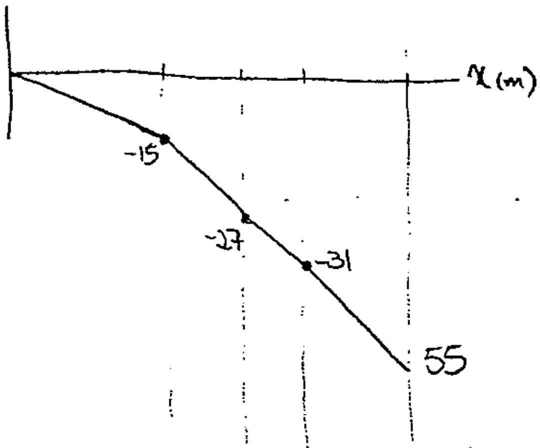
$2.5 \geq x \geq 1.7$

7.35 continued)

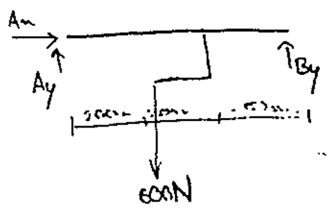
(V)



(M)



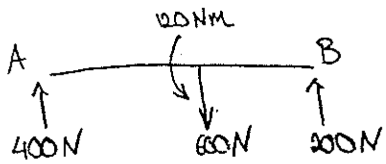
7.49)

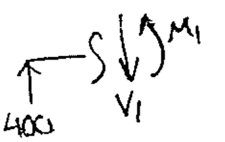


$\circlearrowleft + \sum M_A = 0$: $B_y(0.6) - (600)(0.2) = 0$: $B_y = 200N \uparrow$

$\sum F_x = 0$: $A_x = 0$

$\sum F_y = 0$ $A_y - 600 + 200 = 0$: $A_y = 400N \uparrow$





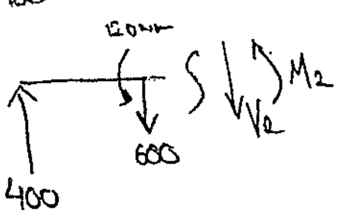
$V_1 = 400N$

$M_1 = 400x$ Nm

just to the left of C $x = 400$mm

$V_1 = 400N$

$M_1 = 160$ Nm

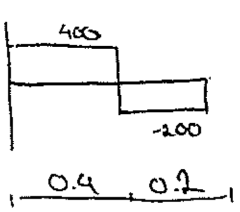


$V_2 + 600 - 400 = 0 \rightarrow V_2 = -200N$

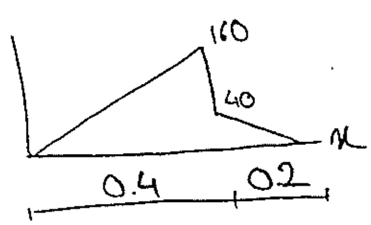
$M_2 - 400x + 600(x - 0.4) + 120 = 0$

$0.6 \geq x \geq 0.4 \Rightarrow$ $\begin{cases} x = 0.4 \rightarrow & M_2 = 40 \text{ Nm} \\ x = 0.6 \rightarrow & M_2 = 0 \text{ Nm} \end{cases}$

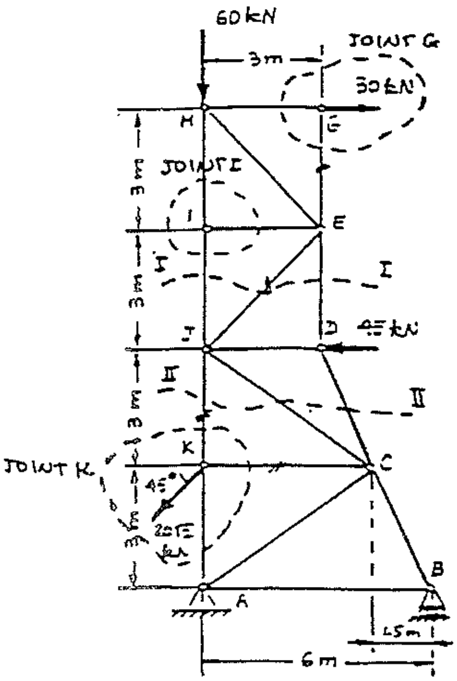
(V)



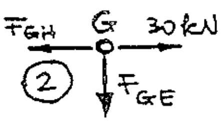
(M)



1.



FBD JOINT G:

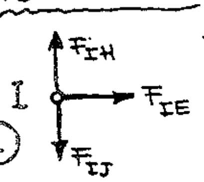


2

$\overset{+}{\downarrow} \Sigma F_y = 0;$

$\rightarrow \boxed{F_{GE} = 0}$ 4

FBD JOINT I:



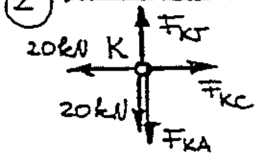
2

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$\rightarrow \boxed{F_{IE} = 0}$ 3

- **METHOD of JOINTS:**

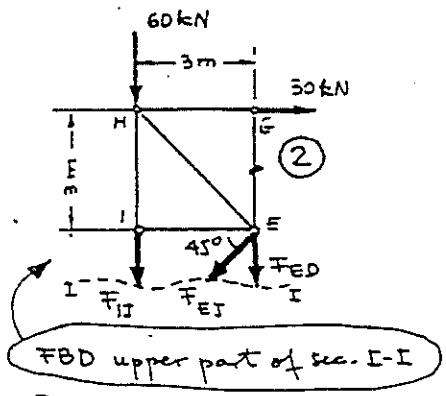
2 **FBD JOINT K:**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{KC} - 20 = 0 \quad \boxed{F_{KC} = 20\,k}$

6 TENSIO

- **METHOD of SECTIONS:**



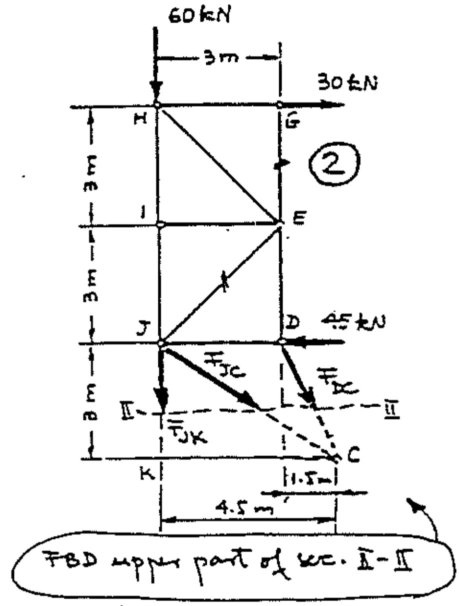
FBD upper part of sec. I-I

$\overset{+}{\rightarrow} \Sigma F_x = 0;$

$-F_{EJ}\frac{\sqrt{2}}{2} + 30 = 0$

$\rightarrow \boxed{F_{EJ} = 42.43\,kN}$ 6

TENSION



FBD upper part of sec. II-II

$\overset{\curvearrowleft}{+} \Sigma M_C = 0;$

$F_{JK}(4.5) + 60(4.5) - 30(9) + 45(3) = 0$

$\rightarrow F_{JK} = -30\,kN \quad \boxed{F_{JK} = 30\,kN}$

COMPRESSION

$\Sigma \quad 35$