Concordia UNIVERSITY

Faculty of Engineering and Computer Science

FINAL EXAMINATION ENGR 242/2 Statics Sections: T,V,X,YY Date: December 17, 2005

Instructors: Professors Bagchi, Dargahi, Hammad, Nokken, Stathopoulos (coordinator)

Materials allowed: Non-programmable calculators

Time allowed: 3 hours

Special instructions: For each problem show the relevant free body diagram if appropriate. Problems carry equal weights. <u>Solve ALL 6 problems given.</u>

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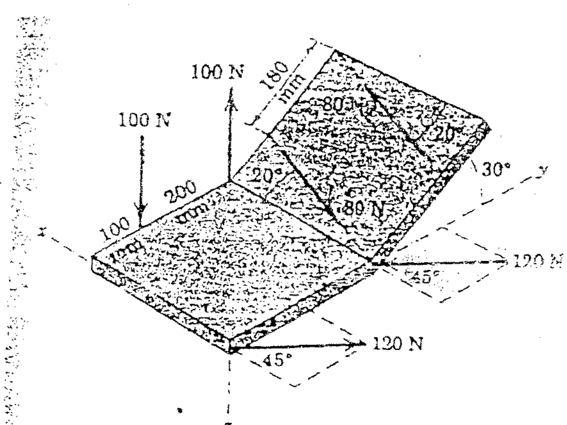
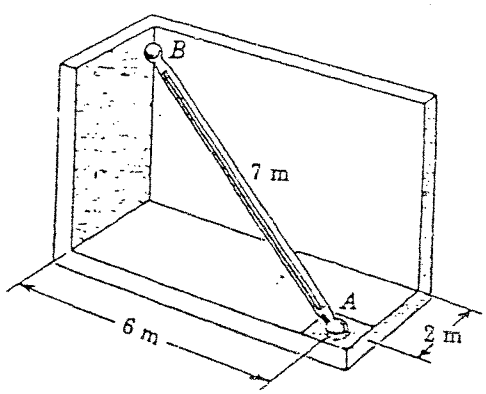
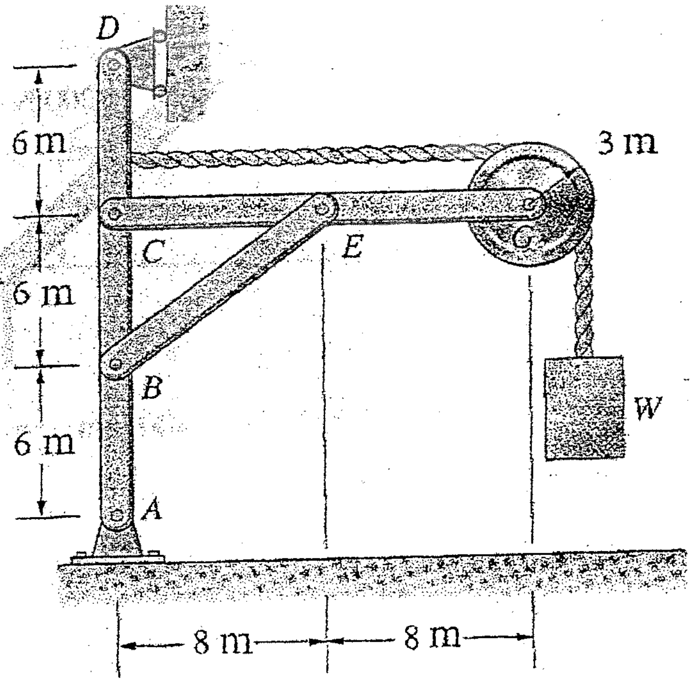
1. Three couples are formed by the three pairs of equal and opposite forces. Determine the resultant of M (moment vector) of three couples.



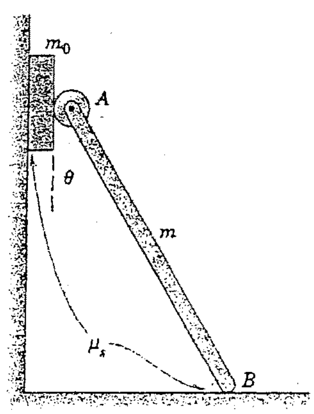
2. The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A on the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.



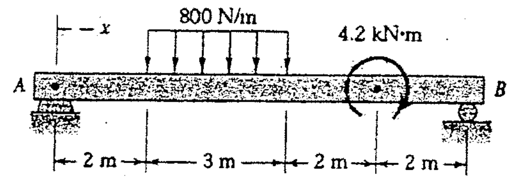
3. The frame in the following figure supports a suspended weight W = 4.0 kN. Determine the forces at each joint in member ABCD.



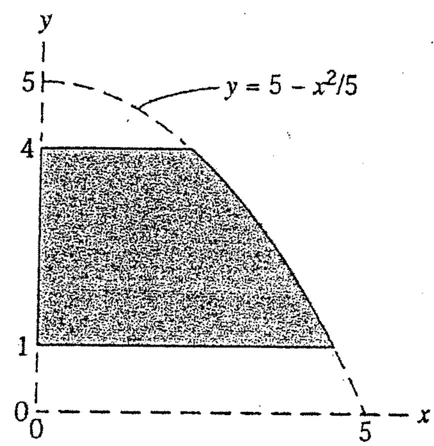
4. A block of mass m_0 is placed between the vertical wall and the small ideal roller at the upper end A of the uniform slender bar of mass m. The lower end B of the bar rests on the horizontal surface. If the coefficient of static friction is μ_s at B and also between the block and the wall, determine a general expression for the minimum value θ_{min} of θ for which the block will remain in equilibrium.



5. (a) Plot the shear and moment diagrams for the beam.
   (b) Find the value and location of the maximum bending moment.



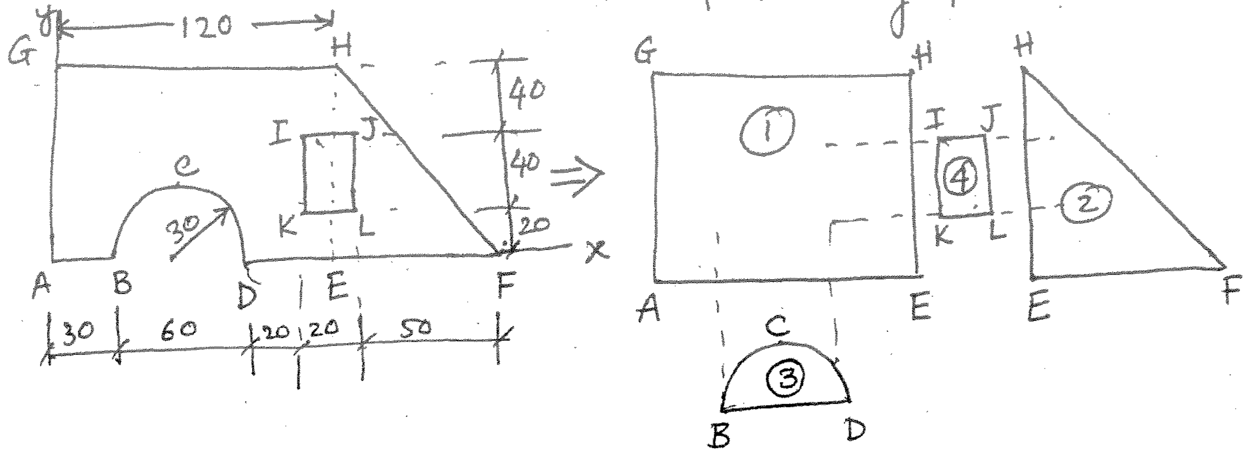
6. Locate the centroid of the shaded area by computing its x and y coordinates; use the method of integration.



ENGR 242: Solution to the Final Exam 2004 (12 pages)

Prob 1.

Break the area into the following parts



Part	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	12000	60	50	720 000	600 000
2	3000	140	$\frac{100}{3}$	420 000	100 000
3	−1413.72	60	$\frac{40}{\pi}$	−84823.2	−18 000
4	−800	120	40	−96000	−32000
	12786.3			959 176.8	650 000

Notes:

① Areas of segments 1 and 2 are positive and The areas of segments 3 and 4 are negative

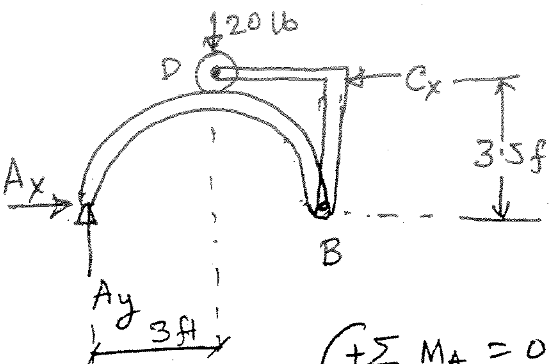
② The distances of the centroids of each segments are determined with respect to the x and y axes as shown.

③ The area of the semi circular part (part 3) is $\frac{1}{2}\pi r^2 = -1413.72$ and $\bar{y} = \frac{4r}{3\pi} = \frac{40}{\pi}$

Now, $\bar{X} = \frac{\Sigma A_i \bar{X}_i}{\Sigma A_i} = \frac{959176.8}{12786.3} = 75$, $\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{650000}{12786.3} = 50.8$

Prob 2 :

① Draw the FBD of the whole frame and find the reactions



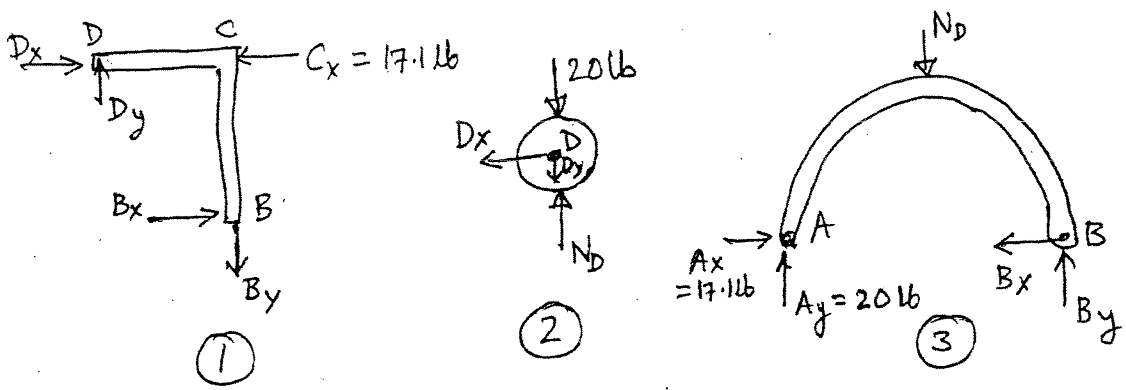
$\circlearrowleft + \Sigma M_A = 0$ gives,

$-20(3) + C_x(3.5) = 0 \quad \Rightarrow C_x = 17.1$ lb

$\underset{+\rightarrow}{\Sigma F_x = 0}$ gives, $A_x - 17.1 = 0 \Rightarrow A_x = 17.1$ lb

$+\uparrow \Sigma F_y = 0$ gives, $A_y - 20 = 0 \Rightarrow A_y = 20$ lb

② Now draw the FBDs of the individual frame elements and the disk.



considering FBD 3 (member AB)

$\underset{+\rightarrow}{\sum F_x = 0}$ gives, $\overset{A_x}{17.1} - B_x = 0 \quad \Rightarrow \quad B_x = 17.1 \text{ lb}$

$\circlearrowleft + \sum M_B = 0$ gives, $-\overset{A_y}{20}(6) + N_D(3) = 0 \quad \Rightarrow \quad N_D = 40 \text{ lb}$

$\uparrow + \sum F_y = 0$ gives $\overset{A_y}{20} - \overset{N_D}{40} + B_y = 0 \quad \Rightarrow \quad B_y = 20 \text{ lb}$

Considering FBD 2 (the disk)

$\underset{+\rightarrow}{\sum F_x = 0}$ gives, $D_x = 0$

$\uparrow + \sum F_y = 0$ gives, $\overset{N_D}{40} - 20 - D_y = 0 \quad \Rightarrow \quad D_y = 20 \text{ lb}$

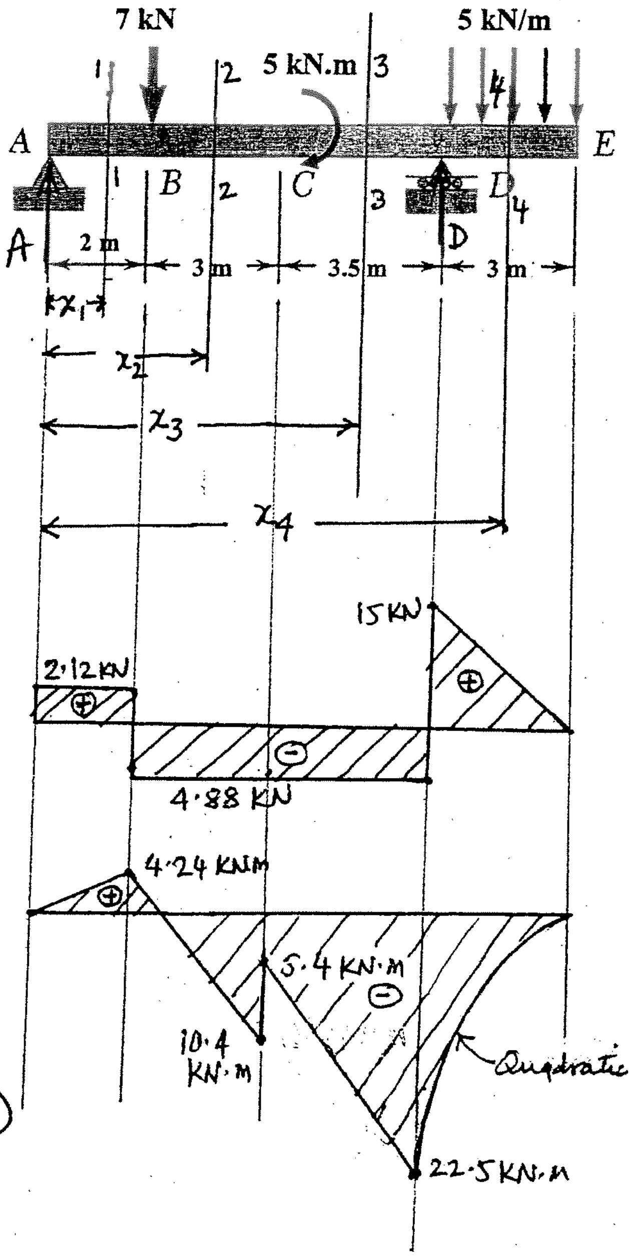
In this case, FBD 1 is not used for the solution; however, it can be used for checking the results.

Ans: Reactions at D: $D_x = 0$, $D_y = 20 \text{ lb} \downarrow$
(considering the disk)

Reactions at B: $B_x = 17.1 \text{ lb} \leftarrow$
$B_y = 20 \text{ lb} \uparrow$
(considering member AB)

Q3

(3) For the beam shown, (a) draw the shear force and bending moment diagrams, and (b) determine the maximum absolute values of shear force and bending moment.



Find the support Reactions

$\circlearrowleft + \Sigma M_A = -(2)(7) - 5 + (8.5)(D) - (10)(5\times3) = 0$

$D = 19.88$ KN

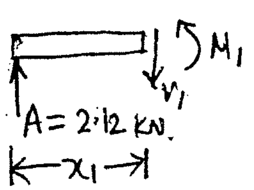
$\Sigma F_y = A + D - 7 - (5\times3) = 0$

$A = 7 + 15 - 19.88$

$= 2.12$ KN

a) shear Force and Bending moment diagrams

1) Consider the left side of section 1-1 $(0 \le x_1 \le 2m)$



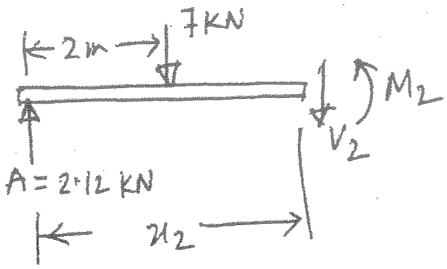
$\Sigma F_y = A - V_1 = 0 \rightarrow V_1 = A = 2.12$ KN

$\circlearrowleft + \Sigma M_1 = M_1 - (x_1) A = 0 \rightarrow M_1 = 2.12 x$

at $x_1 = 0$: $V_1 = 2.12$ KN and $M_1 = 0$

at $x_1 = 2m$: $V_1 = 2.12$ KN and $M_1 = 4.24$ KNm

2) consider the left side of section 2-2 ($2m \le x_2 \le 5m$)



$\Sigma F_y = A - 7 - V_2 = 0$

$\Rightarrow V_2 = A - 7$

$= 2.12 - 7$

$= -4.88 KN$

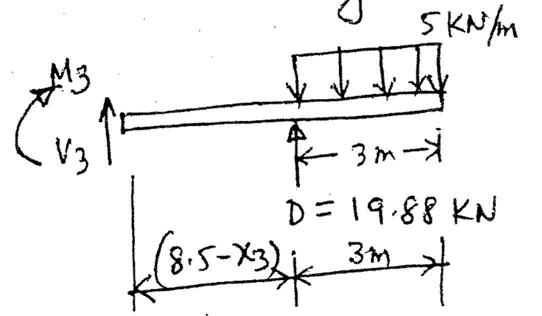
$\circlearrowleft + \Sigma M_2 = M_2 - (x_2)(A) + (x_2 - 2)(7) = 0$

$M_2 = 2.12 x_2 - 7(x_2 - 2)$

at $x_2 = 2m$: $V_2 = -4.88 KN$, $M_2 = 4.24 KN.m$

at $x_2 = 5m$: $V_2 = -4.88 KN$, $M_2 = -10.4 KN.m$

3) Consider the right side of section 3 ($5m \le x_3 \le 8.5m$)



$\Sigma F_y = V_3 + D - (3)(5) = 0$

$V_3 = 15 - 19.88 = -4.88 KN$

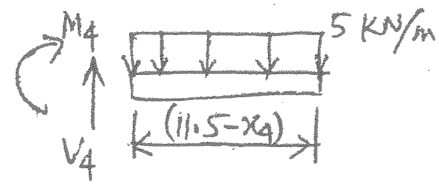
$\circlearrowleft + \Sigma M_3 = -M_3 + (8.5 - x_3) D - (8.5 - x_3 + 1.5) 15 = 0$

$M_3 = 19.88 (8.5 - x_3) - (10 - x_3)(15)$

at $x = 5m$: $V_3 = -4.88 KN$, $M_3 = -5.4 KN.m$

at $x = 8.5m$: $V_3 = -4.88 KN$, $M_3 = -22.5 KN.m$

(4) Consider the right side of section 4 $(8.5\,m \le x_4 \le 11.5\,m)$



$$\Sigma F_y = V_4 - 5(11.5 - x_4) = 0$$

$$V_4 = 5(11.5 - x_4)$$

$$+\Sigma M_4 = -M_4 - \left(\frac{11.5 - x_4}{2}\right)[5(11.5 - x_4)]$$

$$M_4 = -2.5(11.5 - x_4)^2$$

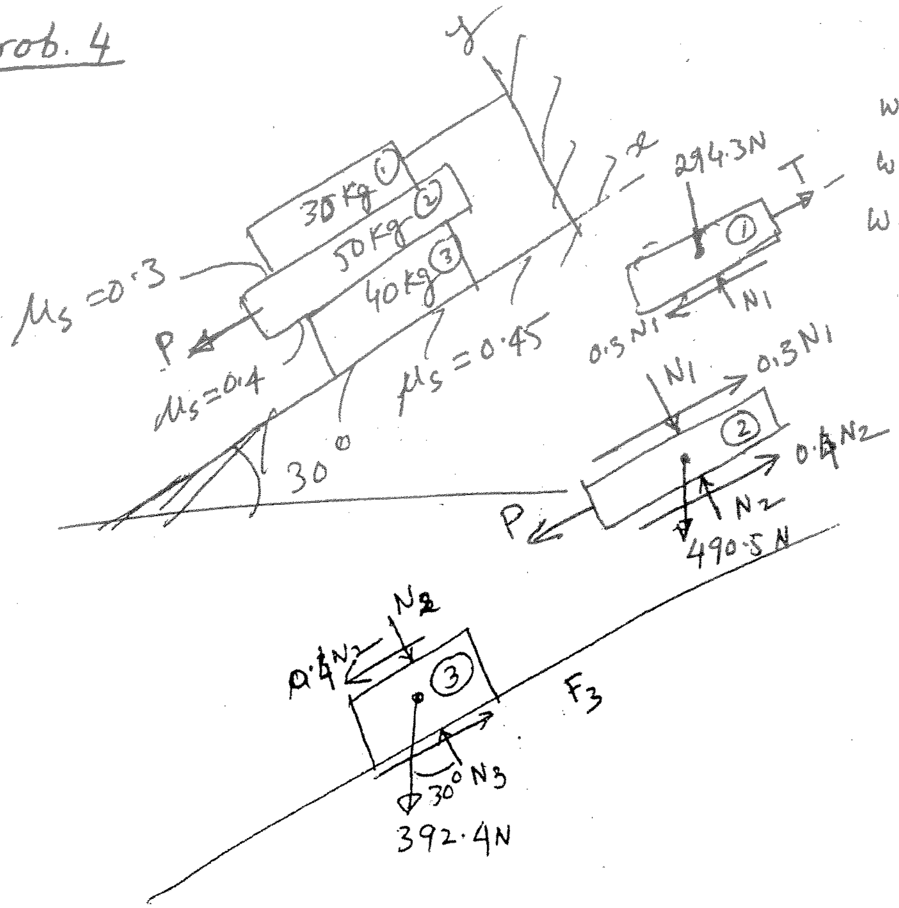
at $x_4 = 8.5\,m$: $V_4 = 15$ KN, $M_4 = -22.5$ KN.m

at $x_4 = 11.5\,m$; $V_4 = 0$, $M_4 = 0$

(b) Maximum shear force = 15 KN at D

Maximum bending moment = 22.5 KN.m at D

Prob. 4



$w_1 = 30 \times 9.81 = 294.3$ N

$w_2 = 50 \times 9.81 = 490.5$ N

$w_3 = 40 \times 9.81 = 392.4$ N

Case 1. Assume slippage on both faces of block 2.

FBD ①

$\Sigma F_x = -294.3 \sin 30^\circ + T - 0.3 N_1 = 0$ — (1)

$\Sigma F_y = -294.3 \cos 30^\circ + N_1 = 0 \rightarrow N_1 = 254.87$ N

substituting N_1 in Eqn. (1) $\rightarrow$

$T = 0.3 N_1 + 294.3 \sin 30^\circ = 223.61$ N

FBD ②

$\Sigma F_x = -P + 0.3 N_1 + 0.4 N_2 - 490.5 \sin 30^\circ = 0$ — (2)

$\Sigma F_y = -N_1 + N_2 - 490.5 \cos 30^\circ = 0$

or, $N_2 = N_1 + 490.5 \cos 30^\circ = 679.66$ N

substituting N_2 in Eqn (2)

$P = 0.3 N_1 + 0.4 N_2 - 490.5 \sin 30^\circ = 103.1$ N

FBD ③

$\Sigma F_y = N_3 - N_2 - 392.4 \cos 30^\circ = 0 \rightarrow N_3 = 679.66 + 392.4 \cos 30^\circ$
$= 1018.83$ N

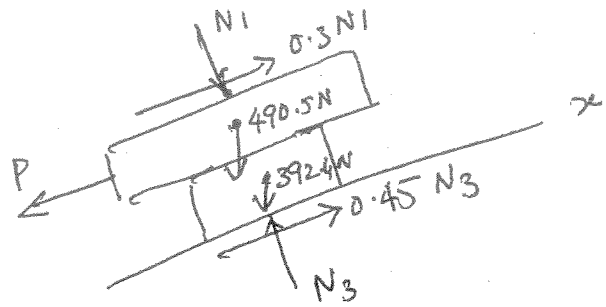
$\Sigma F_x = -0.4 N_2 - 392.4 \sin 30^\circ + \cancel{0.45} F_3 = 0$

$F_3 = 0.4(679.66) + 392.4 \sin 30^\circ = 468$ N

Max possible friction $F_{3\,max} = \mu_s N_3 = 0.45(1018.83) = 458.5$ N < 468 N

So, block 3 moves as well

case (2) Assume No slippage between middle and bottom blocks, block 2 and 3 move together.



$N_1 = 254.87\ N$
from case (1)

$$\Sigma F_x = 0.3N_1 + 0.45N_3 - P - 490.5\sin 30^\circ - 392.4\sin 30^\circ = 0 \quad (3)$$

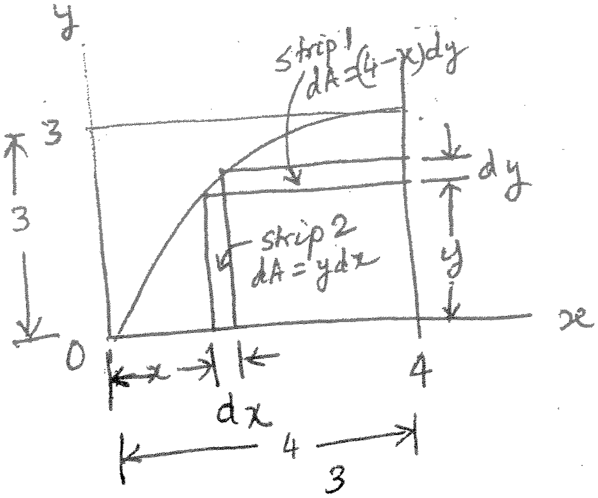
$$\Sigma F_y = -N_1 + N_3 - (490.5 + 392.4)\cos 30^\circ = 0$$

$$N_3 = N_1 + (490.5 + 392.4)\cos 30^\circ = 1019.48\ N$$

Substituting N_3 in (3)

$$P = 0.3(254.87) + 0.45(1019.48) - (490.5 + 392.4)\sin 30^\circ$$
$$= 93.78\ N$$ ✓

Prob. 5



Find the value of k

$x = ky^2$

for $x = 4,\ y = 3$

$4 = k\,9$

$k = 4/9$

then $x = \frac{4}{9}y^2 \rightarrow y = \frac{3}{2}\sqrt{x}$

$$I_x = \int_A y^2\,dA = \int_0^3 y^2(4-x)\,dy = \int_0^3 y^2\left(4-\frac{4}{9}y^2\right)dy$$

Consider strip 1

$$= \int_0^3 \left(4y^2 - \frac{4}{9}y^4\right)dy = 4\left[\frac{y^3}{3}\right]_0^3 - \frac{4}{9}\left[\frac{y^5}{5}\right]_0^3$$

$$= \frac{4}{\cancel{3}}\times \cancel{27}^{\,9} - \frac{4}{\cancel{9}}\times\frac{\cancel{9}\times 27}{5}$$

$$= 36 - 21.6 = 14.4$$

$$I_y = \int_A x^2\,dA = \int_0^4 x^2\cdot y\,dx = \int_0^4 x^2\cdot \frac{3}{2}\sqrt{x}\,dx$$

Consider strip 2

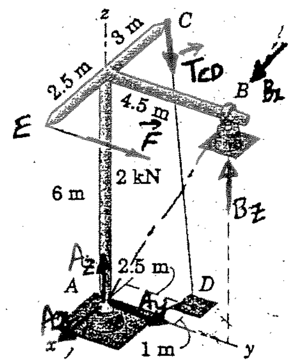
$$= \frac{3}{2}\int_0^4 x^{5/2}\,dx = \frac{3}{2}\cdot\left[\frac{x^{5/2+1}}{\frac{5}{2}+1}\right]_0^4$$

$$= \frac{3}{2}\cdot\left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4 = \frac{3}{2}\times\frac{2}{7}\times(4)^{7/2}$$

$$= \frac{3}{7}\times 2^7 = 54.86$$

prob 6

① Draw the FBD of the frame



6 unknown quantities

a) Cable force T

b) Reactions B_x and B_z at B

c) Reactions A_x, A_z and A_z at A

② To solve the problem, get the coordinates of the key points and set up the equilibrium equations $\Sigma \vec{M}_A = 0$ and $\Sigma \vec{F} = 0$.

$A\ (0, 0, 0)$ m $C\ (-3, 0, 6)$ m $E\ (2.5, 0, 6)$ m

$B\ (0, 4.5, 6)$ m $D\ (-1, 2.5, 0)$ m

$$\vec{CD} = (-1+3)\,\bar{i} + (2.5-0)\,\bar{j} + (0-6)\,\bar{k} = 2\bar{i} + 2.5\bar{j} - 6\bar{k} \text{ m}$$

$$|\vec{CD}| = \sqrt{2^2 + 2.5^2 + 6^2} = 6.8 \text{ m}$$

unit vector $\vec{\lambda}_{CD} = \dfrac{\vec{CD}}{|\vec{CD}|} = \dfrac{2i + 2.5j - 6k}{6.8}$

$$\vec{T}_{CD} = T_{CD}\vec{\lambda}_{CD} = \frac{T_{CD}}{6.8}\left(2\bar{i} + 2.5\bar{j} - 6\bar{k}\right)$$

③ Moment Equation $\Sigma \vec{M}_A = 0$

$$\vec{M}_A = \vec{r}_{C/A} \times \vec{T}_{CD} + \vec{r}_{E/A} \times \vec{F} + \vec{r}_{B/A} \times \vec{B}$$

$\vec{r}_{C/A} = -3\bar{i} + 6\bar{k}$ m $\vec{F} = 2\bar{j}$

$\vec{r}_{E/A} = 2.5\bar{i} + 6\bar{k}$ m $\vec{B} = B_x\bar{i} + B_z\bar{k}$

$\vec{r}_{B/A} = 4.5\bar{j} + 6\bar{k}$ m

$$\vec{M}_A = (-3\bar{i} + 6\bar{k}) \times \frac{T_{CD}}{6.8}(2\bar{i} + 2.5\bar{j} - 6k)$$

$$+ (2.5\bar{i} + 6\vec{k}) \times 2\bar{j}$$

$$+ (4.5\bar{j} + 6\bar{k}) \times (B_x\bar{i} + B_z\bar{k})$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & 0 & 6 \\ 2 & 2.5 & -6 \end{vmatrix} \frac{T_{CD}}{6.8} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2.5 & 0 & 6 \\ 0 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 4.5 & 6 \\ B_x & 0 & B_z \end{vmatrix}$$

$$= \frac{T_{CD}}{6.8}(-15\bar{i} - 6\bar{j} \quad 7.5\bar{k}) + (-12\bar{i} + 5\bar{k})$$

$$+ (4.5B_z\bar{i} \quad 6B_x\bar{j} - 4.5B_x\bar{k})$$

$$= \bar{i}\left[-\frac{15T_{CD}}{6.8} - 12 + 4.5B_z\right] + \bar{j}\left[-\frac{6T_{CD}}{6.8} + 6B_x\right]$$

$$+ k\left[-\frac{7.5T_{CD}}{6.8} + 5 - 4.5B_x\right]$$

With $\vec{M}_A = 0$

$$-\frac{15T_{CD}}{6.8} - 12 + 4.5B_z = 0 \quad — (1)$$

$$-\frac{6T_{CD}}{6.8} + 6B_x = 0 \quad — (2)$$

$$\frac{-7.5T_{CD}}{6.8} + 5 - 4.5B_x = 0 \quad — (3)$$

From Eqn (2) $\quad B_x = \frac{T_{CD}}{6.8}$

Substituting B_x to Eqn (3) $\rightarrow \frac{-7.5T_{CD}}{6.8} + 5 - 4.5\frac{T_{CD}}{6.8} = 0$

$T_{CD} = \underline{2.83 \text{ kN}}$

Then $B_y = \frac{T_{CD}}{6.8} = \frac{2.83}{6.8} = 0.417\ kN \simeq 0.42\ kN$

From Eqn (1) $\quad 4.5 B_z = 12 + \frac{15 T_{CD}}{6.8} = 12 + \frac{15(2.83)}{6.8}$

$B_z = 4.05\ kN \qquad \vec{B} = 0.417\bar{i} + 4.05\bar{k}$ kN

$\vec{T}_{CD} = \frac{2.83}{6.8}(2\bar{i} + 2.5\bar{j} - 6\bar{k}) = (0.83\bar{i} + 1.04\bar{j} - 2.49k)$ kN

(4) The force equation $\quad \Sigma\vec{F} = 0$

$$\Sigma\vec{F} = \vec{A} + \vec{B} + \vec{F} + \vec{T}_{CD} = 0$$

$$\vec{A} = A_x\bar{i} + A_y\bar{j} + A_z k$$

Then
$$(A_x\bar{i} + A_y\bar{j} + A_z\bar{k}) + (0.417\bar{i} + 4.05\bar{k}) + 2\bar{j} + (0.83\bar{i} + 1.04\bar{j} - 2.49\bar{k}) = 0$$

which gives, (isolating the $\bar{i}$, $\bar{j}$ and $\bar{k}$ components)

$A_x + 0.417 + 0.83 = 0 \quad \Rightarrow \quad A_x = -1.25$ kN

$A_y + 2 + 1.04 = 0 \quad \Rightarrow \quad A_y = -3.04$ kN

$A_z + 4.05 - 2.49 = 0 \quad \Rightarrow \quad A_z = -1.56$ kN

$$\left.\begin{aligned} \vec{A} &= -1.25\bar{i} - 3.04\bar{j} - 1.56\bar{k} \text{ kN} \\ \vec{B} &= 0.417\bar{i} + 4.05\bar{k} \text{ kN} \\ \vec{T}_{CD} &= 0.83\bar{i} + 1.04\bar{j} - 2.49\bar{k} \text{ kN} \end{aligned}\right\} \text{Ans.}$$

Engr 242

Final 2002

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Concordia
UNIVERSITY

Faculty of Engineering and Computer Science

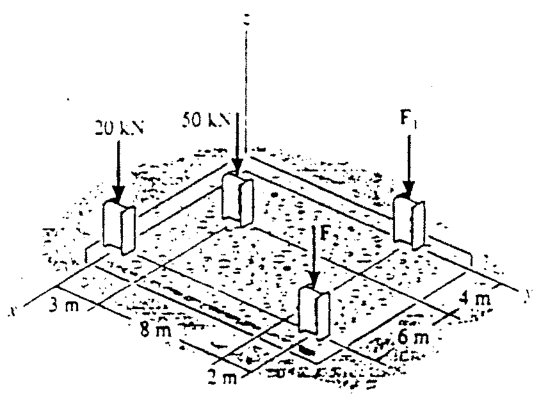
FINAL EXAMINATION ENGR 242/2 Statics Sections: T,V,X,YY **Date:** December 14, 2002

Instructors: Professors Dargahi, Rivard, Sabour, Stathopoulos (coordinator)

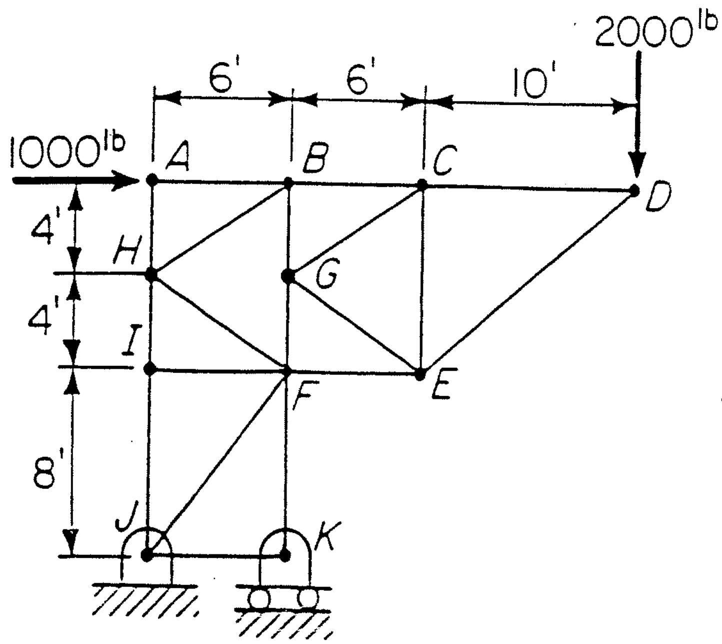
Materials allowed: Non-programmable calculators **Time allowed:** 3 hours

Special instructions: Problems carry equal weights. Solve any FIVE of the given 6 problems.

1. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab, if F_1 = 20 kN and F_2 = 50 kN.

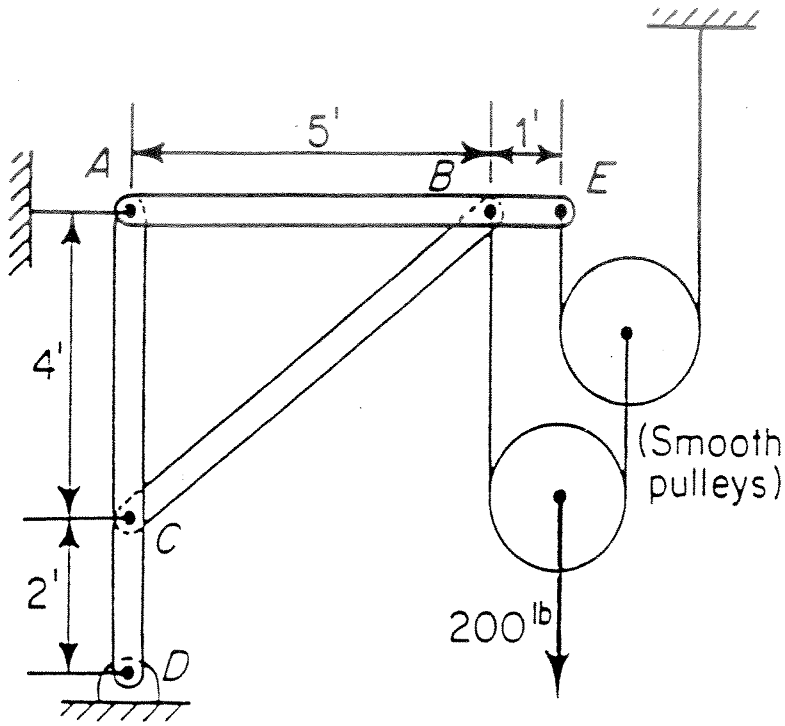


2. Determine the axial forces in members CD, EF, FJ and IF of the pin-connected truss shown. Indicate whether these members are in tension or compression.

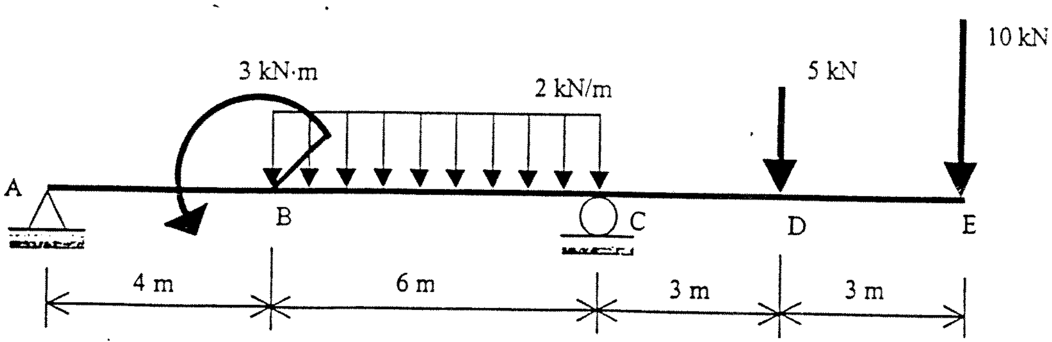


3, 70

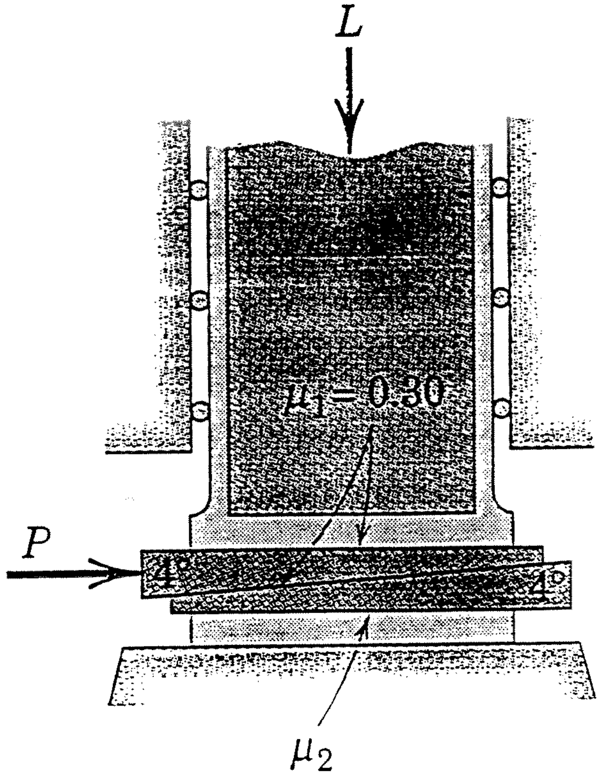
3. Cables are attached to pins at A, B, and E, as shown. Determine the horizontal and vertical components of the pin reaction at C on member DCA. Neglect all weights (members and pulleys).



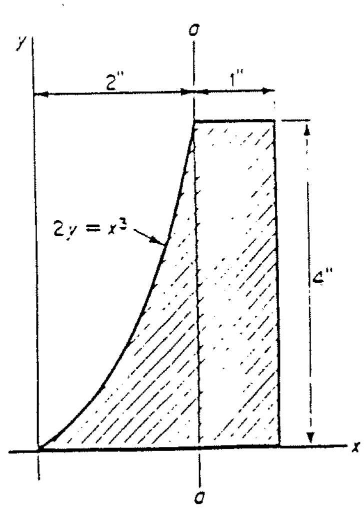
4. Draw the shear force and bending moment diagram for the beam and loading shown. The loading consists of an applied moment (3 kNm) at B, a uniformly distributed load (2 kN/m) between B and C, a point load (5 kN) at D and another one (10 kN) at E.



5. The two 4° wedges are used to position the vertical column under a load L. What is the minimum value of the coefficient of friction μ_2 for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force P to the upper wedge.



6. Determine the moment of inertia of the shaded area with respect to aa axis.



ENGR 242/2 - DEC. 2002 - FINAL EXAM SOLUTIONS

Find Statics Solution 2002 Fall

Q.1

$+\downarrow F_R = \Sigma F_z;$	$F_R = 20 + 50 + 20 + 50 = 140 \text{ kN}$	**Ans**
$M_{ROy} = \Sigma M_y;$	$140(x) = (50)(4) + 20(10) + 50(10)$	
	$x = 6.43 \text{ m}$	**Ans**
$M_{ROx} = \Sigma M_x;$	$-140(y) = -(50)(3) - 20(11) - 50(13)$	
	$y = 7.29 \text{ m}$	**Ans**

$r \times R =$

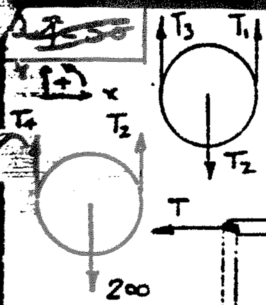
$(xi + yj)R = 900j + 1020i$

$x \times 140i + yj \times 140 =$

$900 = \cancel{x \times 140 i}$ $y \times 140$

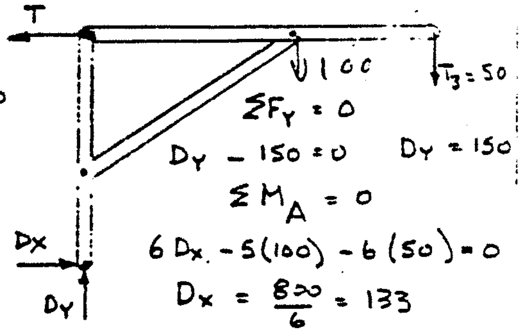
$y = \frac{900}{140}$

A-50



$T_2 = T_4 = 100$

$T_1 = T_3 = 50$



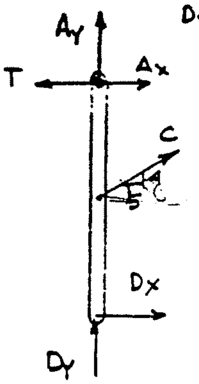
$\Sigma F_Y = 0$

$D_Y - 150 = 0 \quad D_Y = 150$

$\Sigma M_A = 0$

$6D_x - 5(100) - 6(50) = 0$

$D_x = \frac{800}{6} = 133$



$\Sigma M_A = 0$

$6D_x + 4\left(\frac{5}{\sqrt{41}}C\right) = 0$

$C_x = \frac{5}{\sqrt{41}}(40\sqrt{41})$

$C_x = 200 \text{ lb} \leftarrow$

$\underline{C_Y = 160 \text{ lb} \downarrow}$

$400 +$

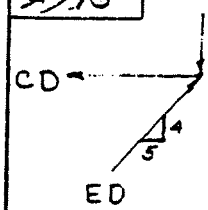
$C = \sqrt{40000 +}$

$= 256$

$C_x = C\cos\theta = \frac{C}{C_x}$

$C_y = C\sin\theta$

Q.2 **A-15**



$\Sigma F_Y = 0$

$\frac{4}{6.4}ED = 2000$

$ED = 3200 \text{ lb } C$

$\Sigma F_x = 0$

$\frac{5}{6.4}3200 = CD$

$JK = 0$

$CD = 2500 \text{ lb } T$ ANS

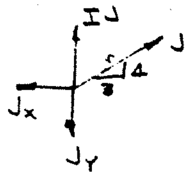
From F.B.D. of entire structure

$\Sigma M_J = 0 \Rightarrow 16 \cdot 1000 + 22 \cdot 2000 - 6K = 0$

$K = 10{,}000 \text{ lb} \uparrow$

$\Sigma F_x = 0 \Rightarrow J_x = 1000 \text{ lb} \leftarrow$

$\Sigma F_Y = 0 \Rightarrow J_Y = 8000 \text{ lb} \downarrow$

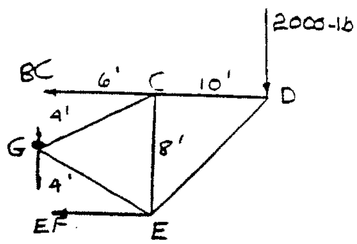


$\Sigma F_x = 0$

$\frac{3}{5}JF = 1000$

$JF = 1667 \text{ lb } T$ ANS

$IF = 0$



$\Sigma M_G = 0; \quad -16(2000) + 4BC - 4EF = 0$

$BC - EF = 8000$

$\Sigma F_x = 0; \quad \underline{-BC - EF = 0}$

$0 - 2EF = 8000$

$EF = 4000\text{-lb } C$

$I_a = \int x^2 dA$

Q-8

Rectangle

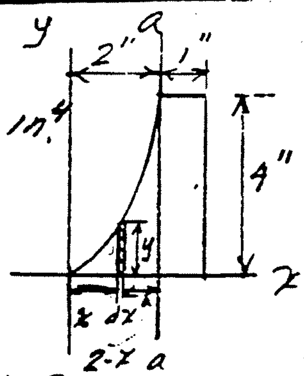
$I_a = \frac{1}{3}(4)(1)^3 = 1.333 \text{ in}^4$

Cubic

$2y = x^3$

$dA = y\,dx$

$dI_a = (2-x)^2 y\,dx$



$I_a = \int_0^2 (4 - 4x + x^2)\frac{x^3}{2}dx$

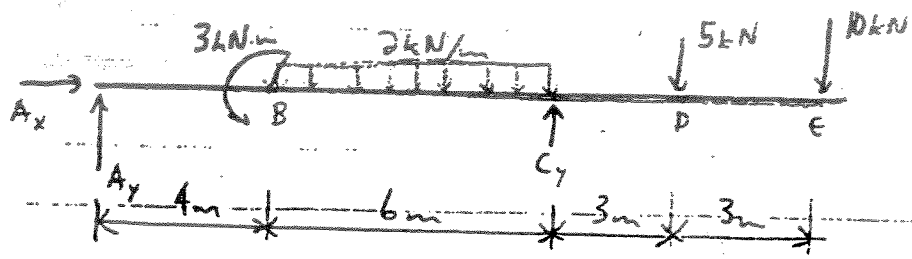
$= \frac{x^4}{2} - \frac{2}{5}x^5 + \frac{x^6}{12}\Big]_0^2$

$= 8 - 12.8 + 5.333 = 0.533 \text{ in}^4$

Total I_a = $\underline{1.867 \text{ in}^4}$ Ans.

2.4 Solution to Beam Question

FBD.

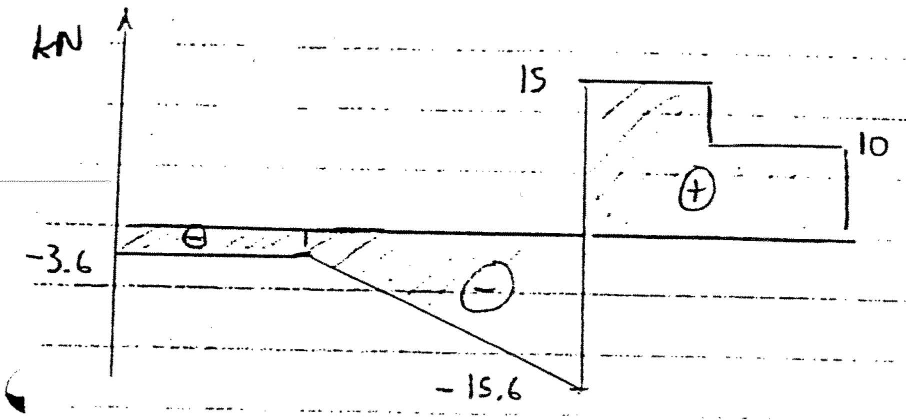


$\circlearrowleft \Sigma M_{@A}: 3 - 2*6*7 - 5*13 - 10*$
$+ 10\,C_y = 0 \rightarrow \boxed{C_y = 30.}$

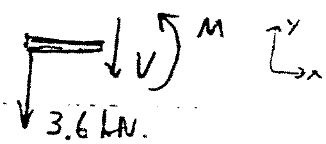
$\Sigma F_x = A_x = 0$

$\Sigma F_y = -2*6 - 5 - 10 + \overset{30.6}{C_y} + A_y = 0$

$\boxed{A_y = -3}$

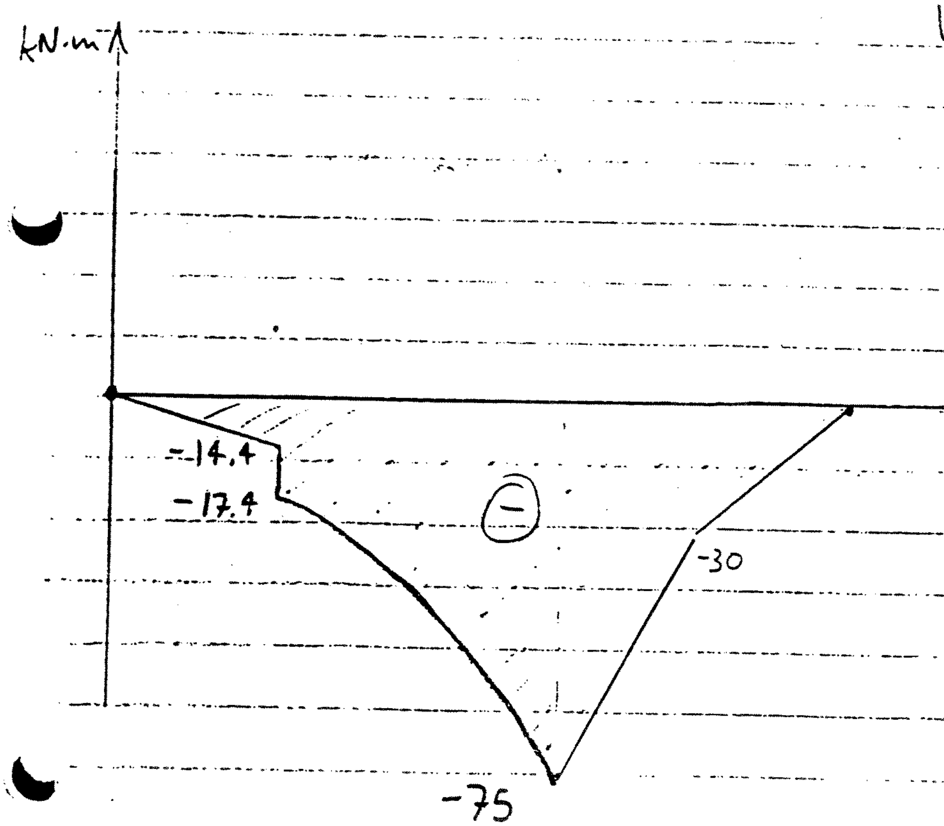


At A)



$\Sigma F_y = -V - 3.6 = 0 \rightarrow V_A = -3.6$

$V_c = V_B - \underset{\text{load curve}}{\text{Area under}} = -3.6 - 2*6 = -$

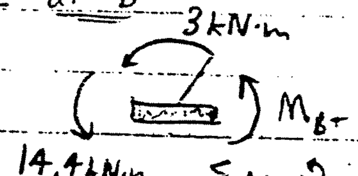


Moments:

$M_{B^-} = M_A + \underset{\text{shear}}{\text{Area under}}$

$= 0 + 4*(-3.6) = -14.$

at B



$\Sigma M_B \circlearrowleft: M_{B+} + 3 + 14.4$

$M_{B+} = -17.$

$M_c^- = M_{B+} + \text{Area} =$

$= -17.4 - \left(\frac{3.6 + 15.6}{2}\right)*6 =$

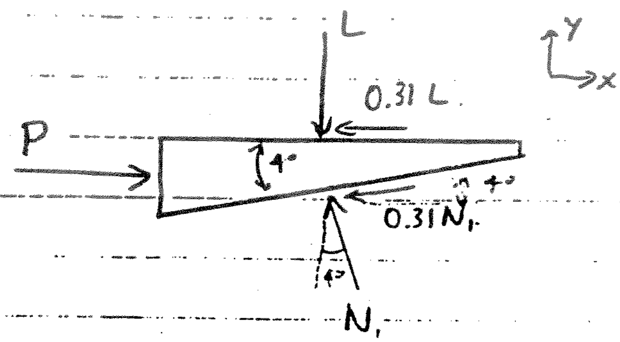
$M_D = -75 + 15*3 = -30$

$M_E = -30 + 10*3 = 0$

Q.5

Solution to Friction Question

FBD top wedge.

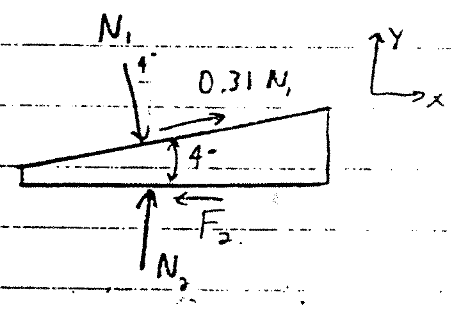


$$\Sigma F_y = -L + \cos 4^\circ N - 0.31 \sin 4 N = 0$$

$$N = \frac{L}{(\cos 4^\circ - 0.31 \sin 4^\circ)}$$

$$N = 1.0247 \ L$$

FBD lower wedge.



$$\Sigma F_x = \sin 4^\circ N_1 + 0.31 \cos 4^\circ N_1 - F_2 = 0$$

$$F_2 = \underbrace{(\sin 4^\circ + 0.31 \cos 4^\circ)}^{0.379} * 1.0247 L$$

$$F_2 = 0.388 L$$

$$\Sigma F_y = N_2 - \cos 4^\circ N_1 + 0.31 \sin 4^\circ N_1 = 0$$

$$N_2 = (\cos 4^\circ - 0.31 \sin 4^\circ) * 1.0247$$

$$N_2 = L$$

At impending motion

$$F_2 = \mu_2 N_2 \Rightarrow \mu_2 = \frac{F_2}{N_2} = 0.388$$

∴ The coefficient of friction μ_2 must be ≥ 0.388.

56

CONCORDIA UNIVERSITY
FACAULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

ENGR 242/T Statics

Time 70 minutes

Date: October 30, 2002 Instructors: Dargahi

Family Name: First Name:
ID:

All questions have equal marks.

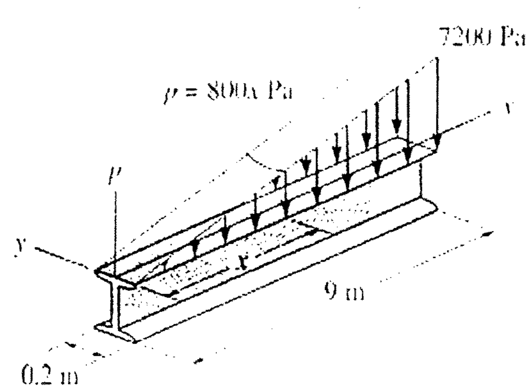
Question # 1......................

Question # 2......................

Question # 3......................

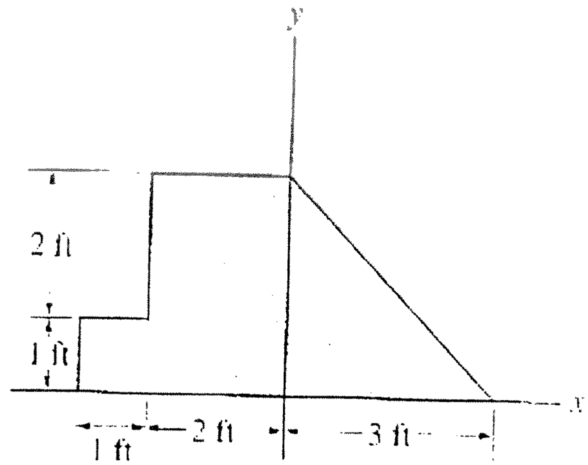
Question1

A distributed loading of $\rho = 800x$ Pa over the top surface of the beam shown below. Determine the magnitude and location of the equivalent resultant force.



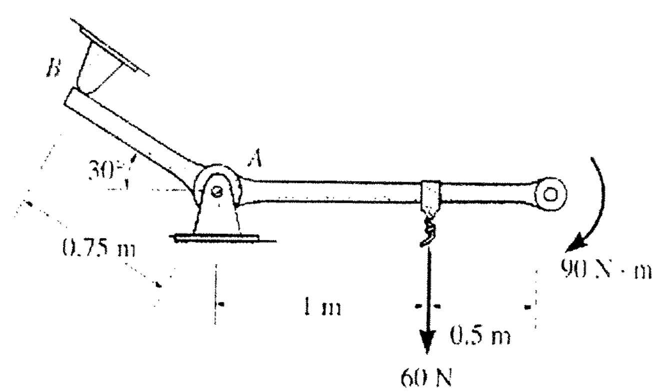
Question2

Locate the centroid of the plate area shown below.



Question3

The link shown below is pin-connected at A and rests against a smooth support at B. Compute the horizontal and vertical components of reaction at the pin A.



CONCORDIA UNIVERSITY
FACAULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

ENGR 242/T Statics

Time 70 minutes

Date: November 22, 2002 Instructors: Dargahi

Family Name: First Name: ID:

All questions have equal marks.

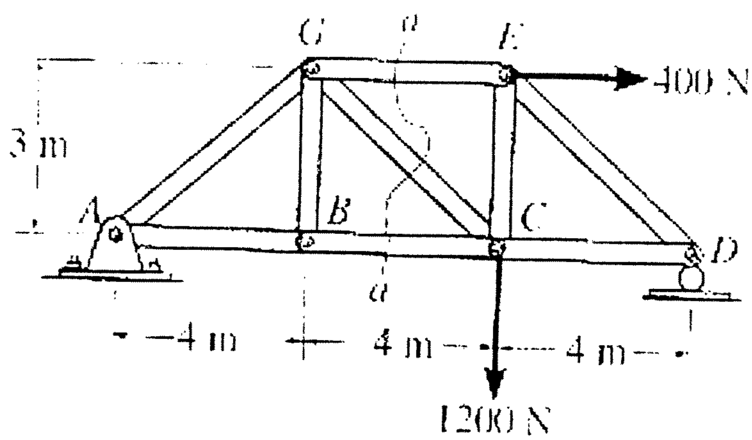
Question # 1.......................

Question # 2.......................

Question # 3.......................

Question1

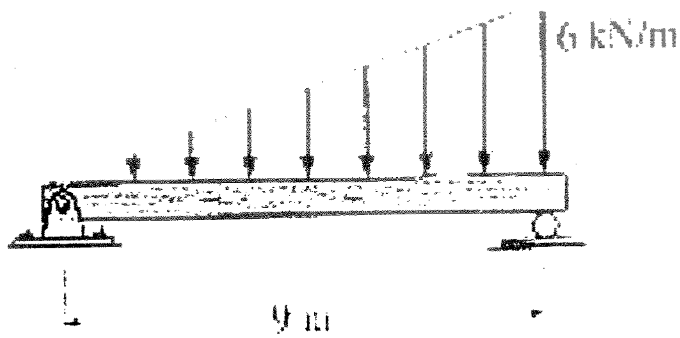
Determine the force in members GE, GC, and BC of the truss shown in Figure below. Indicate whether they are in tension or compression.



3

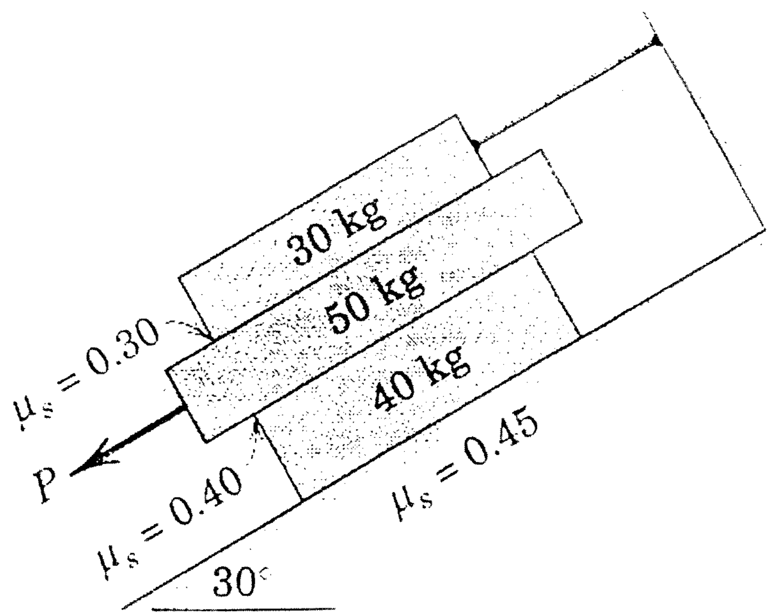
Question2

Draw the shear force and bending-moment diagrams for the beam shown below.



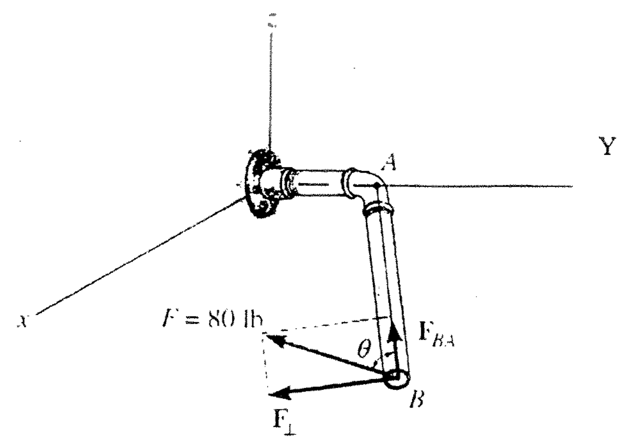
Question3

Three flat blocks are positioned on the 30°inclined as shown, and a force P parallel to the inclined is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pair of mating surfaces is shown. Determine the maximum value which p may have before any slipping takes place



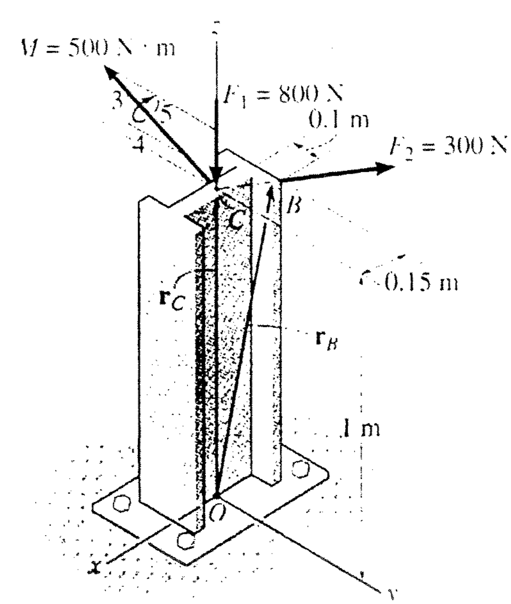
Question1

The pipe in Figure below is subjected to the force of F= 80 lb. Determine the angle θ between **F** and the pipe segment BA, and the magnitudes of the components of **F**, which are parallel and perpendicular to BA.



Question2

A structure member is subjected to a couple moment **M** and forces $\mathbf{F_1}$ and $\mathbf{F_2}$ as shown in Figure below. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.

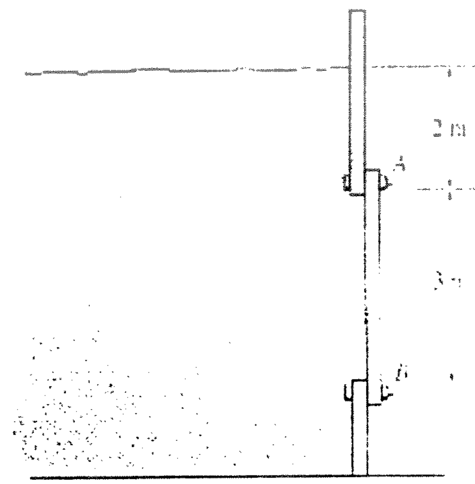


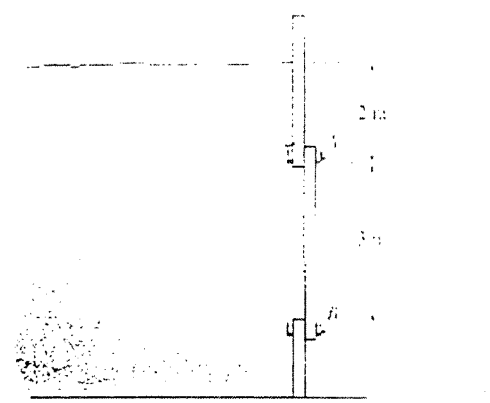
(a)

5

Question3

Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate AB shown in Figure below. The plate has a width of 1.5 m; $\rho_w = 1000\ kg/m^3$.



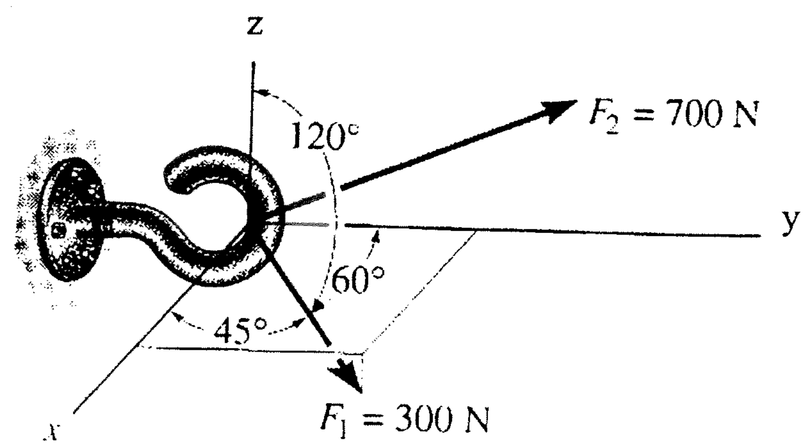


6

Question 1

Two forces act on the hook shown in Fig. 1. Specify the coordinate direction angles of $\mathbf{F_2}$ so that the resultant force $\mathbf{F_R}$ acts along the positive y axis and has a magnitude of 800N.

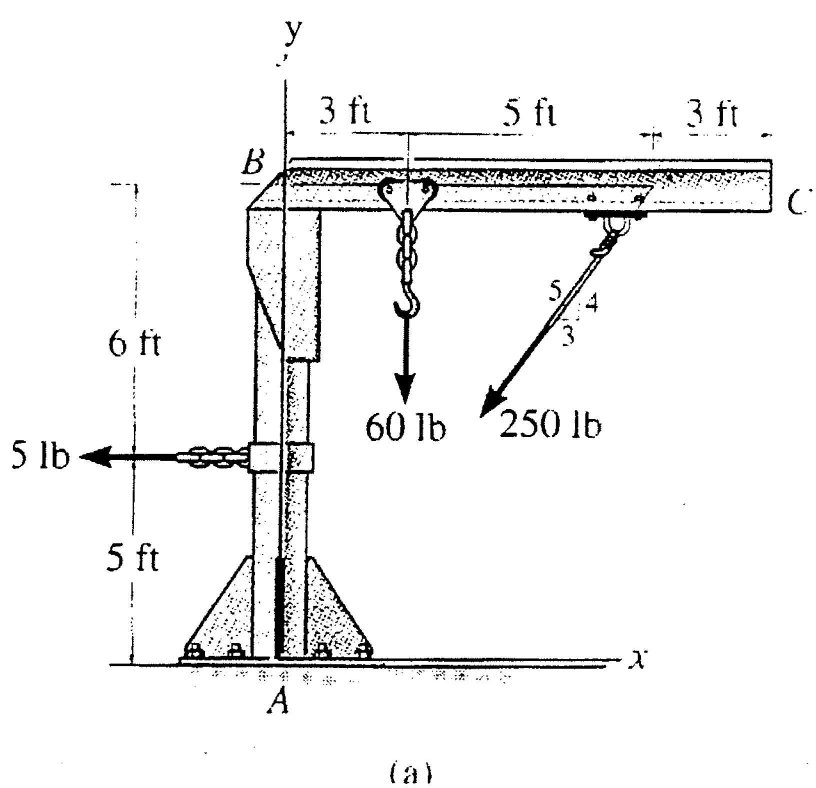
Figure 1



Question 2

The jib crane shown in Fig. 2 is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column AB and boom BC.

Figure 2

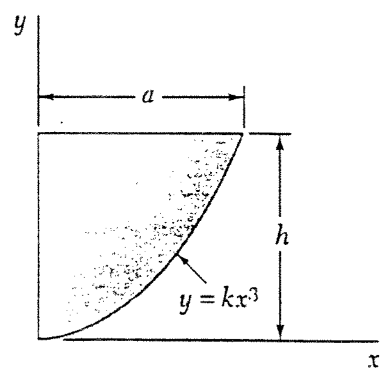


M 03/1

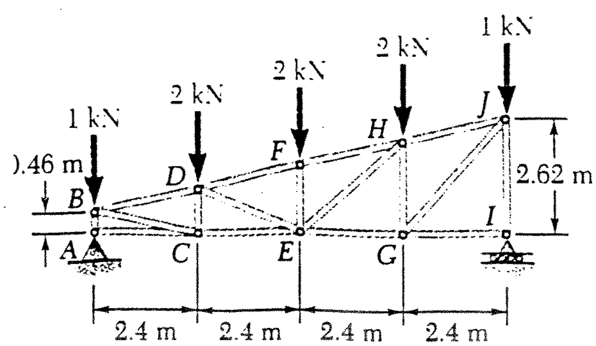
CONCORDIA UNIVERSITY

COURSE	NUMBER	SECTION	
Statics	ENGR 242		
EXAMINATION	DATE	TIME	# OF PAGES
Mid-term	March, 2003		1
INSTRUCTOR			
Dr. F. Haghighat			
MATERIALS ALLOWED:			
Calculator			
SPECIAL INSTRUCTIONS:			
All questions have equal marks. **All questions should be answered.**			

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.



A pitched flat roof truss is loaded as shown. Determine the force in members EG, GH, and HJ.



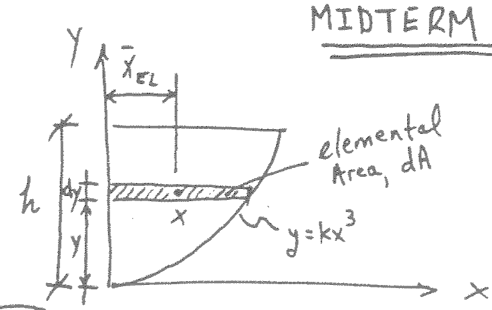
1.60$

MIDTERM #2 — Solutions

M 03/2

1/2

5.42



Given: Area

Find: Centroid by integration

Sol'n: STEP 1

$x=a$
$y=h$

given $y=kx^3$ & transforming:

$h=ka^3$

$\therefore k=\dfrac{h}{a^3}$

So $\therefore$ given $y=kx^3$ & solving for x:

$x=\left(\dfrac{y}{k}\right)^{1/3}$

$=\left(\dfrac{1}{k}\right)^{1/3}\cdot y^{1/3}$ — the same

$\therefore x=\left(\dfrac{1}{h/a^3}\right)^{1/3}\cdot y^{1/3}$

Simplifying, $\boxed{x=\dfrac{a}{h^{1/3}}\cdot y^{1/3}}$

$\because dA=x\,dy$ where dA = area of elemental section (from diagram)

$dA=\left(\dfrac{a}{h^{1/3}}\cdot y^{1/3}\right)dy$

integrating yields,

$A=\int dA=\int_0^h\left(\dfrac{a}{h^{1/3}}y^{1/3}\right)dy=\dfrac{a}{h^{1/3}}\int_0^h y^{1/3}dy$

$=\dfrac{3}{4}\cdot\dfrac{a}{h^{1/3}}\left[y^{4/3}\right]_0^h$

$=\dfrac{3}{4}\cdot\dfrac{a}{h^{1/3}}\cdot h^{4/3}$

$A=\dfrac{3}{4}ah$ //

STEP 2

$\bar{x}_{EL}=\dfrac{1}{2}x$ (from the diagram)

$\therefore \bar{x}_{EL}=\dfrac{1}{2}\cdot\dfrac{a}{h^{1/3}}\cdot y^{1/3}$

$\therefore \int\bar{x}_{EL}dA=\int_0^h\dfrac{1}{2}\cdot\dfrac{a}{h^{1/3}}y^{1/3}\overbrace{\left[\dfrac{a}{h^{1/3}}y^{1/3}\right]}^{x}dy$

$=\dfrac{1}{2}\dfrac{a^2}{h^{2/3}}\left[\dfrac{3}{5}y^{5/3}\right]_0^h$

$=\dfrac{3}{10}a^2h$

STEP 3

$\boxed{\bar{y}_{EL}=y}$

$\int\bar{y}_{EL}dA=\int_0^h y\overbrace{\left[\dfrac{a}{h^{1/3}}y^{1/3}\right]dy}^{dA}=\dfrac{a}{h^{1/3}}\int_0^h y^{4/3}dy$

$=\dfrac{a}{h^{1/3}}\left[\dfrac{3}{7}\cdot y^{7/3}\right]_0^h$

$=\dfrac{3}{7}ah^2$

Step 4

$\therefore \bar{x}A=\int\bar{x}_{EL}dA$

$\bar{x}=\dfrac{\frac{3}{10}a^2h}{\frac{3}{4}ah}=\dfrac{2}{5}a$ //

similarly,

$\bar{Y}A=\int\bar{y}_{EL}dA$

$\bar{Y}=\dfrac{\frac{3}{7}ah^2}{\frac{3}{4}ah}=\dfrac{4}{7}h$ //

$\therefore$ Location of Centroid is:

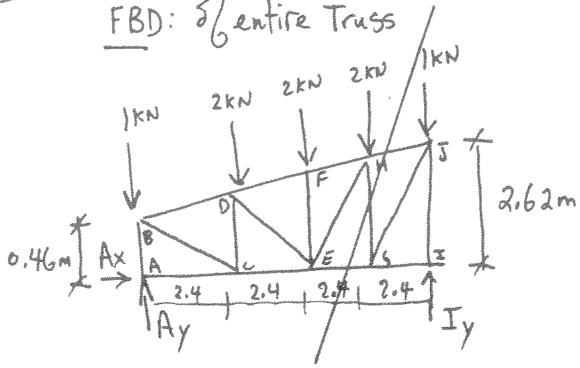
$C(\bar{x},\bar{Y})=\left(\dfrac{2}{5}a,\ \dfrac{4}{7}h\right)$ //

M03/3

2/2

6.52

FBD: of entire Truss



Given: Truss loaded as shown

Find: Member forces,
F_{EG}
F_{GH}
F_{HJ}

Note: Every single member length of this truss can be found with the given info.

$+\uparrow \Sigma F_y = 0 \quad A_y + I_y = 8\,kN$

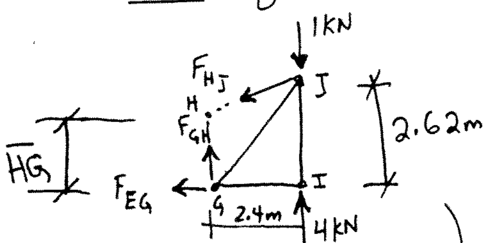
$\xrightarrow{+} \Sigma F_x = 0 \quad \boxed{A_x = 0}$

$\circlearrowleft \Sigma M_A = 0 \quad -2(2.4) - 2(4.8) - 2(7.2) - 1(9.6) + 9.6 I_y = 0$

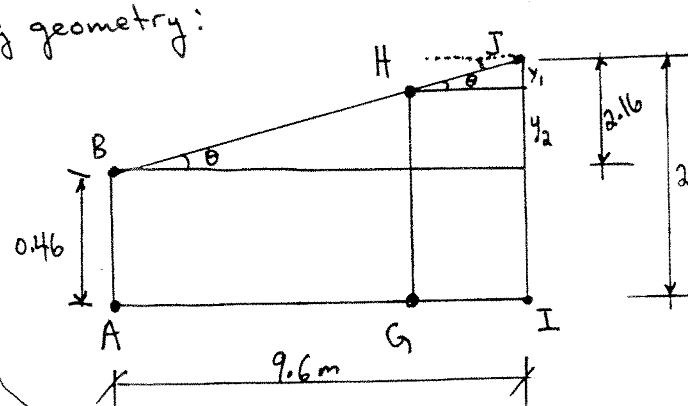
$\boxed{\begin{array}{l} \therefore I_y = 4\,kN \uparrow \\ \therefore A_y = 4\,kN \uparrow \end{array}}$

By Method of Sections: through members EG, GH, HJ (ie. the unknown member forces we are looking for)

FBD: of section



$\boxed{\text{STEP 1}}$: We must find height $\overline{HG}$ (ie. length of member HG) using geometry:



$\tan\theta = \frac{2.16}{9.6}$

$\theta = 12.68°$

$\tan 12.68° = \frac{y_1}{2.4}$

$\therefore y_1 = 0.54\,m$

$\therefore y_2 = 2.16 - 0.54 = 1.62\,m$

Check: $2.62 = 0.46 + y_1 + y_2$
$= 0.46 + 0.54 + 1.62$
$2.62 = 2.62$ ✓

$\therefore \overline{HG} = 0.46 + y_2$
$= 0.46 + 1.62$
$\overline{HG} = 2.08\,m$ //

$\boxed{\text{STEP 2}}$:

$\circlearrowleft \Sigma M_H = 0$

$4(2.4) - (1)(2.4) - F_{EG}(2.08) = 0$

$\boxed{\therefore F_{EG} = 3.46\,kN\ T.}$

$\circlearrowleft \Sigma M_J = 0$ ✓

$-F_{GH}(2.4) - F_{EG}(2.62) = 0$

$\therefore F_{GH} = -3.7788\,kN$

OR $\boxed{F_{GH} = 3.78\,kN\ C.}$

$\xrightarrow{+} \Sigma F_x = 0$ ✓

$-F_{EG} - F_{HJ}\cos 12.68° = 0$

$\therefore F_{HJ} = -3.55$

OR $\boxed{F_{HJ} = 3.55\,kN\ C.}$

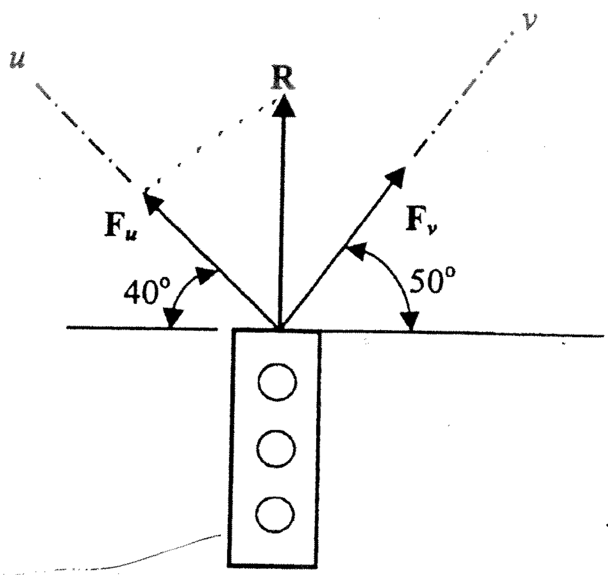
ENGR 242

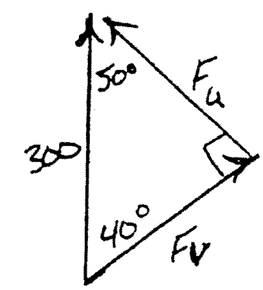
28/03/03

SOLUTION

090127102 – Statics
Quiz 1
September 9, 2002

Two cables are used to support a stop light as shown in the figure. The resultant **R** of the cable forces $\mathbf{F}_u$ and $\mathbf{F}_v$ has a magnitude of 300 lb and its line of action is vertical. Determine the magnitudes of the forces $\mathbf{F}_u$ and $\mathbf{F}_v$.





$$\frac{300}{\sin 90} = \frac{F_u}{\sin 40} = \frac{F_v}{\sin 50}$$

$F_u = 192.8$ lb

$F_v = 229.8$ lb

ALTERNATE SOLUTION – RESOLVE R INTO u & v COMPONENTS

$F_u = R\cos 50 = 192.8$ lb

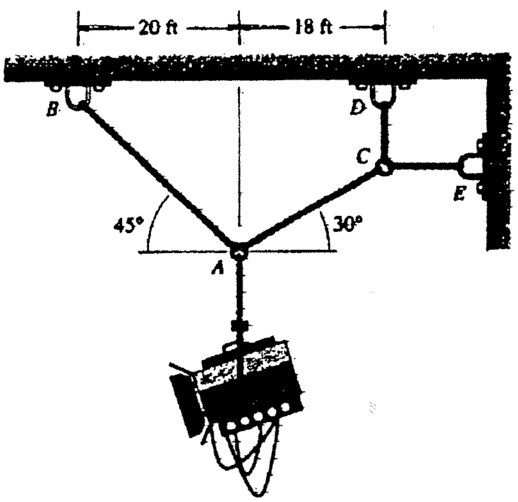
$F_v = R\cos 40 = 229.8$ lb

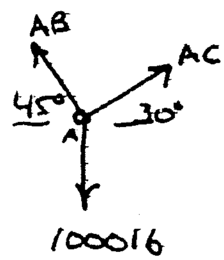
1.0

Statics
090127102
Exam 1
October 2, 2002

General instructions: Show all work and all free body diagrams used for every problem. Don't count on the instructor reading your mind. Each problem is worth 25 points. Think about the problems before you start working to avoid unnecessary work. Work neatly.

1. The system of cables suspends a 1000-lb bank of lights above a movie set. Determine the tension in cables *AB* and *CE*.





$\Sigma F_x = 0$

$-AB\cos 45 + AC\cos 30 = 0$

$AB = \dfrac{AC\cos 30}{\cos 45}$

$\Sigma F_y = 0$

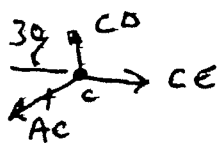
$AB\sin 45 + AC\sin 30 - 1000 = 0$

$\dfrac{AC\cos 30\sin 45}{\cos 45} + AC\sin 30 = 1000$

$\underline{1.366}\ AC = 1000$

$AC = 732\text{ lb}$

$\boxed{AB = 896.6\text{ lb}}$



$\Sigma F_x = 0$

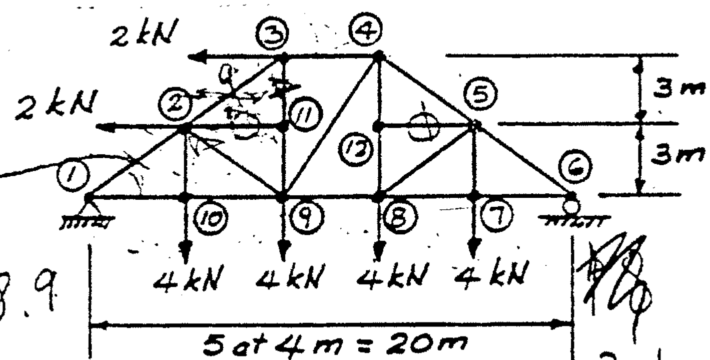
$-AC\cos 30 + CE = 0 \qquad -732\cos 30 + CE = 0$

$CE = 634\text{ lb}$

14.8

4. In the simply-supported truss shown, determine the axial forces in members 5-12, 2-3 and 3-11. Indicate whether these members are under tension or compression.

36.87



8.9

7.1

3-11 → 9.12 T

5-12 → 0

2-3 → 15.2 T

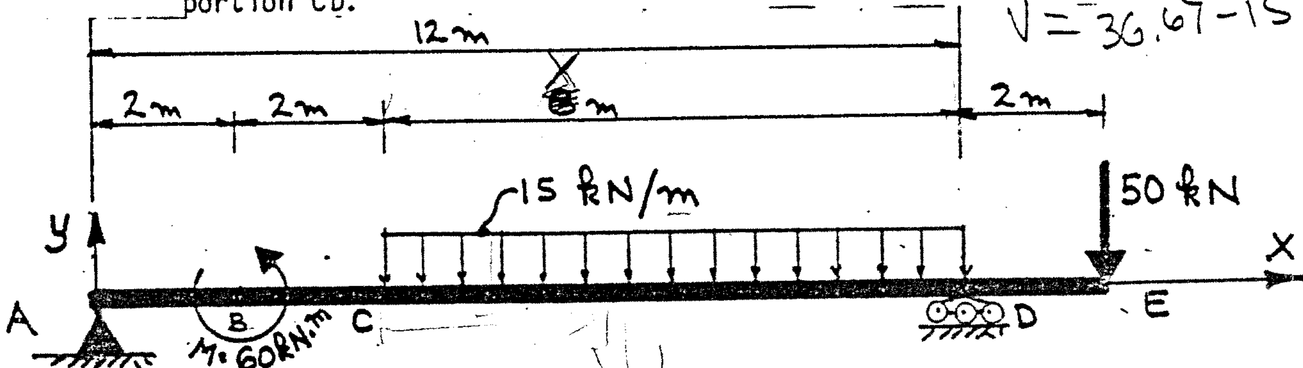
12 → 14.83

5. For the beam shown:

a) Sketch the shear force and bending moment diagram and indicate peak values.

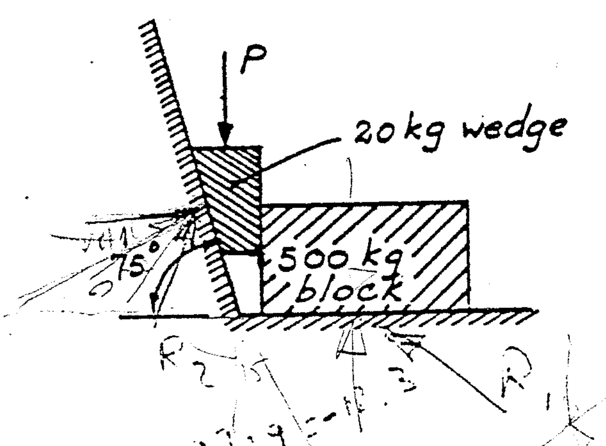
b) Write the equations for the shear force and bending moment for portion CD.

$V = 36.67 - 15x$



6. Determine the smallest force P, applied to the 20 kg wedge, that is required to move the 500 kg block. The coefficient of static friction between all surfaces is 0.15.

$M = \frac{15x^2}{2} - 36.67 - 20$



R_2 R_1

$T_{23} + T_{29} = -12.3$

$-T_{23} - T_{29} = 8.133$

10.21

8.133

CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science

ENGR 242/2 STATICS, Section XX, Fall 1998 Dr. H. Rivard

Test #2

Attempt all questions. Only non-programmable calculators are permitted. Time: 70 minutes.

Marks

10 (1) Three forces are applied on the box as shown in Figure 1.
(a) Replace the forces exerted with an equivalent force-couple system at the origin.
(b) Find the pitch of the wrench. $P = \frac{R \cdot M^R}{R^2}$
(c) Locate the point where the axis of the wrench intersects the XY plane.

10 (2) For the plane area shown in Figure 2, determine
(a) the first moments of area with respect to the X and Y axes, and
(b) the location of the centroid.

10 (3) The U shaped bracket is loaded and supported as shown in Figure 3. Determine the components of the reactions at supports A and B.

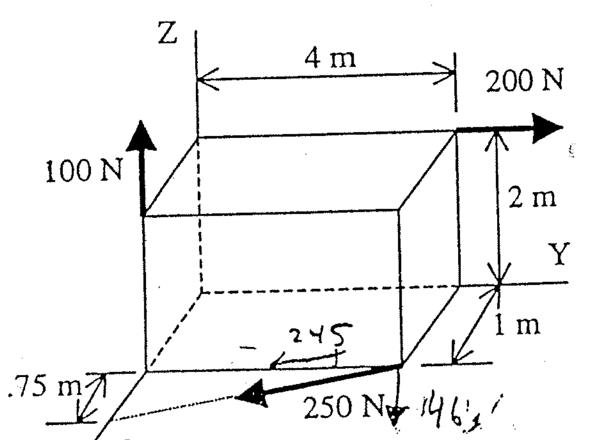


Figure 1

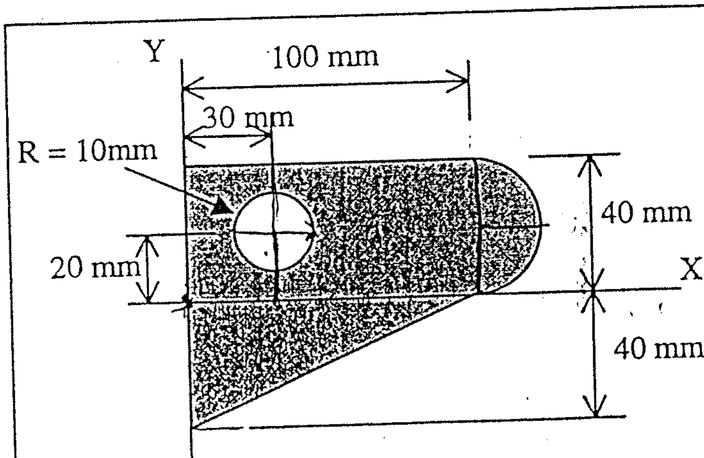


Figure 2

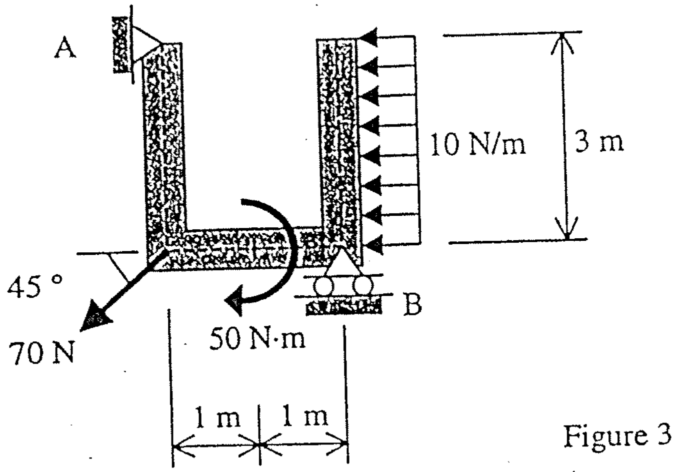


Figure 3

EXAMINATION	DATE	TIME	# OF PAGES
Final	December 15, 1982	19:00 - 22:00	4

INSTRUCTOR	DIVISION
Profs. Goldman, Stathopoulos	Day and Evening

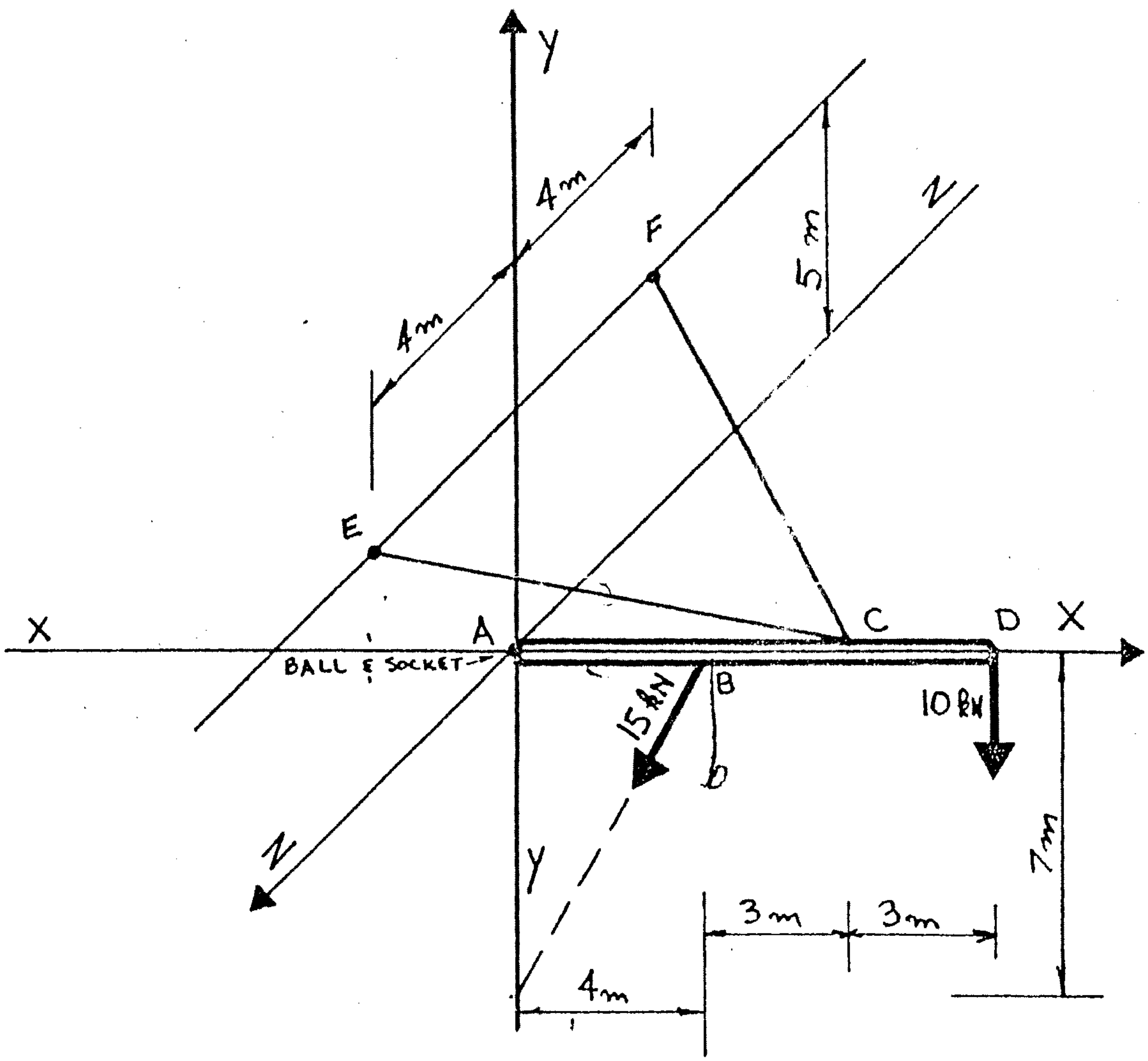
MATERIALS ALLOWED

Any Calculating Device

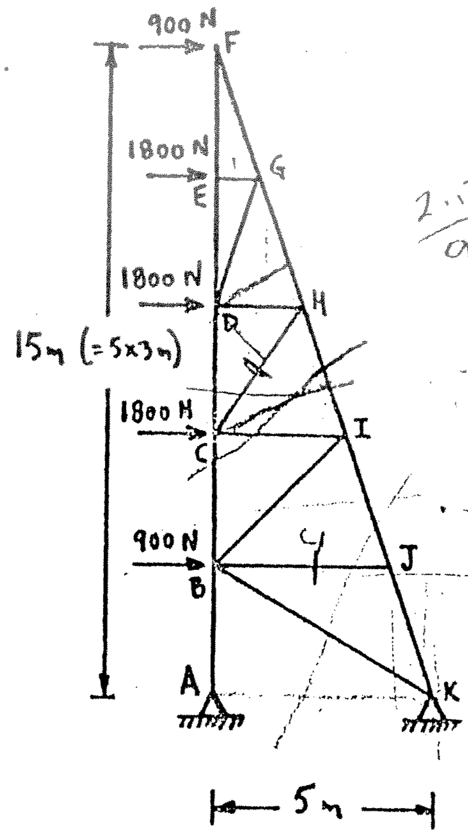
SPECIAL INSTRUCTIONS

Do any five problems. Only the first five problems presented will be graded.
All questions carry equal weight.

1) Calculate the reaction at support A and the tension of the cables CE and CF for the structure in equilibrium shown.



2) The truss shown is used to support an outdoor theater screen. If a wind pressure, acting on the face of the screen, creates the loading shown on the joints, determine the force in members BJ, CH and HI. Indicate whether the members are in tension or compression.



56.4

$\frac{2.5}{9} = \frac{x}{6}$

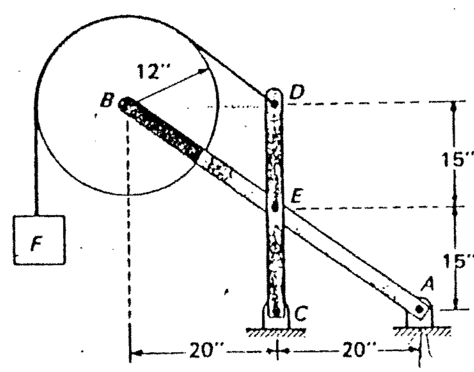
$\frac{56.4}{9} = \frac{B}{1}$

$\frac{5}{x} = \frac{15}{6}$

$\frac{5}{x} = \frac{15}{9}$

$\frac{5}{x} = \frac{15}{12}$

3) Body F weighs 500 lb. The weights of all other members may be neglected. Determine the horizontal and vertical components of the pin-reactions at C and E on member CD and indicate their correct action.



$\frac{b}{556.4} = \frac{3}{1}$

356

336

CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science

ENGR 242/2 STATICS, Section XX, Fall 1999

Test #2

Attempt all questions. Only non-programmable calculators are permitted. Time: 60 minutes.

Marks

10 (1) The windlass in Figure 1 is subjected to a load of 150 N. Determine the horizontal force P needed to hold the handle in the position shown and the components of reaction at the ball-and-socket joint A and the smooth journal bearing B. The bearing at B is in proper alignment and exerts only force reactions on the windlass.

10 (2) For the plane area shown in Figure 2, determine
- (a) the first moments of area with respect to the X and Y axes,
- (b) the location of the centroid, and
- (c) the volume of the body of revolution obtained by rotating the area about the Y axis.

10 (3) A 0.5 x 0.8-m gate is located at the bottom of a tank filled with water, as shown in Figure 3. The gate is hinged along its top edge A and rests on a frictionless stop at B. Determine the minimum tension required in cable BCD to open the gate. The specific weight of water is 9.81 kN/m^3.

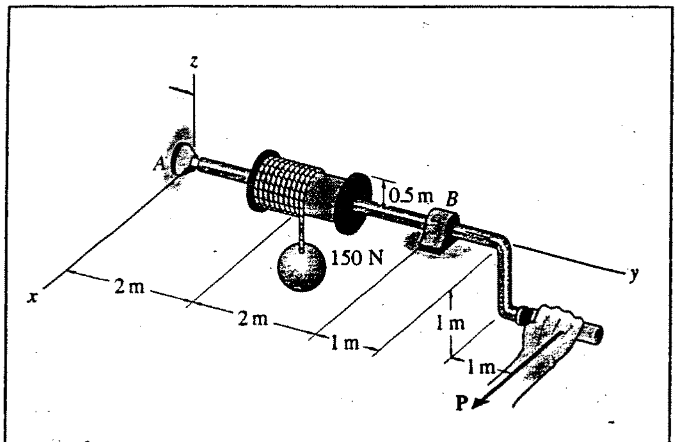


Figure 1

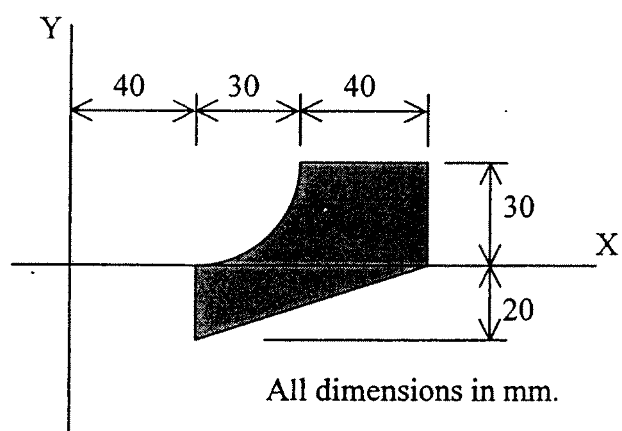


Figure 2

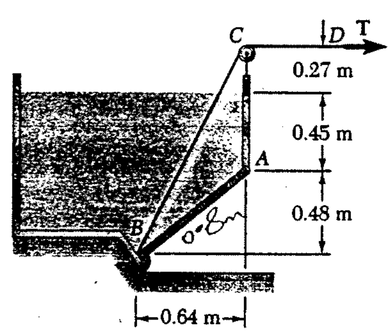


Figure 3

CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science

ENGR 242/2 STATICS, Section XX, Fall 1998 Dr. H. Rivard

Test #2

Attempt all questions. Only non-programmable calculators are permitted. Time: 70 minutes.

Marks

10 (1) Three forces are applied on the box as shown in Figure 1.
(a) Replace the forces exerted with an equivalent force-couple system at the origin.
(b) Find the pitch of the wrench. $p = \frac{R \cdot M}{R^2}$
(c) Locate the point where the axis of the wrench intersects the XY plane.

10 (2) For the plane area shown in Figure 2, determine
(a) the first moments of area with respect to the X and Y axes, and
(b) the location of the centroid.

10 (3) The U shaped bracket is loaded and supported as shown in Figure 3. Determine the components of the reactions at supports A and B.

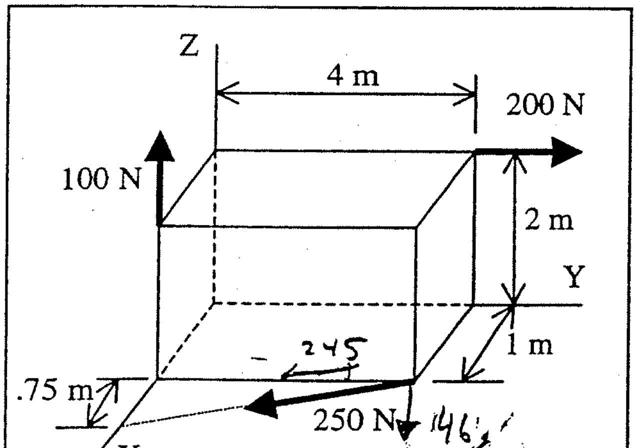


Figure 1

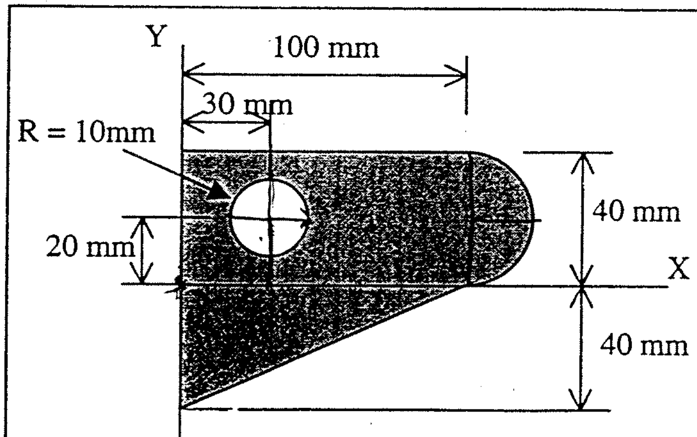


Figure 2

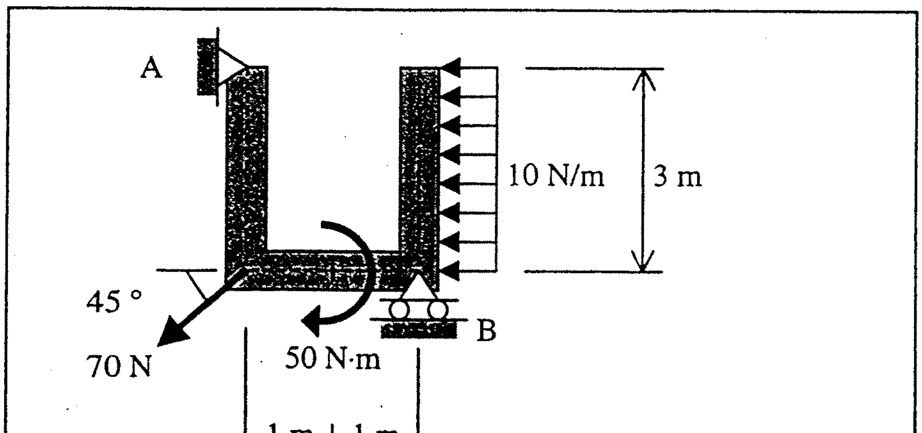


Figure 3

Note: Answers are given for each problem.

If any question (solutions): Please Contact: MILIND PIMPRIKAR H-971-16

CONCORDIA UNIVERSITY

COURSE	NUMBER	SECTION	
Statics	ENGR. C242/2	01, T, V, X, Y, XX	
EXAMINATION	**DATE**	**TIME**	**# OF PAGES**
Alternate	December 16, 1984		3
INSTRUCTOR		**DIVISION**	
		Day and Evening	

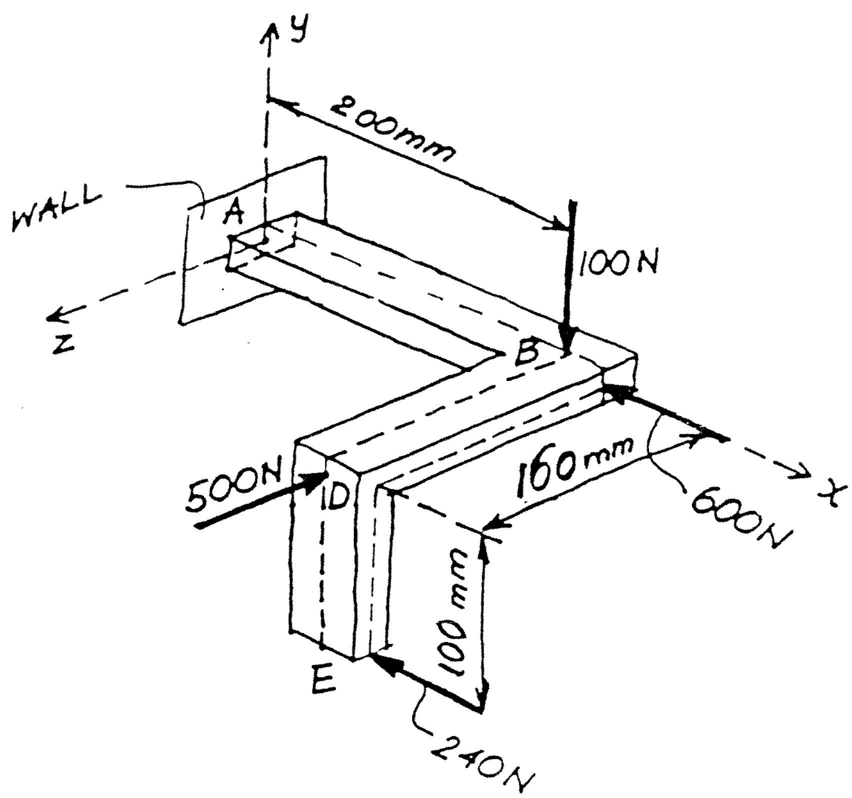
Any calculating device

SPECIAL INSTRUCTIONS

Do all five problems.

All questions carry equal weight.

1. Four forces are applied to the machine component ABDE as shown. Replace these forces by an equivalent force-couple system at A.



Ans. $\mathbf{R} = (-5[illegible]0\ N)\mathbf{i} - (100\ N)\mathbf{j} - (5[illegible])\mathbf{k}$

$\mathbf{M}_A = (61.6\ N\cdot m)\mathbf{j} - ([illegible]\ N\cdot m)\mathbf{k}$

10

3) Find the resultant of the system of forces acting on the section shown in Fig. 3. Show where the resultant acts with respect to point A.

Not in Equilibrium!

Call this Point P.

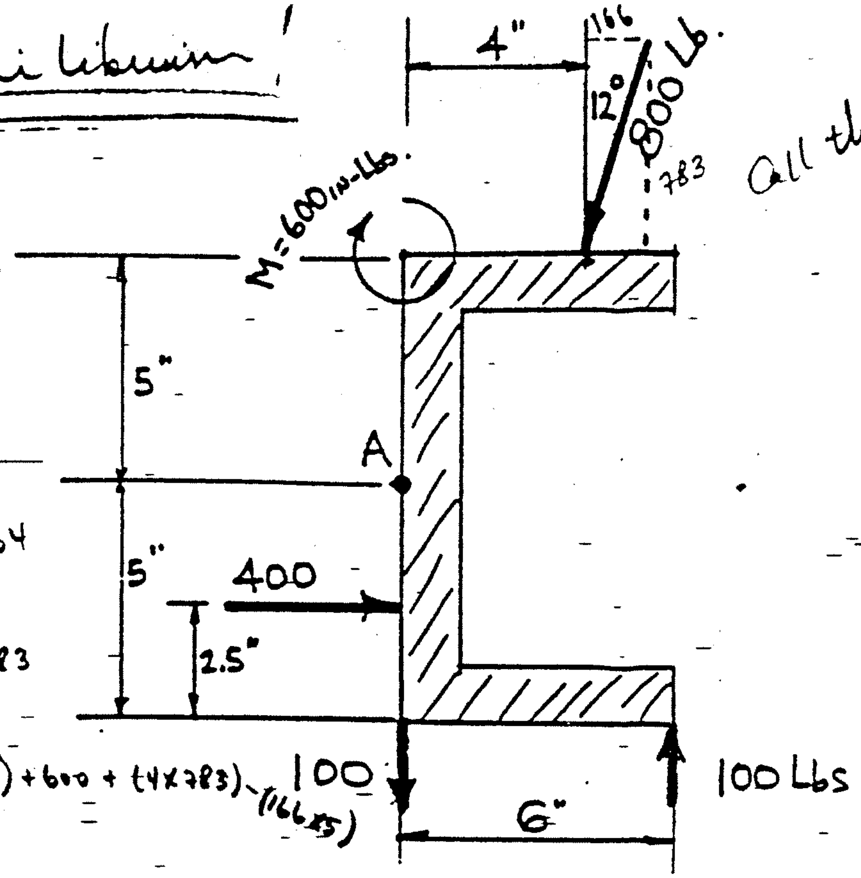


FIG. 3

(A) $\Sigma F_x =$

$\Sigma F_y =$

$\Sigma M_e = M_R$

$d_R R = M_R$

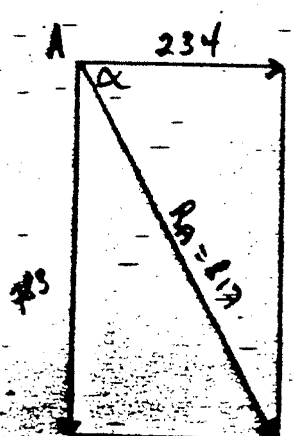
(B) $\xrightarrow{+} \Sigma F_x = +400 - 166 = +234$

$+\uparrow \Sigma F_y = 400 + 100 - 783 = -783$

$\overset{+}{\curvearrowright} \Sigma M_{C_A} = -(6 \times 100) - (2.5 \times 400) + 600 + (4 \times 783) - (166 \times 5)$

$\Sigma M_R = 1302$ in-lbs

(C)



$R_A = \sqrt{234^2 + 783^2} = 817$

$\tan \alpha = \frac{783}{234}$ $\alpha = 73.4°$

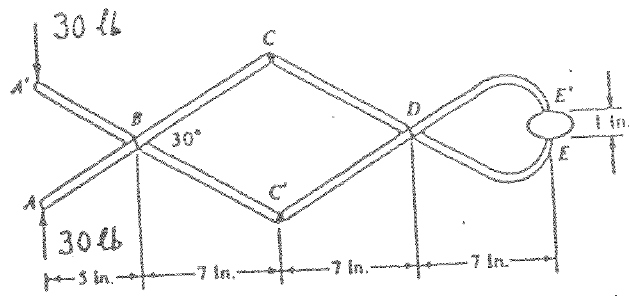
(D)

$d_R R = M_R$

$d_R = \frac{1302}{817} = 1.6"$

(E)

simple tool shown is a system of fireplace tongs. Connections at B, C, ... d D are simple pins. Determine the forces acting at points B, C, D and E.

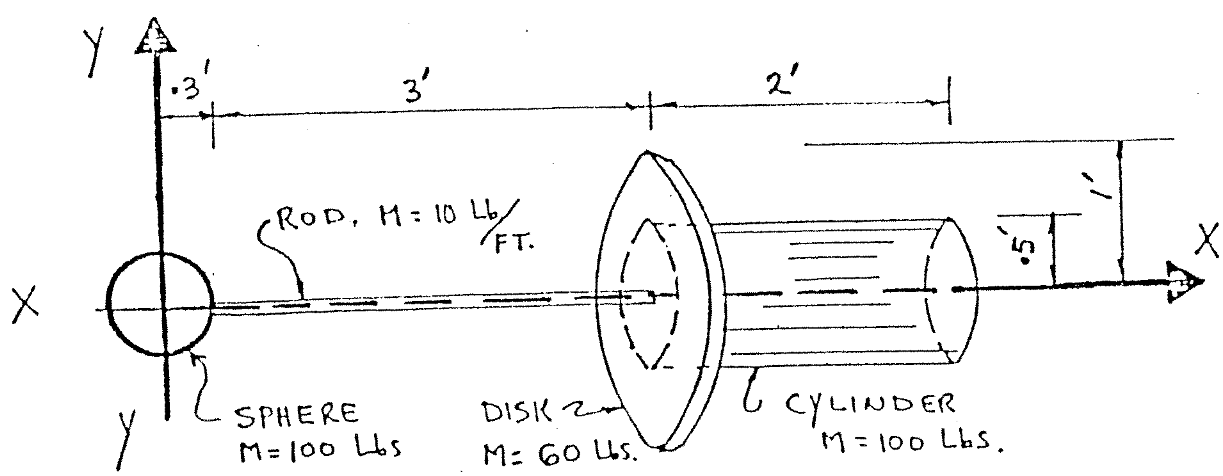


5. A sphere, a rod, a thin disk and a cylinder are welded together as shown in the figure to form one body.

   a) Locate the center of gravity.

   b) Find the mass moment of inertia with respect to the y-y axis; and

   c) Find the radius of gyration with respect to the y-y axis.



CONCORDIA UNIVERSITY

COURSE	NUMBER	SECTION	
Statics	ENGR 242/2	I, V, X, XX, Y,	
EXAMINATION	DATE	TIME	# OF PAGES
Final	December 7, 1991	14:00-17:00	6

INSTRUCTOR
Profs. Babarutsi, Fabrikant, Goldman, Saathoff, Stathopoulos, Wang

MATERIALS ALLOWED: ☐ NO ☒ YES (PLEASE SPECIFY)

Any calculating device.

SPECIAL INSTRUCTIONS:
Do any two questions from PART A and all questions from PART B (total of five).

All questions carry equal weight.

Emad
Nukasha
Section X

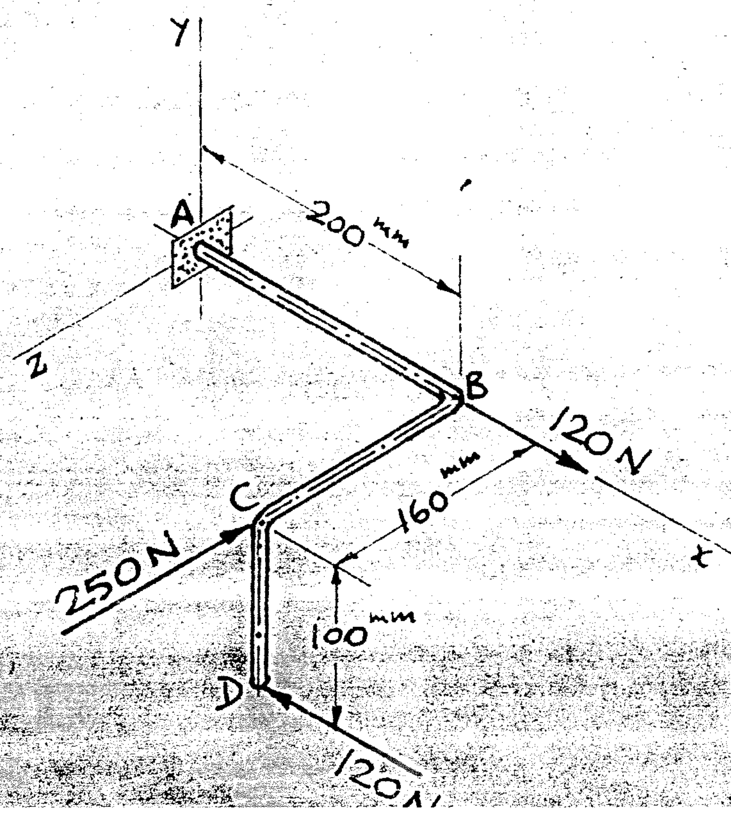
PART A

QUESTION A1:

Replace the three forces shown by:

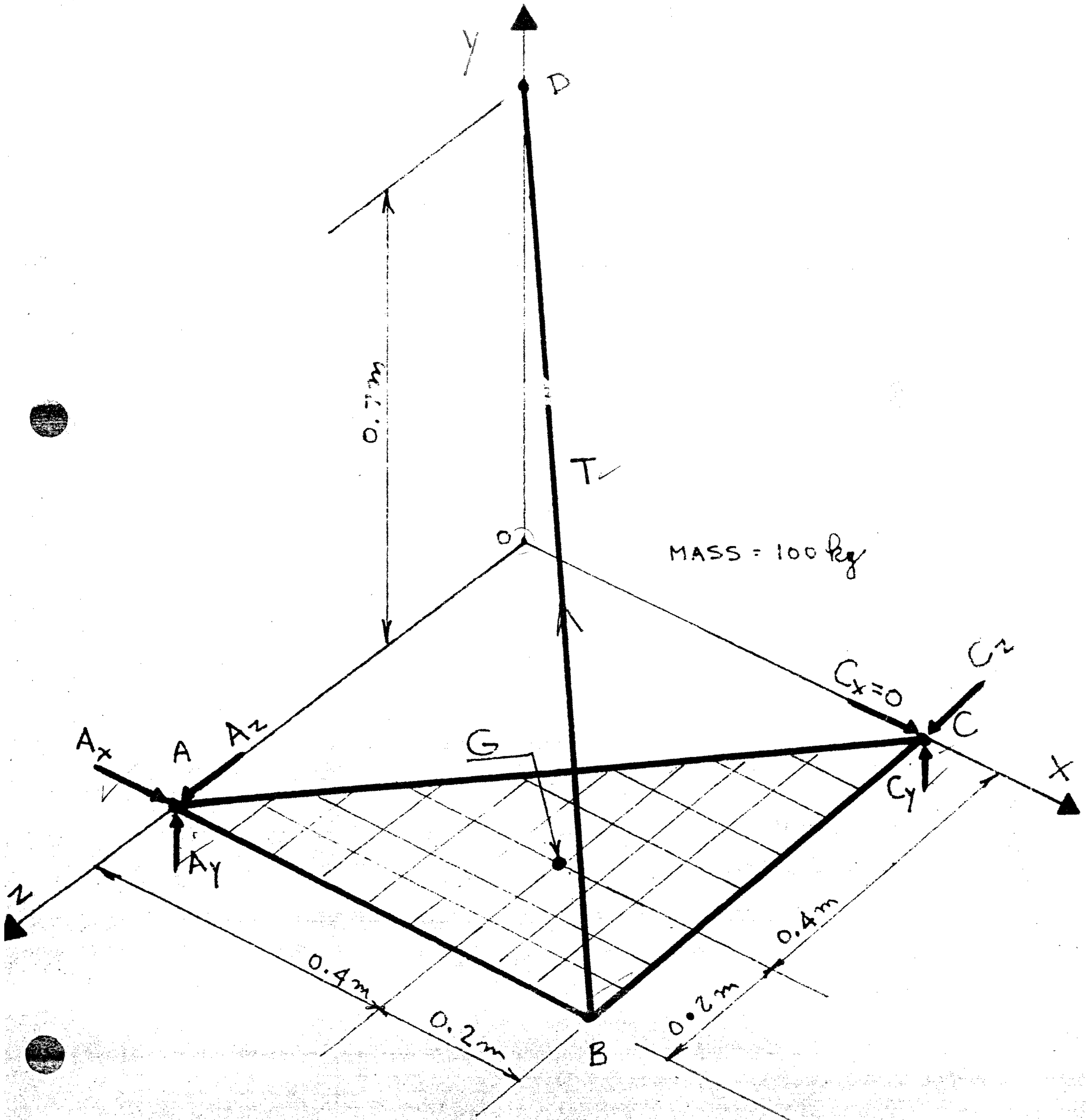
a) A force-couple system at A;

b) A wrench; specify the pitch and the axis of the wrench.



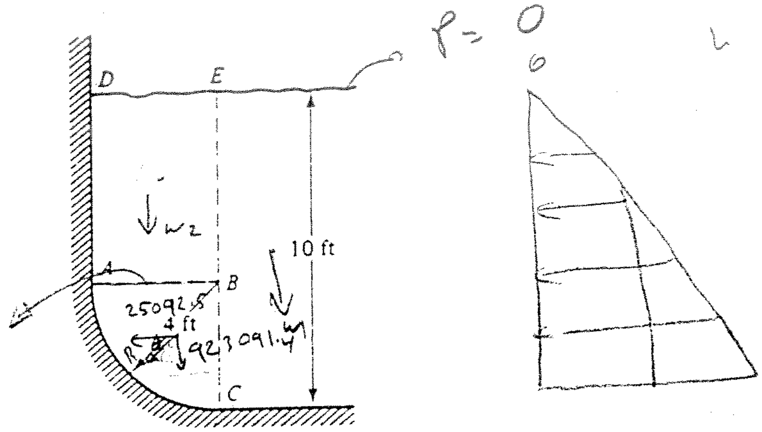
QUESTION A2:

The horizontal triangular plate ABC has a mass equal to 100 kg and is supported at points A and C and by the cable BD. Obtain the tension in cable BD and the reactions at supports A and C. Note that $C_x = 0$. Indicate if the directions of the reactions are correct.



QUESTION A3:

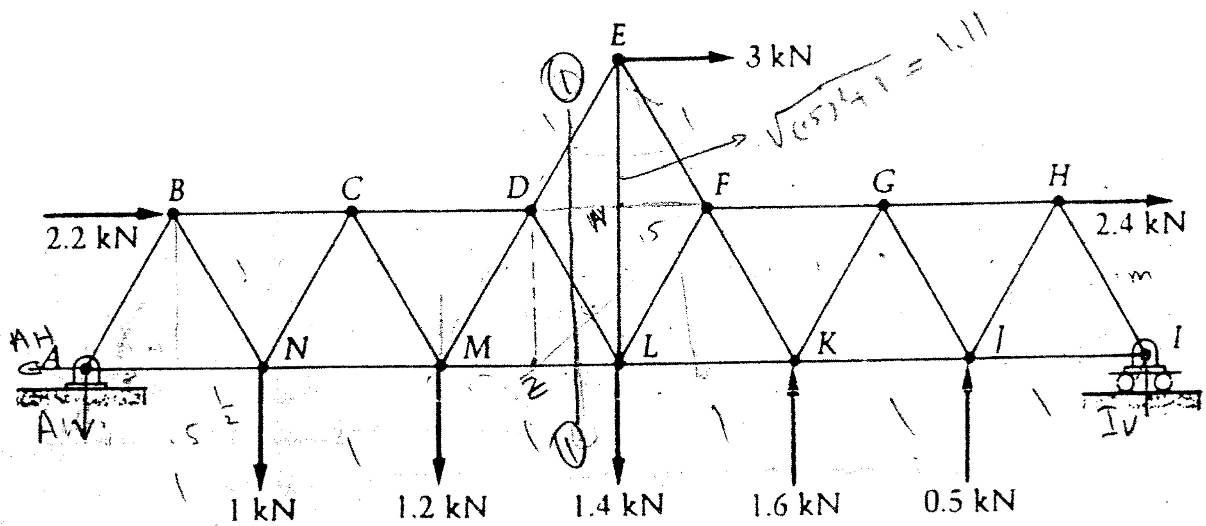
Determine the resultant force R (magnitude, direction, and perpendicular distance of its line of action from point C) acting upon the curved (circular) portion AC of the pool wall shown. The wall is 30 ft. long and the specific weight of water is 62.4 lb/ft^3.



PART B

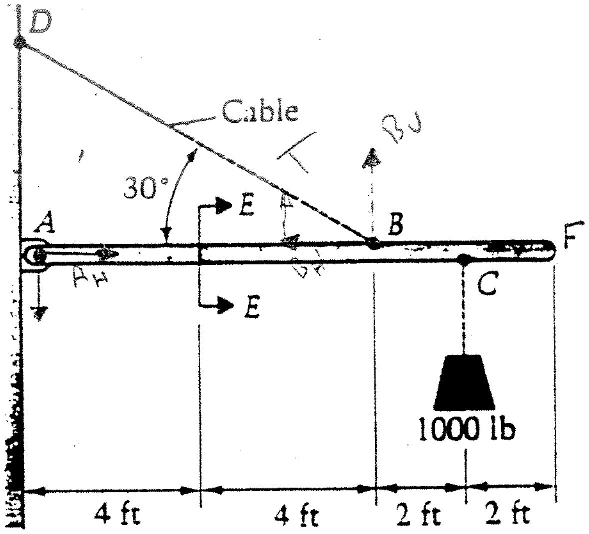
QUESTION B1:

In the truss shown all members are 1 meter long except EL. Find the force in member EL and indicate whether this member is under tension or compression.



QUESTION B2:

The 1000 lb weight is suspended from the beam at point *C*, as shown. The beam is connected to a vertical wall by a frictionless pin at *A* and by a cable between *B* and *D*. Neglecting the weight of the beam, determine the internal forces and moment transmitted across section *E-E*, and draw the shear force and bending moment diagrams for the beam *ABCF*.



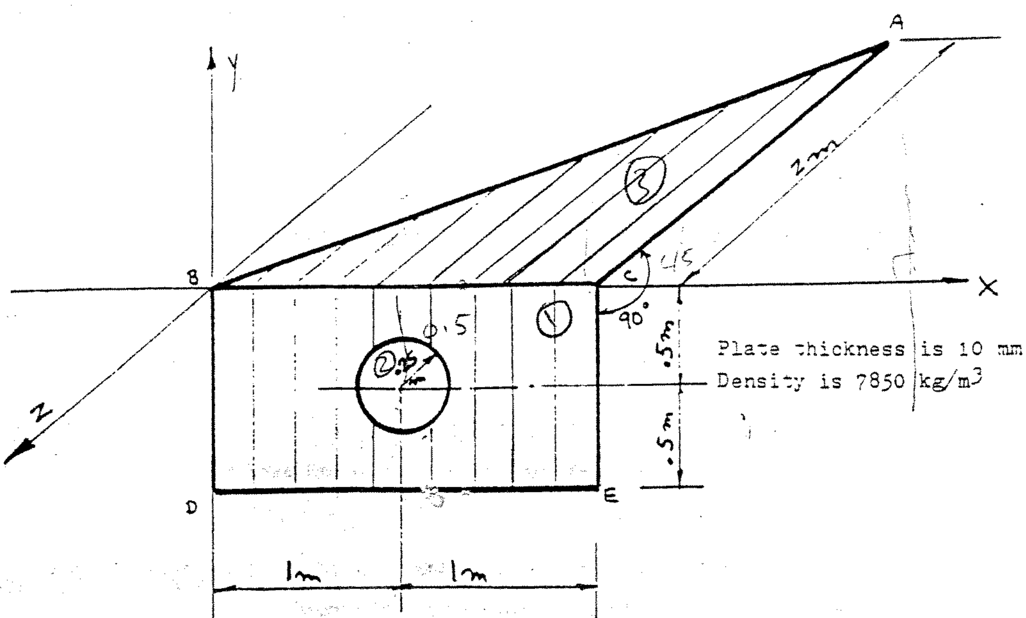
QUESTION B3:

For the composite thin plate shown find:

a) the coordinates of the centre of gravity;

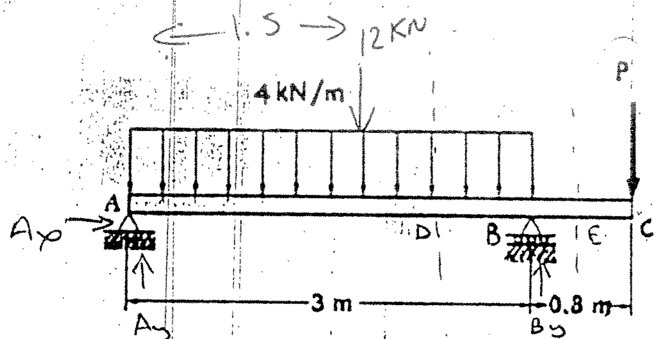
b) the mass moment of inertia with respect to Bz axis; and

c) the radius of gyration with respect to Bz axis.



Question 6:

For the beam ABC shown, draw the shear force and bending moment diagrams. Determine the range of values of P for which the maximum absolute value of the bending moment is equal to or less than 3.6 kN-m.



Sweet Sensation o' the Nation!

$\sum M_A = 0$:

$$(-12)(1.5) + B_y(3) - P(3.8) = 0$$

$$B_y = \frac{18 + 3.8P}{3} = \underline{\underline{6 + 1.27P}}$$

$\sum M_B = 0$:

$$-A_y(3) + (12)(1.5) - P(0.8) = 0$$

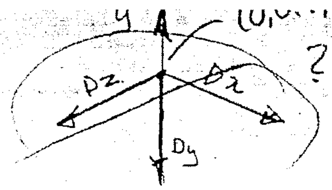
$$\Rightarrow A_y = \frac{18 - 0.8P}{3} = \underline{\underline{6 - 0.27P}}$$

$\sum F_x = 0$ $\quad A_x = 0$

$$-1.58 \leq P \leq 1.58$$

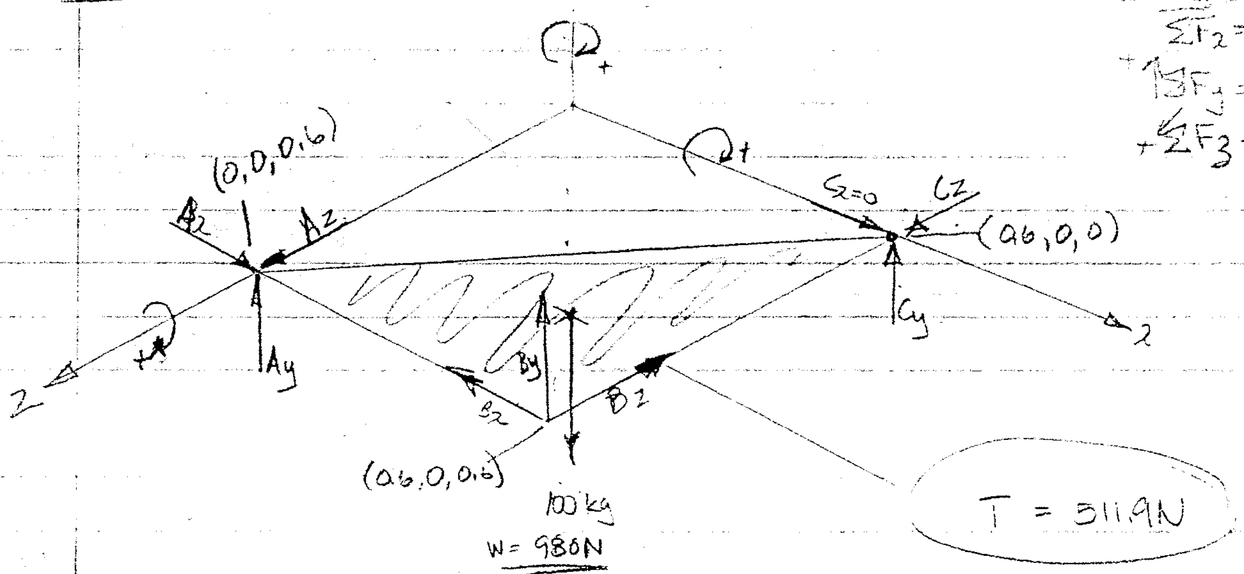
FINAL 91

QUESTION 2



? ARE THESE FORCES INCLUDED?

NOT NEEDED DUE TO TENSION BEING SHOWN AT Pt B



$\Sigma F_x = 0$ | $\Sigma M_x = 0$

$+\uparrow \Sigma F_y = 0$ | $\Sigma F_y = 0$

$+\swarrow \Sigma F_z = 0$ | $\Sigma M_z = 0$

$T_{PB} = d = \sqrt{(-0.6)^2 + (0.7)^2 + (-0.6)^2}$

$= 1.1$

$T_x = T_{BD}(-0.6/1.1) = -0.545\,T_{BD}$

$T_y = T_{BD}(0.7/1.1) = 0.636\,T_{BD}$

$T_z = T_{BD}(-0.6/1.1) = -0.545\,T_{BD}$

① $\vec{\Sigma F_x} = A_x + (-0.545 T_{BD})$

② $+\uparrow \Sigma F_y = A_y + 0.636 T_{BD} + C_y - 980N$

③ $\swarrow \Sigma F_z = A_z + (-0.545 T_{BD}) + C_z$

④ $\Sigma M_{@x} = -0.6C_y - 0.636T_{BD}(0.6) + 980(0.4)$

⑤ $\Sigma M_{@y} = 0.6C_z + 0.6(-0.545 T_{BD})$

⑥ $\Sigma M_{@z} = 0.6A_y + 0.636T_{BD} - 980(0.4)$

④ $= -0.6C_y - 0.636T_{BD}(0.6) + 980(0.4)$

$= -0.6C_y - 0.382T_{BD} + 392$

$C_y = (392 - 0.382T_{BD})/0.6$

$= 653.3 - 0.637T_{BD}$

⑥ $0.6A_y + 0.636T_{BD}(0.6) - 980(0.4)$

$A_y = (392 - 0.382T_{BD})/0.6$

$A_y = 653.3 - 0.637T_{BD}$

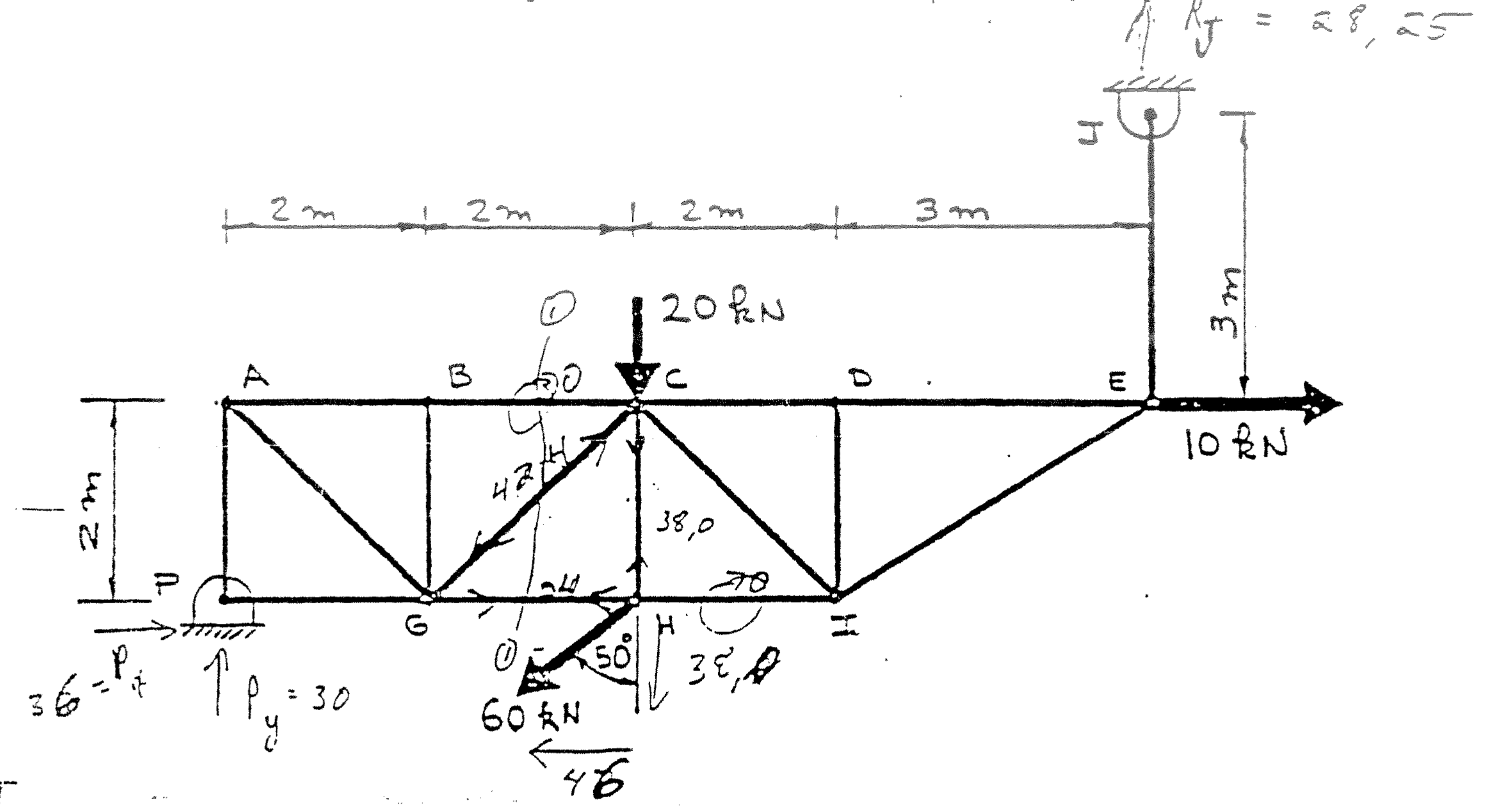
② $\Sigma F_y = A_y + 0.636T_{BD} + C_y - 980$

$= 653.3 - 0.637T_{BD} + 0.636T_{BD} + 653.3 - 0.637T_{BD} - 980$

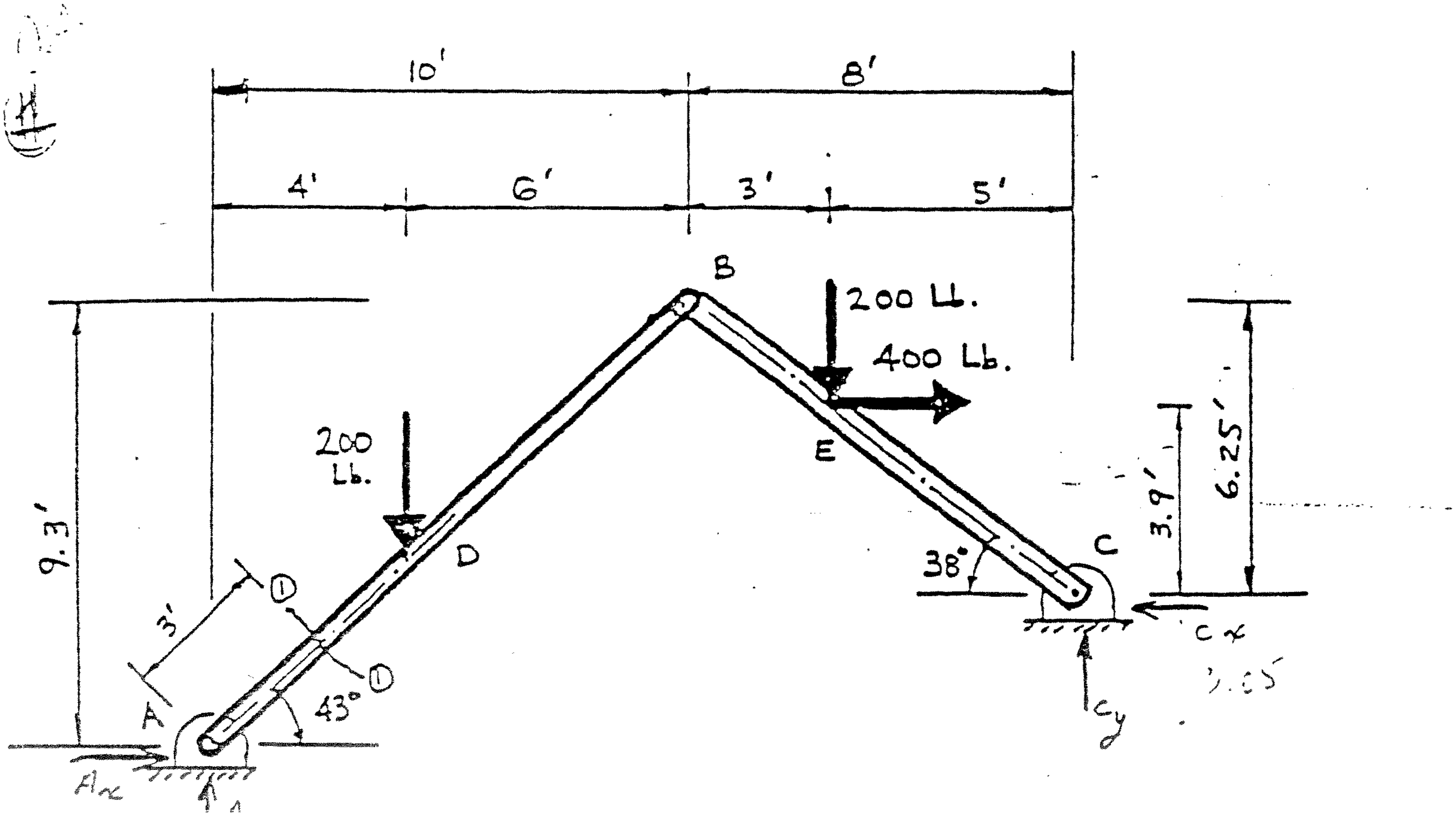
$= 653.3 + 653.3 - 980 - 0.637T_{BD} + .636T_{BD} - 0.637T_{BD}$

$= 326.6 - .638T_{BD} \Rightarrow T_{BD} = \frac{326.6}{.638} = 511.9N \; T$

- For the truss shown, calculate the reactions at J and P.
Calculate the forces in members BC, CH, CG and HI. Show the values on the sketch of the truss indicating whether tension or compression.



4.- For the frame shown obtain the reactions at A and C. Check if the frame is in equilibrium. At section 1-1, obtain axial and shear forces and bending moment.



CONCORDIA UNIVERSITY

COURSE	NUMBER	SECTION
STATICS	ENGR C 242/2	01, V, W, X, AA

EXAMINATION	DATE	TIME	# OF PAGES
SUPPLEMENTAL			3

INSTRUCTOR	DIVISION
Profs. Goldman, Stathopoulos	Day and Evening

MATERIALS ALLOWED

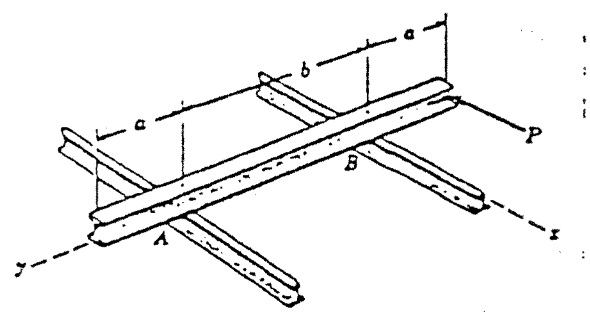
Any Calculating Device

SPECIAL INSTRUCTIONS

Attempt all problems.

All questions carry equal weight.

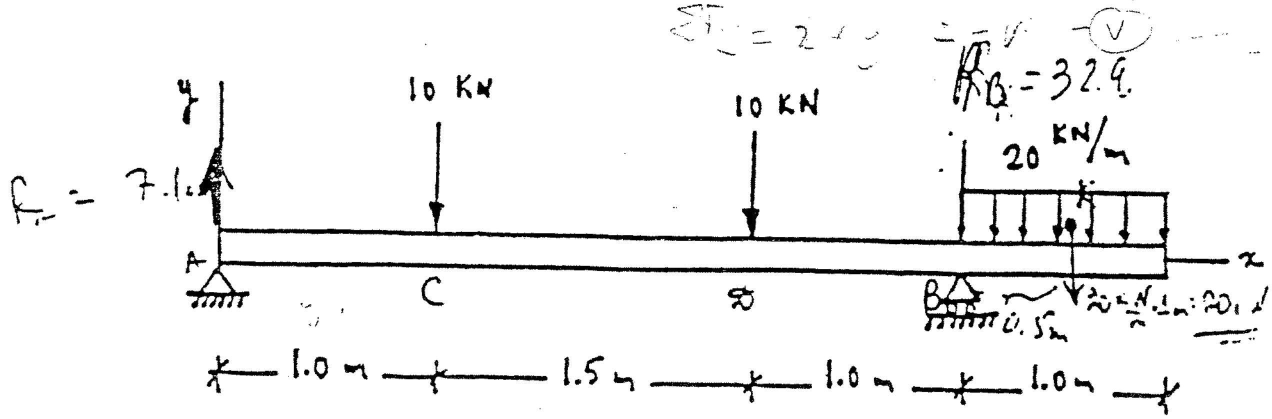
1.- An I-beam of mass m is supported by two fixed horizontal rails as shown. Compute th applied load P that is just sufficient to cause the beam to slip and determine the corresponding friction force at A as slippage begins. The coefficient of friction between the beam and the rails is f.



2.- For the beam shown:

a.- Draw the shear and bending moment diagrams.

b.- Write the shear and bending moment equations for $3.5 < x \leq 4.5$



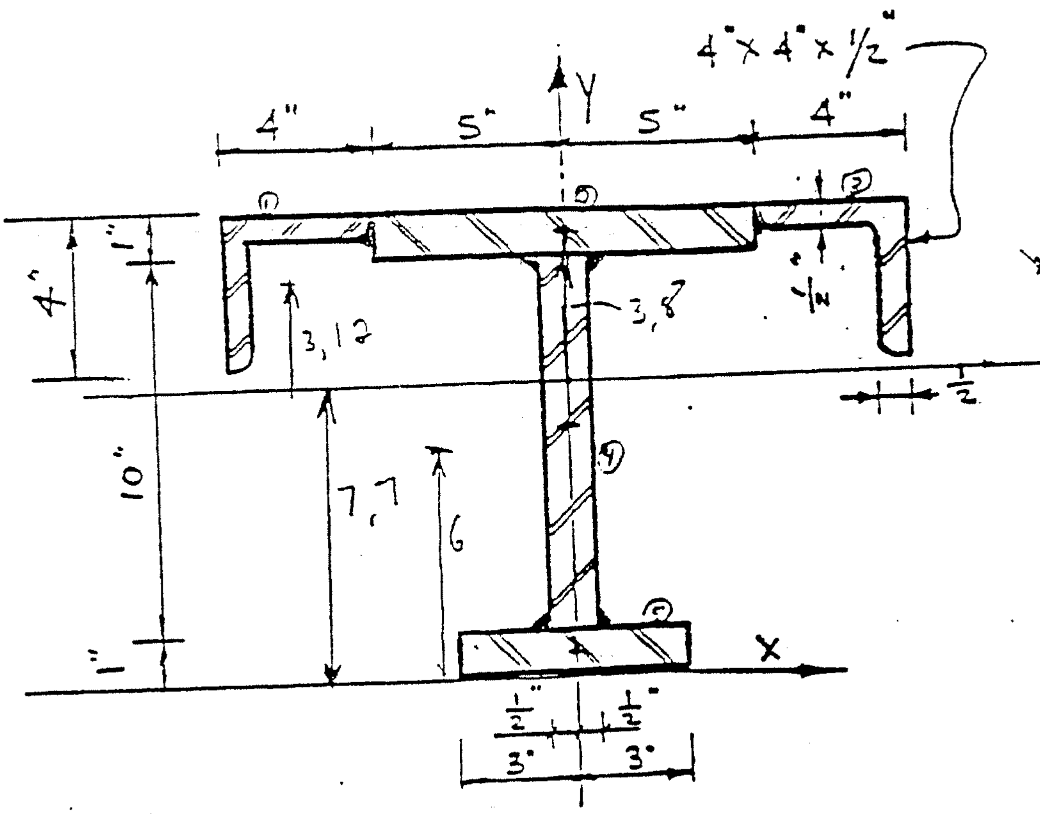
5.- For the cross-section of the beam indicated:

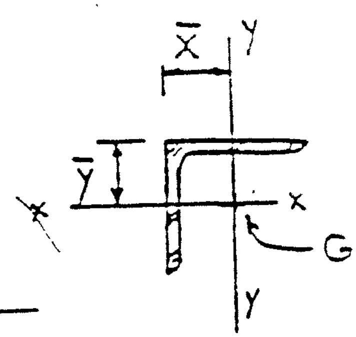
a.- Obtain the location of the centroid.

b.- Obtain the moment of inertia of the area with respect to an axis parallel to the X axis and passing through the centroid.

c.- Obtain the radius of gyration of the area with respect to the same axis as in b.- above.

$I = k^2 A \qquad k = \sqrt{\frac{I}{A}}$

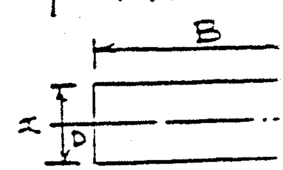




AREA = 3.75 IN^2

I_x = 5.6 IN^4 =

$\bar{X} = \bar{Y}$ = 1.18 IN.



$I_{xx} = \frac{BD^3}{12}$

Dr. Haţeganu

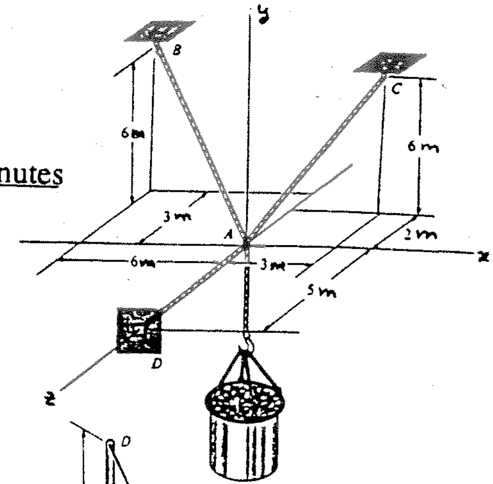
CONCORDIA UNIVERSITY

Faculty of Engineering and Computer Science

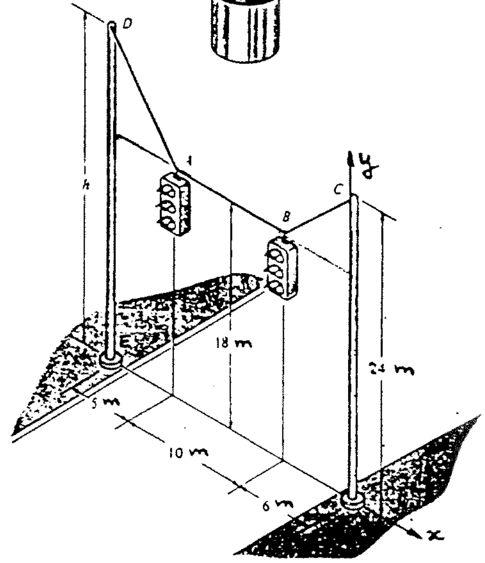
ENGR 242/2 STATICS, Section T
TEST # 1

<u>Attempt all questions, only calculators permitted.</u> <u>Time: 70 minutes</u>

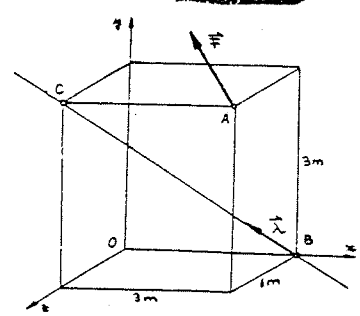
MARKS



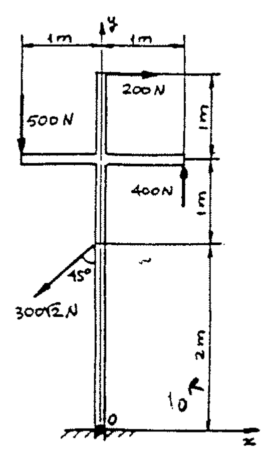
30 1. A bucket is supported by three cables, as shown. Determine the weight W of the bucket, knowing that the magnitude of the tension in cable AD is 194.4 kN. As well, determine the magnitude of the tensions in cables AB and AC.



20 2. The traffic-lights at A and B are suspended from the two poles as shown. If each light has a weight of 50 kN, determine the tension in each of the three supporting cables and the height h of the pole DE. Cable AB is horizontal. *Hint:* First analyze the equilibrium at point B; then, using the result for the tension in AB, analyze the equilibrium at A. (The problem is in 2D.)

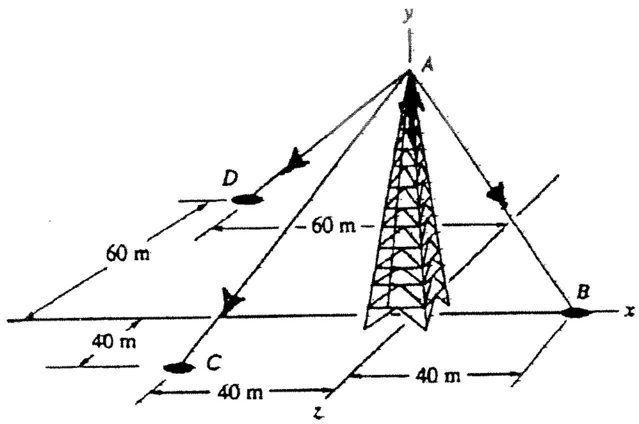


30 3. A force $\mathbf{F} = (-300\,\mathbf{i} + 300\,\mathbf{j} - 100\,\mathbf{k})$ N is acting at point A, as shown. Determine the moments M_x, M_y and M_z of the force about the coordinate axes passing through O, as well as the moment of the force about axis BC. Plot the results.



20 4. Determine the equivalent force-couple system at point O, consisting of the resultant force **R** and the resultant moment $\mathbf{M}_O^R$, if all forces acting on the pole are in the xy-plane. Indicate **R** and $\mathbf{M}_O^R$ on-the figure.

2. Three cables support the 70-m-tall tower. The resultant of the three cable forces on the tower is vertical. If the magnitude of force $\mathbf{F}_{AD}$ is 2000 N, what is the magnitude of $\mathbf{F}_{AC}$? (Look at the problem carefully before you start calculating.)



$$\bar{F}_{AD} = 2000\left(\frac{-60\bar{i}-70\bar{j}-60\bar{k}}{\sqrt{60^2+70^2+60^2}}\right) N$$

$$\bar{F}_{AC} = F_{AC}\left(\frac{-40\bar{i}-70\bar{j}+40\bar{k}}{\sqrt{40^2+70^2+40^2}}\right) N$$

NO OTHER FORCE HAS A Z COMPONENT

$$\Sigma F_z = 0 \qquad \frac{-2000(60)}{\sqrt{60^2+70^2+60^2}} + \frac{40\,F_{AC}}{\sqrt{40^2+70^2+40^2}} = 0$$

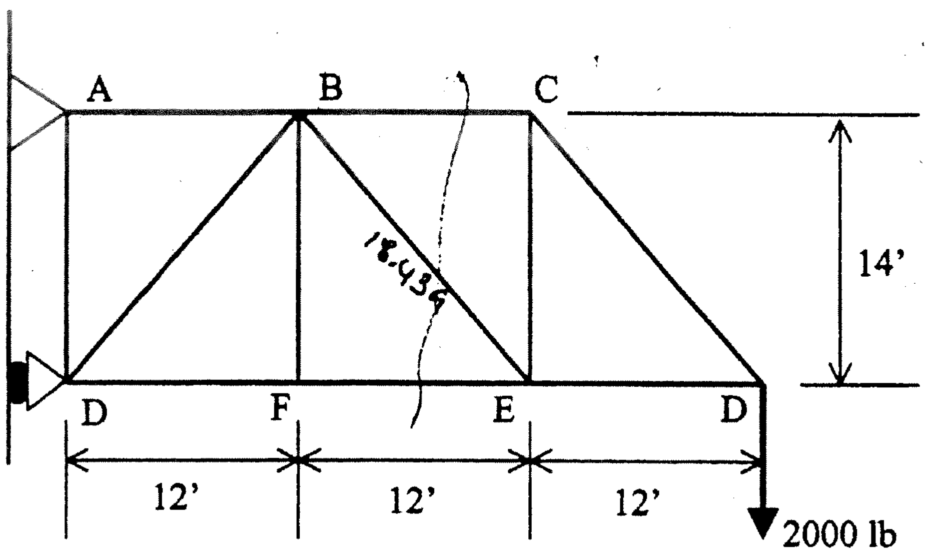
$$\boxed{F_{AC} = 2455\ N}$$

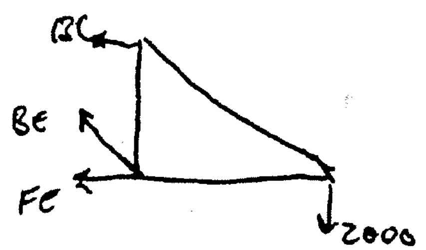
Statics **Final Exam**

Name ______________________

For all problems, it is assumed that the systems are in static equilibrium and lie in the x-y plane. For all problems you must draw all free body diagrams and show all of your work.

1. Determine the forces in the members BC, BE and FE of the truss shown. You must indicate whether the member is in tension or compression.





$\Sigma M_E = 0$ $\quad 14BC - 2000(12) = 0$

$\boxed{BC = 1714.3 \text{ lb } (T)}$

$\Sigma M_B = 0$ $\quad -14FE - 24(2000) = 0$

$FE = -3428.6 \text{ lb}$

$\boxed{3428.6 \ (C)}$

$\Sigma F_y = 0$

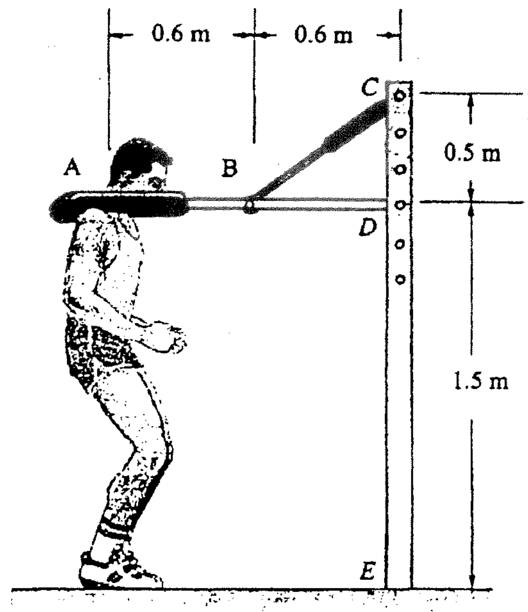
$\frac{14}{18.439} BE - 2000 = 0$

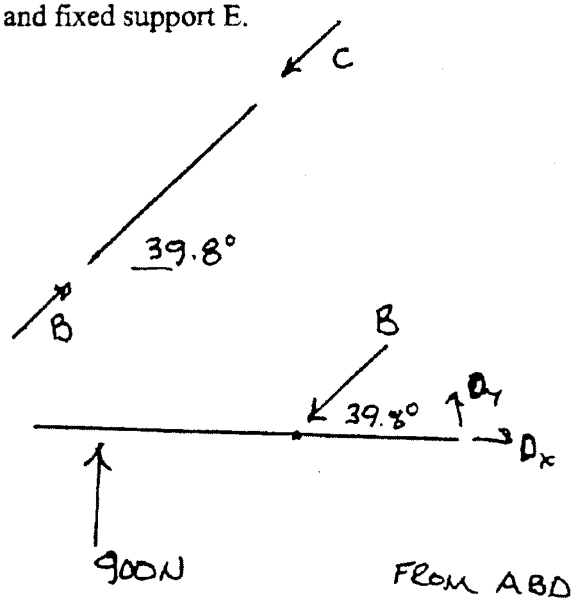
$\boxed{BE = 2634.2 \text{ lb } (T)}$

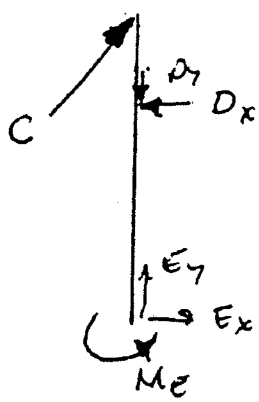
Name ______________________

2. A Rowan student works out at the Rec Center with a squat machine. Just before the bar ABD rotates, the student is exerting a vertical force of 900 N at A with his shoulder. Note that B, C and D are pinned joints and member CDE is fixed at the base E. You may neglect the weight of members ABD, BC and CDE in your analysis.

a) Draw free-body diagrams of each member in the structure.
b) Determine the reactions on pins B, C and D and fixed support E.







FROM ABD

$\Sigma M_A = 0 \qquad -900(1.2) + .6\, B \sin 39.8 = 0$

$B = 2812\ N = C$

$\Sigma F_x = 0 \qquad D_x = B\cos 39.8 = 2160\ N \rightarrow$

$\Sigma F_y = 0 \qquad 900 - B\sin 39.8 + D_y = 0$

$D_y = 900\ N \uparrow$

FROM EDC

$\Sigma F_y = 0 \qquad -D_y + E_y + C\sin 39.8 = 0$

$\Sigma F_x = 0 \qquad -D_x + E_x + C\cos 39.8 = 0$

$-900 + E_y + 2812 \sin 39.8 = 0$

$E_y = -900\ N \downarrow$

$-2160 + E_x + 2812 \cos 39.8 = 0$

$E_x = 0$

FULL STRUCTURE

$\Sigma M_E = 0$

$M_E - 900(1.2) = 0$

$M_E = 1080\ N\text{-}m$ ↺

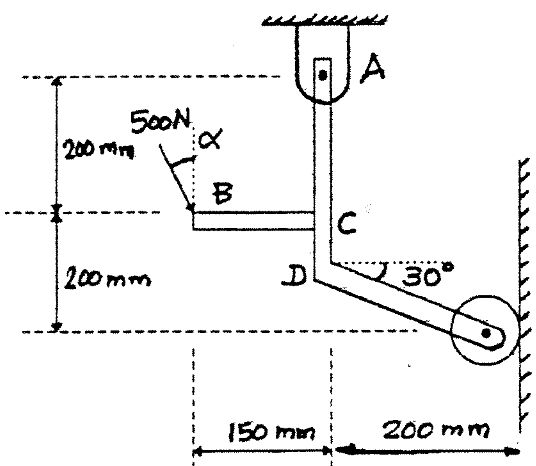
MAE 206 TEST #2, FALL 1996, 10/18/96

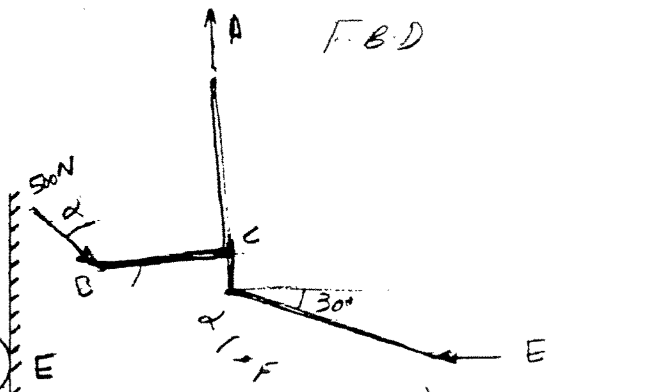
NAME:____________________ SS#________________

PROBLEM 1: (30 POINTS)

FBD: 10 POINTS
$A_x = 0$
EQUATIONS: 10 (Force Δ)
Solution completeness) 10

Determine (a) the value of α for which the reaction at A is vertical, (b) the corresponding reactions at A and E.





Alito:

$\circlearrowleft + \sum M_C = 0$
$(500)\cos\alpha(150) - E(200) = 0$

$\xrightarrow{+} \sum F_x = 0 \quad +(500)\sin\alpha - E = 0$

$+\uparrow \sum F_y = 0 \quad -500\cos\alpha + A = 0$

$E = 500\sin\alpha$

$\therefore \quad 500\cos\alpha(150) - (200)(500\sin\alpha) = 0$

$\frac{\sin\alpha}{\cos\alpha} : \tan\alpha = \frac{150}{200}$

$\alpha = 36.9^\circ$

$E: \ 500\sin\alpha = 300\text{ N} \leftarrow$

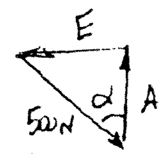
$A: \ 500\cos\alpha = 400\text{ N} \uparrow$

3 FORCE BODY: CONCURRENT AT F.

CF = 200 mm ; BC = 150 mm

$\tan\alpha = \frac{150}{200}$ $\boxed{\alpha = 36.9^\circ}$

Force Δ:



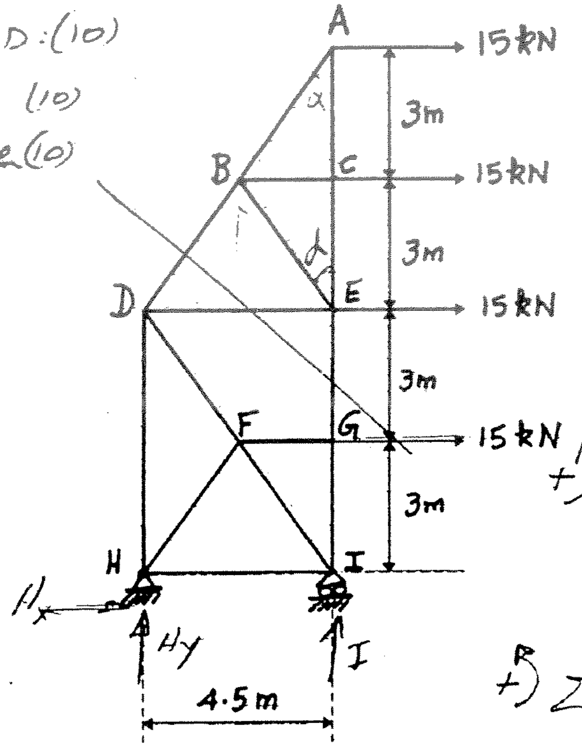
$E: \ 500\sin\alpha = 300\text{ N} \leftarrow$

$\boxed{A: \ 500\cos\alpha = 400\text{ N} \uparrow}$

PROBLEM 2: (30 POINTS)

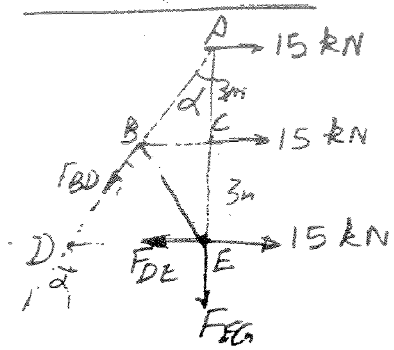
Determine the forces in BD and DE of the truss shown.

Section FBD: (10)
Equations (10)
Completion (10)



Method of sections

Take upper part



$\tan\alpha = \frac{DE}{AE} = \frac{4.5}{6} = \frac{3}{4}$

$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

$\overset{+}{\circlearrowleft} \sum M_E = 0$

$-15(6) - 15(3) + F_{BD} \cos\alpha \cdot (4.5) = 0$ $\quad \frac{4}{5}$

$\boxed{F_{BD} = 37.5 kN\ T}$

$\overset{+}{\circlearrowleft} \sum M_A = 0$

$15(3) + 15(6) - F_{DE}(6) = 0$

$F_{DE} = \frac{135}{6} = 22.5\ kN\ T$

Method of Joints (Calculation not necessary in this problem)

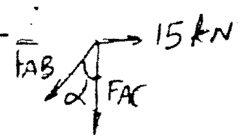
$\circlearrowleft \sum M_H = 0 \quad -15(12) - 15(9) - 15(6) - 15(3) + I(4.5) = 0$

$I = 100\ kN$

$\sum F_x = 0 \quad H_x + 60\ kN = 0 \quad H_x = 60\ kN \leftarrow$

$\sum F_y = 0 \quad H_y + I = 0 \quad \boxed{H_y = +100\ kN \downarrow}$

Joint A:



$\sum F_y = 0$

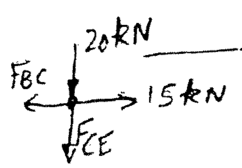
$- F_{AB} \sin\alpha + 15 = 0$

$F_{AB} = (15)\left(\frac{5}{3}\right) = 25\ kN\ T$

$\downarrow + \sum F_{AB} \cos\alpha + F_{AC} = 0$

$F_{AC} = -F_{AB}\cos\alpha = 20\ kN\ C$

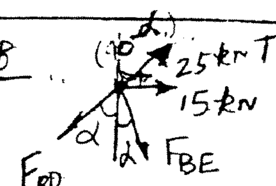
Joint C



$F_{BC} = 15\ kN\ T$

$F_{CE} = 20\ kN\ C$

Joint B



$F_{BD}\cancel{\cos\alpha} + F_{BE}\cancel{\cos\alpha} = 25\cancel{\cos\alpha}$

$F_{BD}\sin\alpha - F_{BE}\sin\alpha - 15 - 25\sin\alpha = 0$

$F_{BD} + F_{BE} = 25$

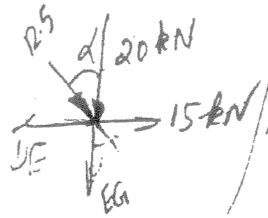
$F_{BD} - F_{BE} = \frac{15 + 25\sin\alpha}{\sin\alpha} = 50$

$2F_{BD} = 75$

$F_{BD} = 37.5\ kN\ T$

$F_{BE} = +12.5\ kN\ C$

Go to Joint E



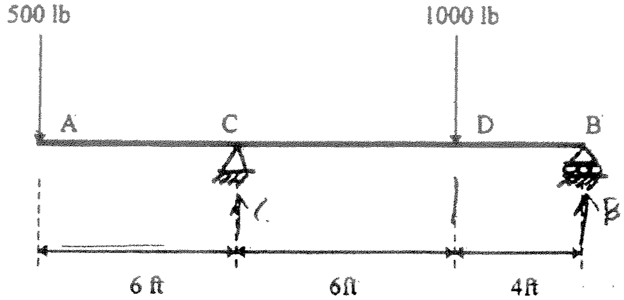
$\xrightarrow{+}\sum F_x = 0$

$15 + 12.5\sin\alpha - F_{DE} = 0$

$$\boxed{F_{DE} = 15 + 12.5\left(\frac{3}{5}\right) = 22.5 \text{ kN } T}$$

PROBLEM 3: (30 POINTS)

Draw the shear force-bending moment diagrams for the following beam.



$\Sigma M_B = 0$

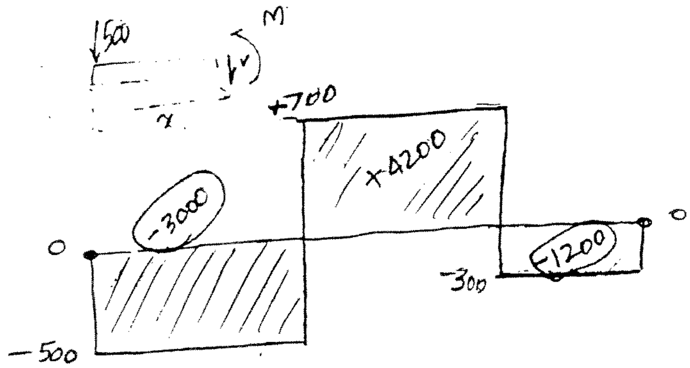
$-C(10) + 1000(4) + 500(16) = 0$

$C = 1200\ lb \uparrow$

$+\uparrow \Sigma F_y = 0 \quad B = 300\ lb \uparrow$

Shear

V(lb)



$-V - 500 = 0$

$V = -500$

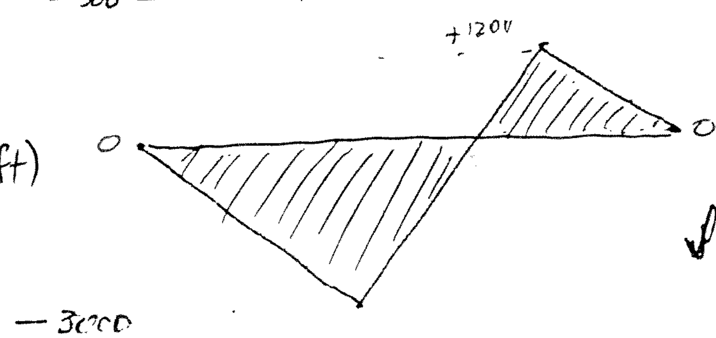
B M: AC

$M + 500x = 0$

$m = -500x$

BMD

M(lb-ft)



Bm. C.D

$M + 500(x) - 1200(x-6) = 0$

(1,114) at D $m = 1200(12-6) - 500(12) = 0$

$= 7200 - 6000 = +1200$

Showing Eqns 10

Sh. force 10

B.m. 10

PROBLEM 4: (10 POINTS)

a. The truss in problem 2 is a SIMPLE TRUSS TRUE FALSE

b. The bracket in problem 1 is a three-force body TRUE FALSE

c. The beam in problem 3 is a:

1. Cantilever beam

2. Simply supported beam

3. Overhanging beam

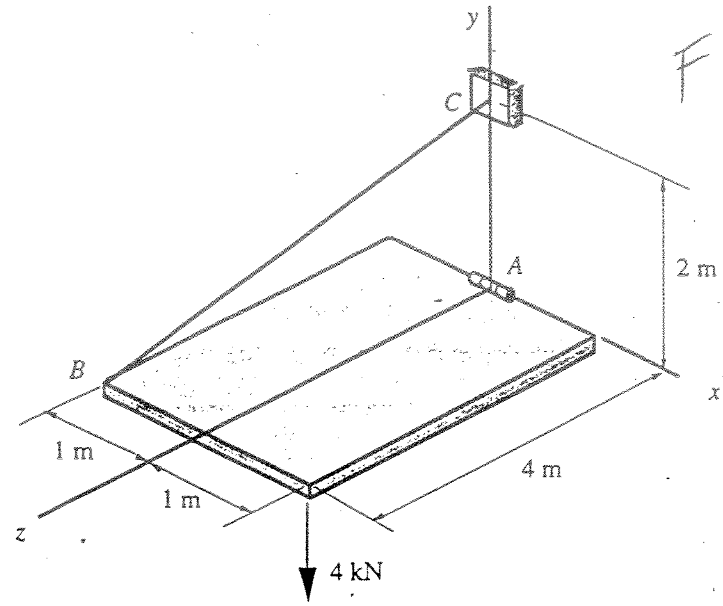
4. Continuous beam

d. If the shear force diagram is a linear function of x, the bending moment diagram is

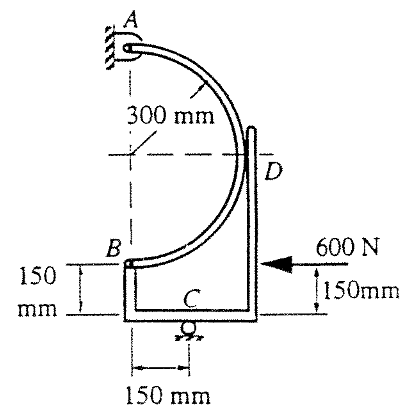
1. linear
2. quadratic
3. cubic
4. constant

F 98/2

4. A 300-kg homogeneous trap door of uniform thickness is subject to a vertical force of 4 kN as shown. It is supported by a frictionless door-hinge at A and by the cable BC. Determine the reaction forces and moments provided by the hinge A.



5. The frame BCD is in contact with the smooth semi-circular member ADB at point D while the two members are connected by the pin B. Determine the forces acting on the semi-circular member (Note: neglect the weight of the members).

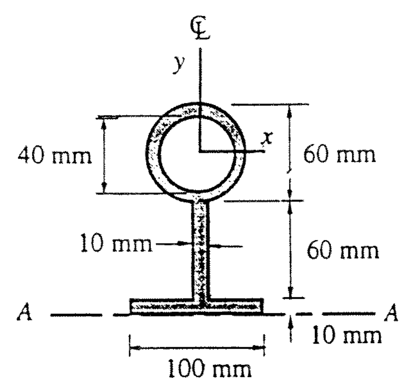


6. For the shaded area shown:

a) Locate its centroid with reference to the axes x, y.

b) Determine its moment of inertia with respect to the x-axis (Note: For a circular area $I_x = I_y = \pi\ r^4/4$).

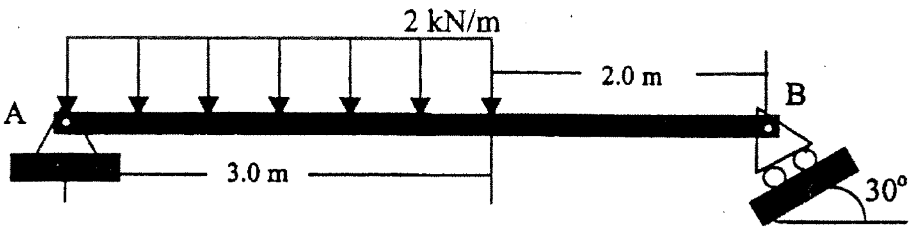
c) Determine the volume of the solid of revolution generated by a 360°-rotation of the shaded area about the axis AA.

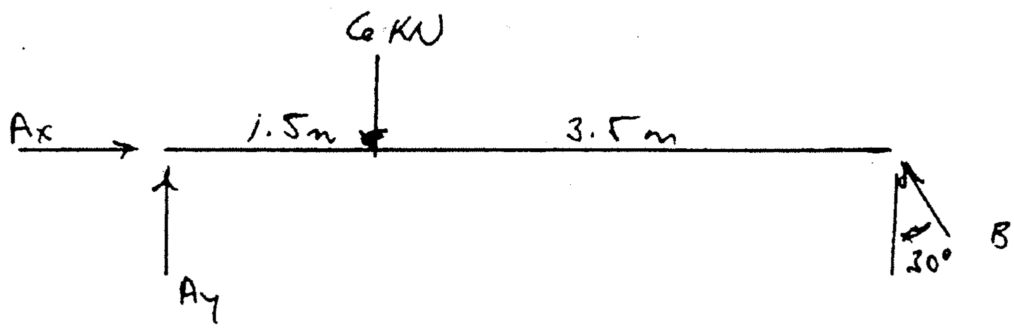


Statics **Final Exam**

Name ____________________________

3. The bar is pinned at point A and on a roller at point B. A 2 kN/m distributed load is applied as shown. Draw the free body diagram and determine the reactions at A and B.





$\Sigma M_A = 0 \qquad -1.5(6) + B\cos 30(5) = 0$

$B = 2.08\ KN$

$\Sigma F_y = 0 \qquad A_y - 6 + 2.08\cos 30 = 0$

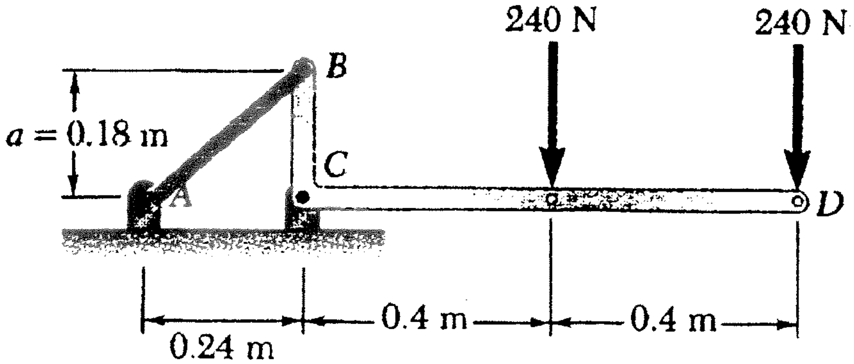
$A_y = 4.2\ KN \uparrow$

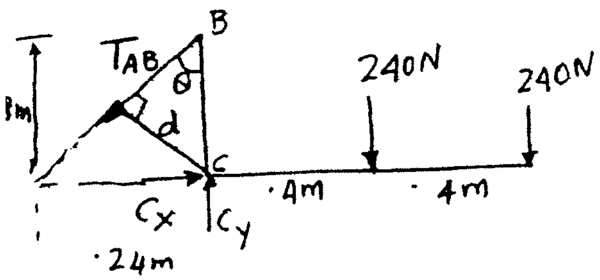
$\Sigma F_x = 0 \qquad A_x - 2.08\sin 30 = 0$

$A_x = 1.04\ KN \rightarrow$

PROBLEM #1 (30 POINTS)

The bracket BCD is hinged at C and attached to a control cable at B. For the loading shown, determine a) the tension in the cable, (b) the reaction at C.





$\tan\theta = \frac{.24}{.18}$ $\theta = 53.13^\circ$

$AB = \sqrt{.24^2 + .18^2} = 0.3$ also: $\sin\theta = \frac{.24}{.3}$

$\cos\theta = \frac{.18}{.3}$

$\circlearrowleft + \sum M_C = 0$ $\quad -240(.4) - 240(.8) + T_{AB} \cdot d = 0$

$d = (CB)\sin\theta = .18\left(\frac{.24}{.3}\right)$ Therefore

$-240(.4) - 240(.8) + T_{AB}(.18)\left(\frac{.24}{.3}\right) = 0$

$\boxed{T_{AB} = 2000\text{N}}$

$+\uparrow \sum F_y = 0$

$C_y - T_{AB}\cos\theta - 240 - 240 = 0$

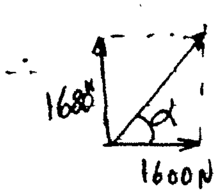
$C_y - 2000\left(\frac{.18}{.3}\right) - 240 - 240 = 0$

$\boxed{C_y = 1680\text{N}\uparrow}$

$\xrightarrow{+} \sum F_x = 0$ $\quad -T_{AB}\sin\theta + C_x = 0$

$\boxed{C_x = 1600\text{N}\rightarrow}$

$C_x = (2000)\left(\frac{.24}{.3}\right) = 1600\text{ N}$



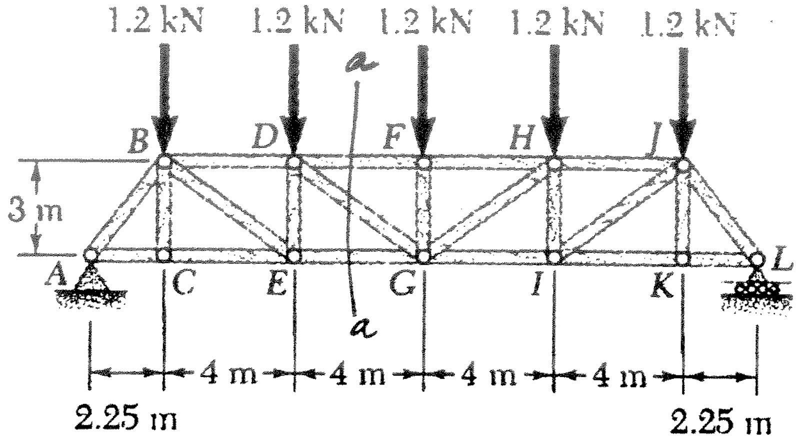
$|C| = \sqrt{1680^2 + 1600^2} = 2320\text{N}$

$\tan\alpha = \frac{1680}{1600}$ $\alpha = 46.4^\circ$

$\boxed{C = 2320\text{N} \measuredangle 46.4^\circ}$

PROBLEM #2 (25 POINTS)

A roof truss is loaded as shown. Determine the force in DF. *Hint: Use Method of Sections.*



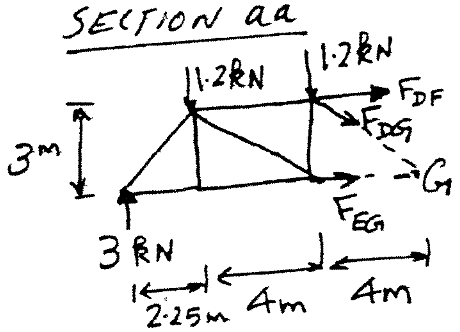
REACTION AT SUPPORTS $\boxed{A_x = 0}$

$+\circlearrowleft \Sigma M_A = 0 = -(1.2)(2.25) - 1.2(6.25) - 1.2(10.25) - 1.2(14.25) - 1.2(18.25) + L(20.5)$

$\therefore -1.2[2.25 + 6.25 + 10.25 + 14.25 + 18.25] + L(20.5) = 0$ $\boxed{L = 3\ kN \uparrow}$

$+\uparrow \Sigma F_y = 0$ $-1.2(5) + 3 + A = 0$ $\boxed{A = 3\ kN \uparrow}$

SECTION aa



$+\circlearrowleft \Sigma M_G = 0$

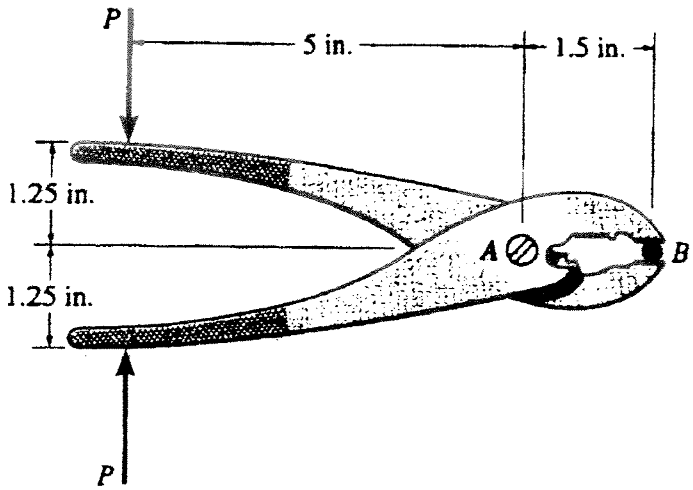
$-F_{DF}(3) + 1.2(4) + 1.2(8) - 3(10.25) = 0$

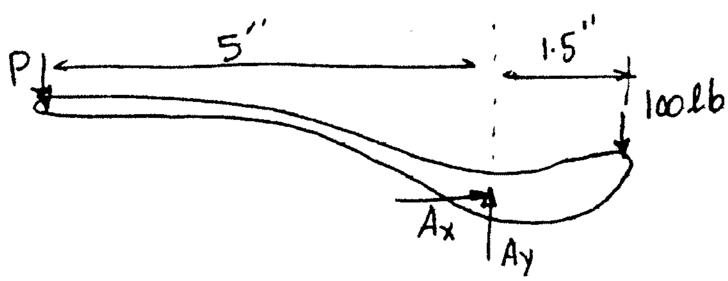
$-F_{DF} = +5.45\ kN$ or $F_{DF} = -5.45\ kN$

or $\boxed{F_{DF} = 5.45\ kN\ C}$

PROBLEM #3 (20 POINTS)

Determine the force P that must be applied to the handles of the pliers so that it develops a force of 100 lb on the smooth bolt B. Also what is the magnitude of the resultant force acting on the pin at A?





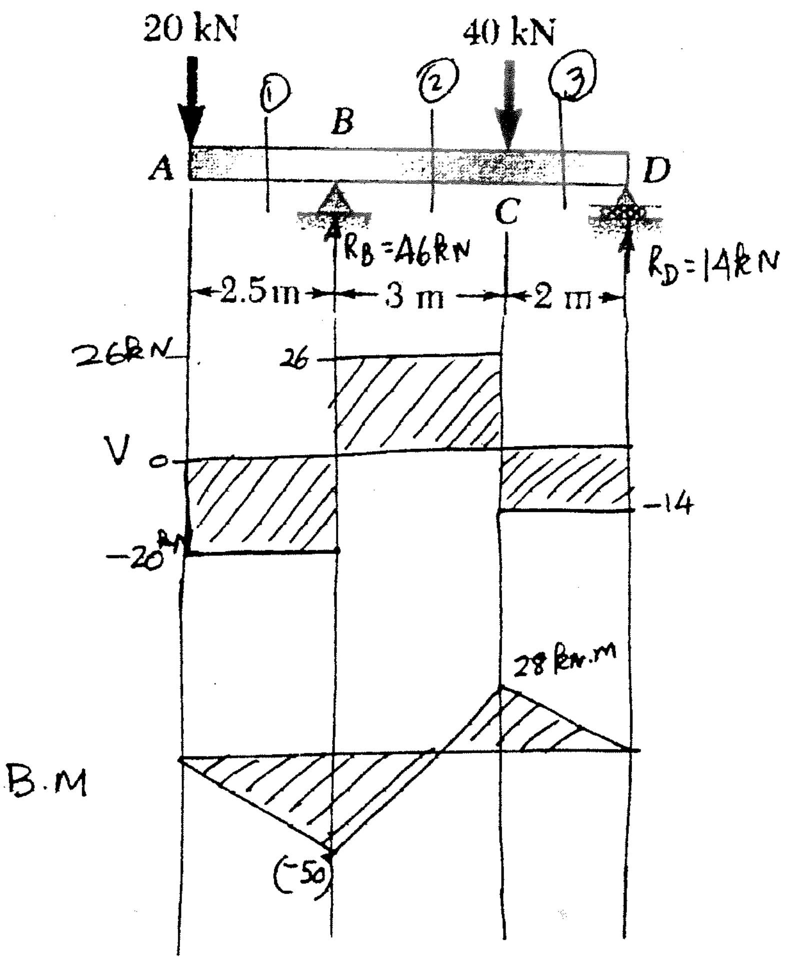
$\circlearrowleft + \sum M_A = 0 \qquad 5P - 100(1.5) = 0 \qquad \boxed{P = 30 \text{ lb}}$

$+\uparrow \sum F_y = 0 \qquad -30 - 100 + A_y = 0 \qquad \boxed{A_y = 130 \text{ lb} \uparrow}$

$\overset{+}{\rightarrow} \sum F_x = 0 \qquad \boxed{A_x = 0}$

PROBLEM #4 (25 POINTS)

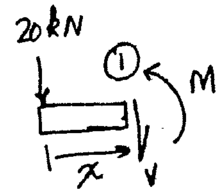
Draw the shear force and bending-moment diagrams for the beam and loading shown. The reaction at B turns out to be 46kN ↑ and the reaction at D turns out to be 14 kN ↑. Show all your free body diagrams and equations.



at B: $B \cdot M: -20(2.5) = -50\,kN.m$

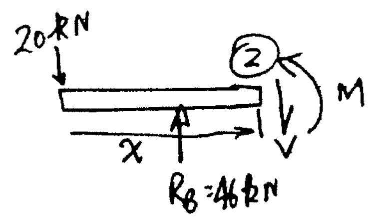
at C: $B \cdot M = 26(5.5) - 115 = +28\,kN.m$

at D: $B \cdot M = -14(7.5) + 105 = 0$



$+\uparrow \Sigma F_y = 0 \quad -20 - V = 0 \quad \boxed{V = -20kN}$

$+\circlearrowleft \Sigma M_1 = 0 \quad 20x + M = 0 \quad \boxed{M = -20x}$



$+\uparrow \Sigma F_y = 0 \quad -20 + 46 - V = 0$

$26 - V = 0$

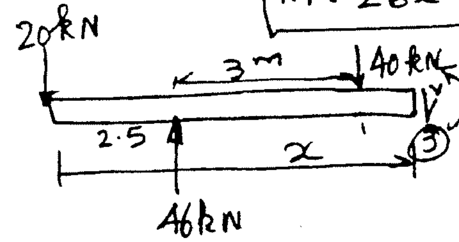
$\boxed{V = 26kN}$

$+\circlearrowleft \Sigma M_2 = 0$

$20(x) - 46(x - 2.5) + M = 0$

$-26x + 46(2.5) + M = 0$

$\boxed{M = 26x - 115}$



$+\uparrow \Sigma F_y = 0 \quad -20 + 46 - 40 - V = 0$

$\boxed{V = -14kN}$

$+\circlearrowleft \Sigma M_3 = 0$

$20(x) - 46(x - 2.5) + 40(x - 5.5) + M = 0$

$14x - 105 + M = 0$

$\boxed{M = -14x + 105}$

EHG 242 Fanal

11/04/03

F98/1

Concordia UNIVERSITY

Faculty of Engineering and Computer Science

FINAL EXAMINATION ENGR 242/2 Statics (Fall 98) Sections: T, V, X, XX

Instructors: Profs. Ha (Coordinator), Haseganu, Megri, Rivard

Materials allowed: Non-programmable calculators **Time allowed**: 3 hours

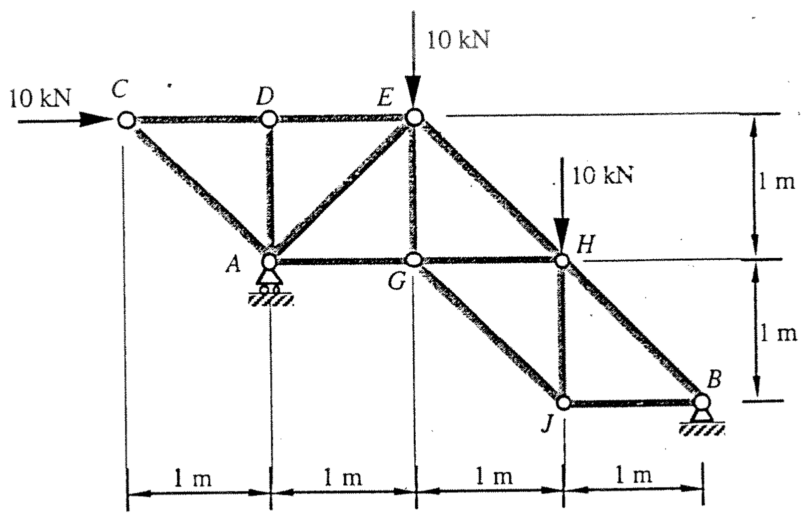
Special instructions: Problems carry equal weights. Solve any FIVE of the given 6 problems.

1. For the truss shown:

a) Use the method of joints to determine the forces in members *BJ* and *BH*;

b) Use the method of sections to determine the forces in members *EG* and *EH*;

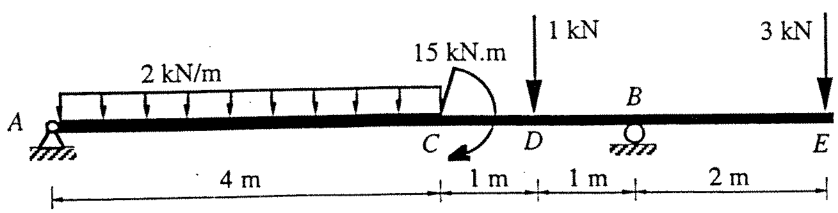
c) Indicate (with justification) the zero-force members.



2. For the given beam and loading:

a) Draw the shear force and bending moment diagrams and indicate the peak values as well as points of zero moment;

b) Derive the expressions for the shear force and the bending moment valid in the interval *AC*.

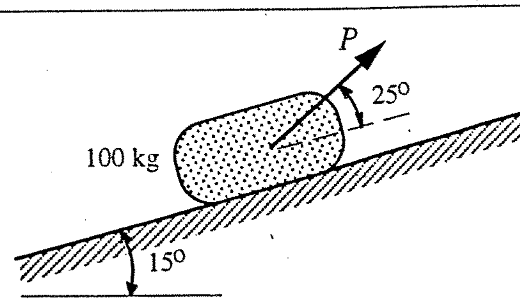


3. The force *P* is used to keep a 100-kg stone in equilibrium on a sloping track. The coefficients of friction between the stone and the track are $\mu_s = 0.40$ and $\mu_k = 0.30$. Determine the (minimum) force *P* required:

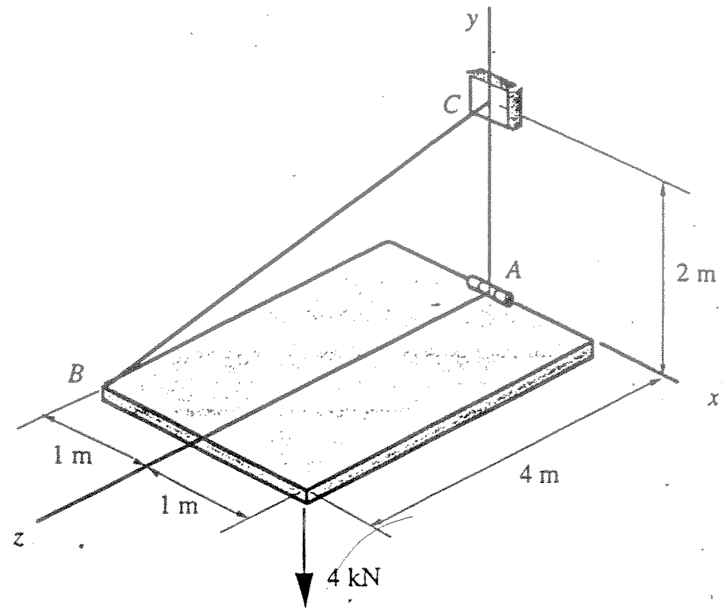
a) To cause impending motion up the slope;

b) To keep the stone in motion once an upward motion has started;

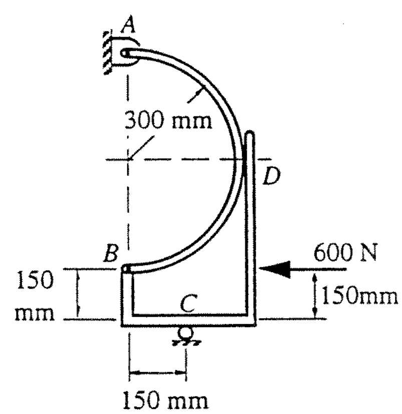
c) To prevent the stone from sliding down.



4. A 300-kg homogeneous trap door of uniform thickness is subject to a vertical force of 4 kN as shown. It is supported by a frictionless door-hinge at A and by the cable BC. Determine the reaction forces and moments provided by the hinge A.



5. The frame BCD is in contact with the smooth semi-circular member ADB at point D while the two members are connected by the pin B. Determine the forces acting on the semi-circular member (Note: neglect the weight of the members).

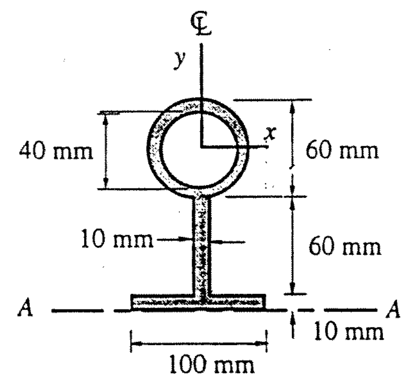


6. For the shaded area shown:

a) Locate its centroid with reference to the axes x, y.

b) Determine its moment of inertia with respect to the x-axis (Note: For a circular area $I_x = I_y = \pi r^4/4$).

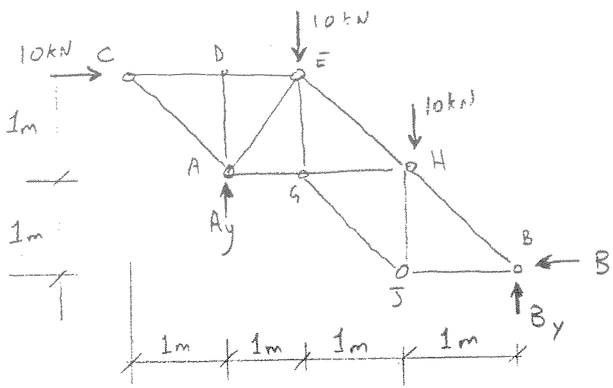
c) Determine the volume of the solid of revolution generated by a 360°-rotation of the shaded area about the axis AA.



Solution Example

F98 3

(1.) FBD : entire truss



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad B_x = 10\,kN \leftarrow$

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y = 10\,kN + 10\,kN = 20\,kN$

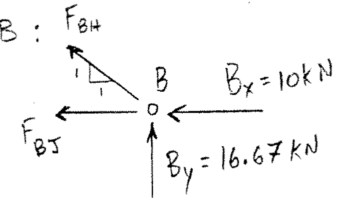
$\circlearrowleft\!+ \Sigma M_B = 0$

$10kN(1m) + 10kN(2m) - 10kN(2m) - A_y(3m) = 0$

$A_y = 3.3\overline{3}\ kN \uparrow$

$\therefore\ B_y = 16.6\overline{6}\ kN \uparrow$

a.) Joint B :



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -F_{BJ} - F_{BH}\cos 45° - 10kN = 0$

$F_{BJ} = -F_{BH}\cos 45° - 10\,kN$

$+\uparrow \Sigma F_y = 0 \qquad F_{BH}\sin 45° + 16.67kN = 0$

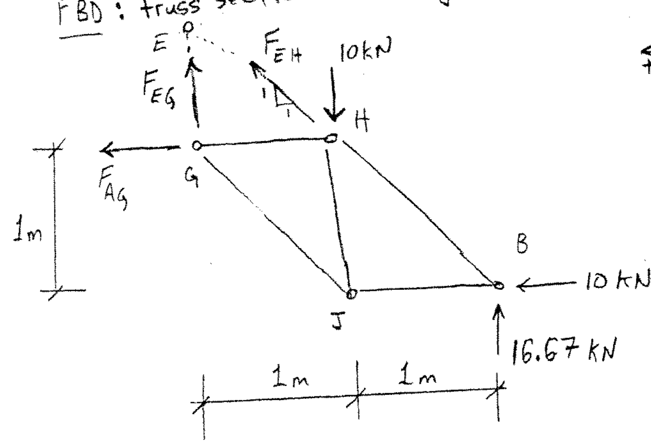
$F_{BH} = -23.57\,kN$

$\therefore\ F_{BJ} = 6.67\,kN$

Force in member BH is 23.57kN in compression.
Force in member BJ is 6.67 kN in tension.

b.) Method of sections:

FBD : truss section on the right



$\circlearrowleft\!+ \Sigma M_H = 0$

$-F_{EG}(1m) - 10kN(1m) + 16.67\,kN(1m) = 0$

$\boxed{F_{EG} = 6.67\,kN \uparrow\ T.}$

$\circlearrowleft\!+ \Sigma M_G = 0$

$-10kN(1m) - 10kN(1m) + 16.67\,kN(2m) + F_{EH}\sin 45° = 0$

$F_{EH} = -18.87\,kN$

$\boxed{\therefore\ F_{EH} = 18.87\,kN \nwarrow C.}$

c.) Zero-force members:

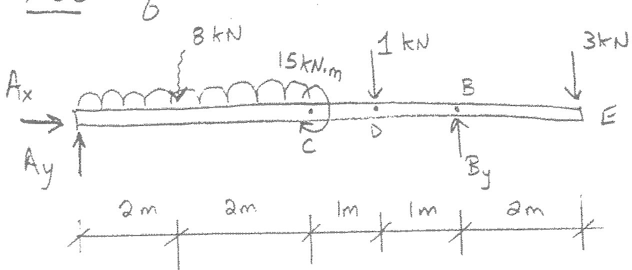
$F_{CA} = 0$ (from equilibrium of pin C.)

$F_{DA} = 0$ (from equilibrium of pin D.)

Note: Members CA and DA are zero-force members, however, they help maintain the truss form.

F98/4

② FBD: of entire beam



$\xrightarrow{+} \sum F_x = 0 \quad A_x = 0$

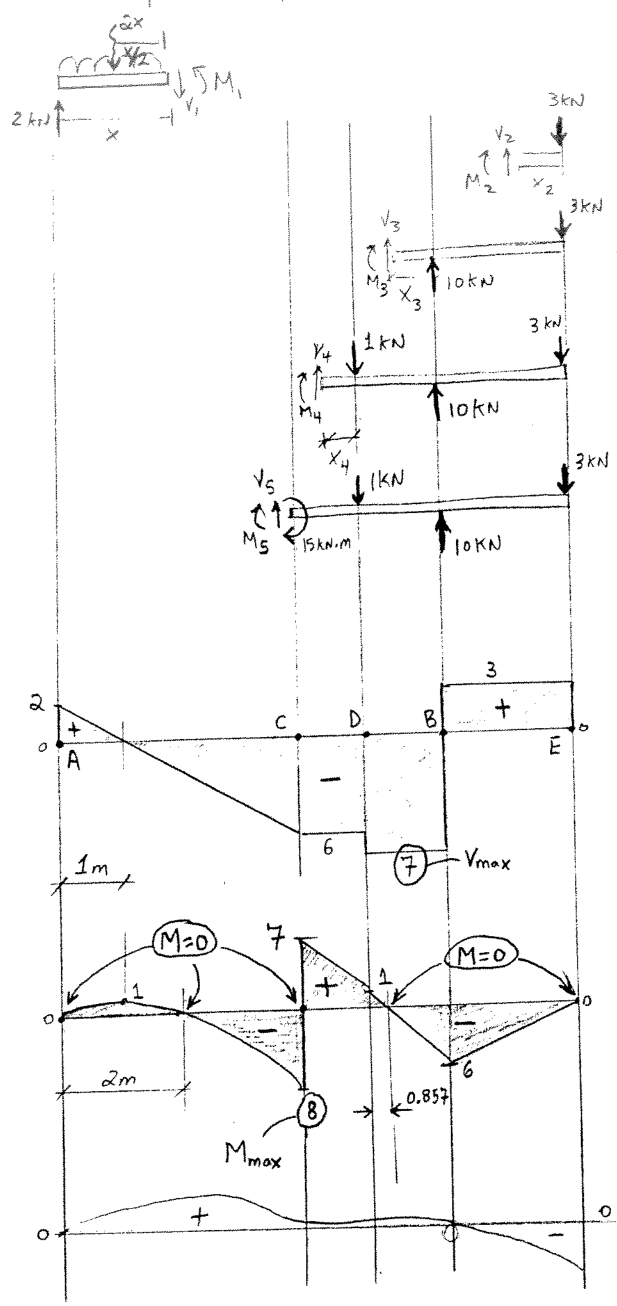
$+\uparrow \sum F_y = 0 \quad A_y + B_y = 12\text{ kN}$

$\circlearrowleft + \sum M_A = 0$

$-8(2) - 15 - 5(1) + B_y(6) - 3(8) = 0$

Reactions at supports A and B $\rightarrow$ $\boxed{B_y = 10\text{ kN}\uparrow \quad \therefore A_y = 2\text{ kN}\uparrow}$

(a)



$V_1 = -2x + 2 \quad \therefore V_1(0) = +2\text{ kN}$
$V_1(4) = -6\text{ kN}$

$M_1 = 2x - 2x \cdot \frac{x}{2} \quad M_1(0) = 0$
$= -x^2 + 2x \quad M_1(4) = -8\text{ kN·m}$

$V_2 = 3\text{ kN}$ (constant between points B & E)

$M_2 = -3x_2 \quad M_2(0) = 0$
$M_2(2) = -6\text{ kN·m}$

$V_3 + 10 - 3 = 0$
$V_3 = -7\text{ kN}$ (constant between points D & B)

$-M_3 + 10x_3 - 3(2 + x_3) = 0$
$M_3 = 7x_3 - 6 \quad \therefore M_3(0) = -6\text{ kN·m}$
$M_3(1) = 1\text{ kN·m}$

$V_4 + 10 - 1 - 3 = 0 \quad V_4 = -6\text{ kN}$ (constant between points C & D)

$-M_4 - 1x_4 + 10(x_4 + 1) - 3(x_4 + 3) = 0$
$M_4 = 6x_4 + 1 \quad \therefore M_4(0) = 1\text{ kN·m}$
$M_4(1) = 7\text{ kN·m}$

@ point C on the beam:

$V_5 = -6\text{ kN}$ (as above)

$-M_5 - 15 - (1)(1) + 10(2) - 3(4) = 0$
$M_5 = -8\text{ kN·m}$ (as above)

checks ✓

Peak Values $\boxed{V_{max} = 7\text{ kN (between points D & B)} \quad M_{max} = 8\text{ kN·m (@ point C)}}$

Zero Moment $\boxed{M = 0 \text{ @ } x = \{0, 2, 5.857, 8\}\text{ m.}}$

(b.) From the first section cut (see above):

we have

Shear:

$-V_1 - 2x + 2 = 0$

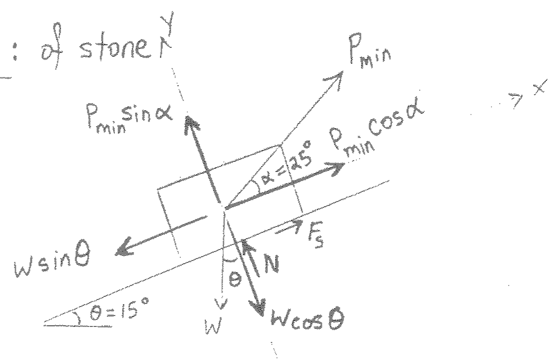
$\therefore \boxed{V_1 = -2x + 2}$ kN (shear is linear with negative slope and y-intercept equal to 2kN)

Bending Moment:

$M_1 + 2x \cdot \frac{x}{2} - 2x = 0$

$\therefore \boxed{M_1 = -x^2 + 2x}$ kN·m (Bending Moment is a parabolic relationship concave down)

(3) FBD: of stone



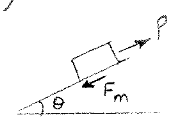
Given: $\mu_s = 0.4$
$\mu_K = 0.3$
$m_{stone} = 100\,kg$

Find: P_{min} for a) impending motion
b.) motion
c.) no motion

F98/5

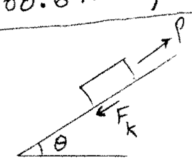
Soln: $W = mg = 100kg\left(9.81\frac{m}{s^2}\right) = 981N \downarrow$; $F_s = \mu_s N = \mu_s\left(W\cos\theta - P_{min}\sin\alpha\right)$

a) Impending motion $(P_x = F_m)$



$+\nearrow \sum F_x = 0 \quad P_{min}\cos\alpha - W\sin\theta - \mu_s\left(W\cos\theta - P_{min}\sin\alpha\right) = 0$

$$\therefore P_{min} = \frac{W\sin\theta + \mu_s W\cos\theta}{(\cos\alpha + \mu_s\sin\alpha)} = \boxed{588.6\,N \; \angle 40°}$$



b.) Motion $(P_x > F_m)$ ∴ replace μ_s with μ_K

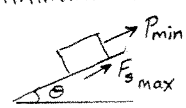
no motion or 'static'

motion or 'Kinetic', or 'Dynamic'

$+\nearrow \sum F_x = 0 \quad P_{min}\cos\alpha = W\sin\theta + \mu_K\left(W\cos\theta - P_{min}\sin\alpha\right)$

$$\therefore P_{min} = \frac{W\sin\theta + \mu_K W\cos\theta}{(\cos\alpha + \mu_K\sin\alpha)} = \boxed{520.9\,N \; \angle 40°}$$

c.) No motion $(P_x < F_m)$ use μ_s where P is minimum when F_s is maximum



$+\nearrow \sum F_x = 0 \quad P_{min}\cos\alpha + \mu_s\left(W\cos\theta - P_{min}\sin\alpha\right) - W\sin\theta = 0$

$$\therefore P_{min} = \frac{W\sin\theta - \mu_s W\cos\theta}{(\cos\alpha - \mu_s\sin\alpha)} = -169.7\,N$$

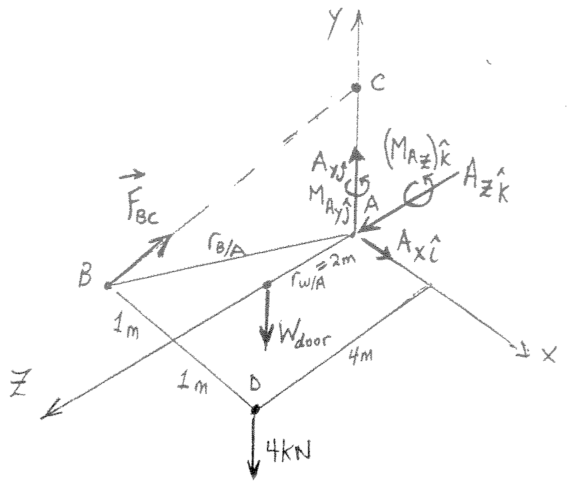
The negative sign indicates that no force is required to prevent the stone from sliding down.

$\therefore \boxed{P_{min} = 0}$ stone will remain stationary due to its own weight.

F98/6

(4) Soln: FBD: of entire system

Given: trap door system
Find: reaction forces & moments at hinge A



Note: The frictionless door hinge at 'A' produces no moment resistance about x-axis

$\therefore (\vec{M}_{Ax})_{\hat{i}} = 0$

$$\vec{P}_D = -(4kN)\hat{j}$$

$$\vec{W}_{door} = mg = 300\,kg\left(9.81\,\frac{m}{s^2}\right) = 2.943\,kN \quad \text{OR} \quad \vec{W}_{door} = -(2.943\,kN)\hat{j}$$

$$\overline{BC} = \left[(1)^2_{\hat{i}} + (2)^2_{\hat{j}} + (-4)^2_{\hat{k}}\right]^{1/2} = 4.5826\,m$$

$$\vec{F}_{BC} = F_{BC}\vec{\lambda}_{BC} = F_{BC}\frac{\overrightarrow{BC}}{BC} = \frac{F_{BC}}{4.5826}\left[\hat{i} + 2\hat{j} - 4\hat{k}\right]$$

$$\vec{r}_{B/A} = -\hat{i} + 4\hat{k}$$

$$\vec{r}_{W/A} = 2\hat{k}$$

$$\vec{r}_{D/A} = \hat{i} + 4\hat{k}$$

$$\Sigma\vec{M} = \Sigma(\vec{r}\times\vec{F}) = 0$$

$$\therefore \left(\vec{r}_{B/A}\times\vec{F}_{BC}\right) + \left(\vec{r}_{W/A}\times\vec{W}_{door}\right) + \left(\vec{r}_{D/A}\times\vec{P}_D\right) + (\vec{M}_{Ay})_{\hat{j}} + (\vec{M}_{Az})_{\hat{k}} = 0$$

$$\frac{F_{BC}}{4.5826}\left[(0-8)_{\hat{i}} - \left(\cancelto{0}{(-1)(-4) - 4(1)}\right)_{\hat{j}} + \left((-1)(2)-0\right)_{\hat{k}}\right] + \left[\left(0-(2)(-2.943)\right)_{\hat{i}} + 0\hat{j} + 0\hat{k}\right] + \left[\left(0-(4)(-4)\right)_{\hat{i}} - 0\hat{j} + \left((-4)(1)\right)_{\hat{k}}\right] + (\vec{M}_{Ay})_{\hat{j}} + (\vec{M}_{Az})_{\hat{k}} = 0$$

coeff. i $\quad \frac{F_{BC}}{4.5826}(-8) + 5.886 + 16 = 0 \quad \boxed{\therefore F_{BC} = 12.54\,kN}$

coeff. j $\quad \cancelto{0}{\frac{F_{BC}}{4.5826}(0)} + 5.886 + (\vec{M}_{Ay})_{\hat{j}} = 0 \quad \therefore \boxed{\vec{M}_{Ay} = -(5.886\,kN\cdot m)\hat{j}}$

coeff. k $\quad \frac{\cancelto{12.54}{F_{BC}}}{4.5826}(-2) - 4 + (\vec{M}_{Az})_{\hat{k}} = 0 \quad \therefore \boxed{\vec{M}_{Az} = (9.47\,kN\cdot m)\hat{k}}$ and $\boxed{(\vec{M}_{Ax})_{\hat{i}} = 0}$

$$\therefore \vec{M}_A = -(5.886\,kN\cdot m)\hat{j} + (9.47\,kN\cdot m)\hat{k}$$

$$\Sigma F = 0 \quad \vec{A} + \vec{F}_{BC} - 2.943\,kN\hat{j} - 4\,kN\hat{j} = 0$$

coeff. i $\quad A_x + \frac{12.54(1)}{4.5826} = 0$

coeff. $\hat{j}$ $\quad A_y + \frac{12.54(2)}{4.5826} - 2.943 - 4 = 0$

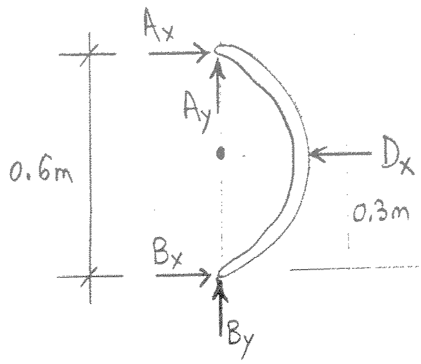
coeff. $\hat{k}$ $\quad A_z + \frac{12.54(-4)}{4.5826} = 0$

$$\boxed{\begin{aligned} A_x &= -2.74\,kN \\ A_y &= 1.47\,kN \\ A_z &= 10.95\,kN \end{aligned}}$$

(5) <u>FBD</u>: semi-circular member ADB

F98/7

<u>Find</u>: forces on semi-circular member ADB



$\overset{+}{\rightarrow}\sum F_x = 0 \quad A_x + B_x = D_x$

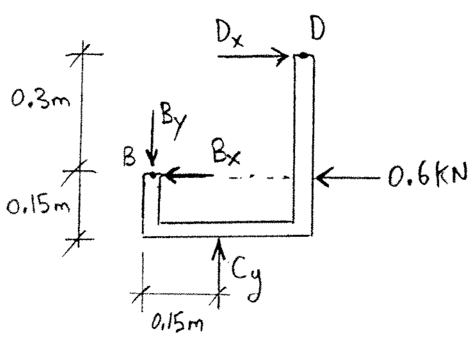
$+\uparrow\sum F_y = 0 \quad A_y + B_y = 0$

$\circlearrowleft\!+ \sum M_B = 0 \quad 0.3D_x - 0.6A_x = 0$

$D_x = 2A_x$

$\therefore B_x = A_x$

<u>FBD</u>: frame BCD



$\overset{+}{\rightarrow}\sum F_x = 0 \quad D_x = B_x + 0.6 \quad \Rightarrow 2A_x = B_x + 0.6$

$+\uparrow\sum F_y = 0 \quad C_y = B_y$

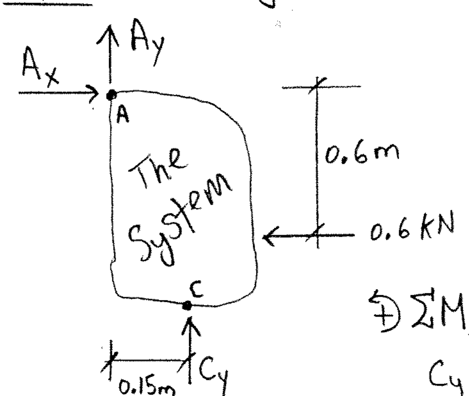
$\circlearrowleft\!+ \sum M_B = 0 \quad 0.15C_y - 0.3D_x = 0$

$D_x = \frac{1}{2}C_y$

$B_x = 0.6\text{kN}\rightarrow$
$A_x = 0.6\text{kN}$
$D_x = 1.2\text{kN}$
$C_y = 2.4\text{kN}$
$B_y = 2.4\text{kN}$
$A_y = -2.4\text{kN}$

OR $\boxed{A_y = 2.4\text{kN}\downarrow}$

<u>Check</u>: <u>FBD</u>: entire system



$\circlearrowleft\!+ \sum M_A = 0$

$C_y(0.15\text{m}) - 0.6\text{kN}(0.6\text{m}) = 0$

$C_y = 2.4\text{kN}\uparrow$

$+\uparrow\sum F_y = 0 \quad C_y + A_y = 0$

$A_y = -C_y$

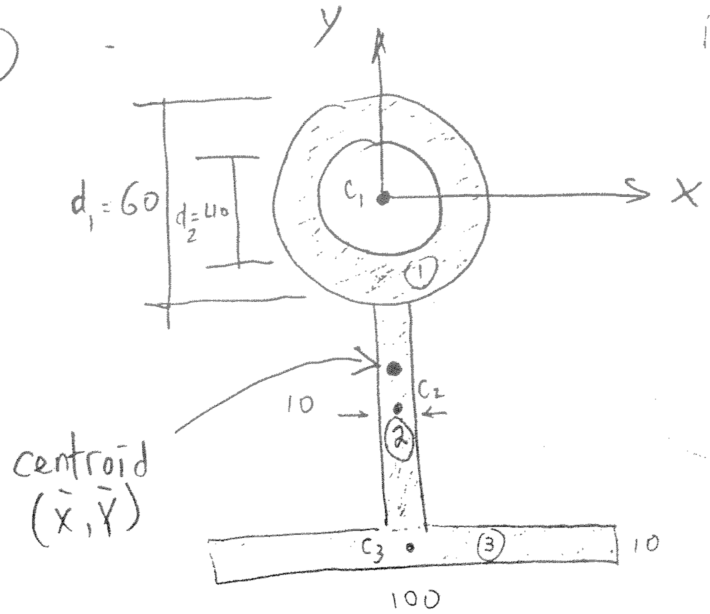
$\therefore A_y = -2.4\text{kN}$

OR $A_y = 2.4\text{kN}\downarrow$ ✓

$\therefore$ Forces acting on semi-circular arc are:

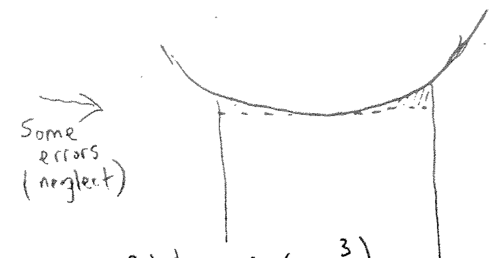
$$\boxed{\begin{aligned} A_x = B_x &= 0.6\text{kN}\rightarrow \\ A_y &= 2.4\text{kN}\downarrow \\ B_y &= 2.4\text{kN}\uparrow \\ D_x &= 1.2\text{kN}\leftarrow \end{aligned}}$$

(6)



Given: Area Shown
Find: a) Centroid
b.) $\bar{\bar{I}}_x$
c.) V_{A-A}

F98/8 ?



	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
(1)	$\pi r_1^2 - \pi r_2^2 = 1570.8$	0	0	0	0
(2)	$10 \times 60 = 600$	0	$-(30+30) = -60$	0	-36×10^3
(3)	$10 \times 100 = 1000$	0	$-(30+60+5) = -95$	0	-95×10^3

$\Sigma' A = 3170.8\ mm^2$ $\qquad \Sigma \bar{x} A = 0 \qquad \Sigma' \bar{y} A = -131 \times 10^3\ mm^3$

first moments of the area

$$Q_y = \overset{?}{\bar{X}} \Sigma A = \Sigma' \bar{x} A \Rightarrow \bar{X} = \frac{0}{\Sigma' A} = 0$$

$$Q_x = \bar{Y} \Sigma A = \Sigma \bar{y} A \Rightarrow \bar{Y} = \frac{-131 \times 10^3\ mm^3}{3170.8\ mm^2} = -41.3\ mm$$

$\therefore$ w/r/t x & y axii, centroid is at $(0, -41.3\ mm)$

b.)

$\boxed{\text{Recall: } I_{rectangle} = \frac{1}{12} b h^3 \quad I_{circle} = \frac{1}{4} \pi r^4}$

Parallel Axis Theorem

$$\bar{\bar{I}} = I + A d^2$$

$$\therefore I_{x_1} = \frac{\pi}{4}(r_1^4 - r_2^4) + 1570.8(41.3)^2 = 3.19 \times 10^6\ mm^4$$

$$I_{x_2} = \frac{1}{12}(10)(60)^3 + 600(60 - 41.3)^2 = 0.39 \times 10^6\ mm^4$$

$$I_{x_3} = \frac{1}{12}(100)(10)^3 + 1000(100 - 41.3 - 5)^2 = 2.89 \times 10^6\ mm^4$$

$$\Sigma' = \bar{\bar{I}}_x = 6.47 \times 10^6\ mm^4$$

c.) By Pappus-Guldinus Theorem (pg 226)

$$V = 2\pi \bar{y} A = 2\pi (100 - 41.3)(3170.8) = 1.17 \times 10^6\ mm^3$$