The following are solutions to some Old Final Exam questions based on the Fundamental Counting Principle, permutations and combinations.

Summer 2014 - Final Exam.

Q:#9. A group of 100 people is divided into 2 teams with 45 people in team A and 55 people in team B.

a) If 20 people are mathematicians and 80 are non-mathematicians, in how many ways can the teams be formed if all mathematicians are in team A?

b) In how many ways can the teams be formed if none of the mathematicians are in team A?

Solution: ⓐ

	Team A	Team B	Total
M: Math.	20		20
NM: Non-Math	25	55	80
Total	45		100

Team A can be chosen in two steps.

Step 1. Choose all 20 mathematicians

Step 2. then choose 25 non-mathematicians from the group of 80 non mathematicians.

Step 1 can be done in $C_{20,20}$ (ways).

Step 2 can be done in $C_{80,25}$ ways.

[Note that once the team A is formed remaing 55 will belong to team B.]. Thus by the FCP the team A can be chosen in $C_{20,20} \times C_{80,25} = 1 \times \frac{80!}{55!\,25!}$

Similarly the team B can also be selected in

$C_{20,1} \times C_{80,55} = 1 \times \frac{80!}{25!\,55!}$ ways. This is same as the number of ways the team A can be formed.

#9 (b): In this case the team A is formed selecting 45 non-mathematicians from the group of 80 non-mathematicians. As in part 9 (a) above, the team A' can be chosen in $C_{80,45} \times C_{20,0} = \frac{80!}{35!\,45!}$ ways. The remaining 55 will belong to the team B. Also, if we decide to choose the team B. Then this can be done in $C_{20,20} \times C_{80,35} = \frac{80!}{45!\,35!}$ ways. Which is same as the number of ways the team A is formed.

Q #10 A research company does a survey involving 18 manufacturers of motor vehicles. The result of survey will be a ranking of the manufacturers from most popular to least popular.

(a) If no companies are equally ranked, what is the total number of possible outcomes of the survey?

Solution: Here arrangement or order in which the manufacturers are ranked matters. So this is a permutation problem of arranging 18 manufacturers choosing all of them at a time. So this can be done in $P_{18,18} = 18!$ ways.

(b). If one of the company is the Acme another is Low Quality, how many possible outcomes of the survey have if Acme is ranked as most popular and Low Quality as the least popular. (No company is equaly ranked)

Solution: Two positions 1^{st} and the 18^{th} positions are taken by Acme and Low Quality. The remaining 16 positions are to be arranged. This can be done in $1 \times P_{16,16} \times 1 = 16!$ ways.

CONCORDIA UNIVERSITY

DEPARTMENT OF MATHEMATICS & STATISTICS

MIDTERM TEST Math-208/1AC

INSTRUCTOR: U. Tiwari Date: May 23, 2013 Time: 75 minutes

Total: 50 marks, 10 marks each.

1. Joanne Ha sells silk-screened –shirts at a community festivals and craft fairs. Her marginal cost to produce one T- shirt is $3.50. Her total cost to produce 60 T– shirts is $300, and sells for $8.50 each.

   (A) Find the linear cost production for Joanne's T-shirt production.
   (B) How many T-shirts must she produce and sell in order to break even?
   (C) How many T-shirts must she produce and sell to make a profit of $500

2. Solve for x in the following equations:

   (A) $\left(\frac{27}{8}\right)^{-x^2} = \left(\frac{2}{3}\right)^{12}$
   (B) $\log(x^2 + x - 2) = 1$
   (C) $\log_5(x+6) + \log_5(x+2) = 1$
   (D) $\log_x\left(\frac{64}{27}\right) = -3$

3. For f(x) = -12x +4 and g(x) = $3(0.4)^x$, find the following:

$$\sum_{h=0}^{h=49} f(h) = f(0) + f(1) + f(2) + \cdots + f(49)$$

$$\sum_{h=0}^{h=19} g(h) = g(0) + g(1) + g(2) + \cdots + g(19)$$

4. Joe Barker bought a rare stamp for his collection. He agreed to pay a lump sum of $2, 500 after 4 years. Until then, he pays 6% simple interest semi-annually.
   (A) Find the amount of each semi-annual payment.
   (B) Joe sets up a sinking fund so that enough money will be present to pay off the $2, 500. He wants to make annual payments into the fund. The account pays 8% compounded annually. Find the amount of each payment.
   (C) Prepare a table showing the amount in the sinking fund after each deposit.

5. The Beys plan to purchase a home for $240, 000. They will pay 20% down and finance the remainder for 30 years at 7.2% interest, compounded monthly.
   (A) How large are their monthly payments?
   (B) What will be their approximate loan balance right after they have made their 96Th payment?
   (C) How much interest will they pay during the 8th year of loan.

May 23, 2013

Math 208/~~1~~ Midterm Test. (Solution)

1 Note that if cost function is linear then marginal cost is same as variable cost.
Here variable cost $= 3.5$
$\therefore$ Cost function $C(x) = 3.5x + b$
where x is number of T-shirts.
Now revenue function is $R(x) = 8.5x$
Profit function $P(x) = R(x) - C(x)$.
Now, to find b in $C(x) = 3.5x + b$.
For $x = 60$, $C(60) = \$300$. Thus
$300 = 3.5 \times 60 + b \Rightarrow b = 90$

(A) Hence $C(x) = 3.5x + 90$

(B) $P(x) = R(x) - C(x) = 5x - 90$
For break-even $P(x) = 0$
$5x - 90 = 0 \Rightarrow x = 18$ shirt for break even.

(C) Now $P(x) = 5x - 90$ for $P(x) = 500$
$500 = 5x - 90 \Rightarrow x = \frac{590}{5} = 118.$

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2 (a) $\left(\frac{27}{8}\right)^{-x^2} = \left(\frac{2}{3}\right)^{12} \Rightarrow \left(\frac{8}{27}\right)^{x^2} = \left(\frac{2}{3}\right)^{12} \Rightarrow$
$\left[\left(\frac{2}{3}\right)^3\right]^{x^2} = \left(\frac{2}{3}\right)^{12} \Rightarrow \left[\frac{2}{3}\right]^{3x^2} = \left[\frac{2}{3}\right]^{12}$
$\therefore 3x^2 = 12 \Rightarrow x = \pm 2$

(b) $\log_{10}(x^2 + x - 2) = 1 \Rightarrow x^2 + x - 2 = 10^1$
or $x^2 + x - 12 = 0 \Rightarrow (x+4)(x-3) = 0$
$x = -4, 3$

(c) $\log_5(x+6) + \log_5(x+2) = 1$
$\log_5 (x+6)(x+2) = 1$
$(x+6)(x+2) = 5$
$x^2 + 8x + 12 = 5 \Rightarrow x^2 + 8x + 7 = 0$
or $(x+7)(x+1) = 0$, $x = -1, \cancel{-7}$
$x = -7$ not valid. Hence $x = -1$

(d) $\log_x \frac{64}{27} = -3 \Rightarrow \frac{64}{27} = x^{-3}$
or $x^3 = \frac{27}{64} \Rightarrow x^3 = \left(\frac{3}{4}\right)^3 \Rightarrow x = \frac{3}{4}$

3 (a) $\sum_{h=0}^{49} (-12h + 4) = 4 - 8 - 20 - \cdots - 584$
$= \frac{n}{2}[a_1 + a_{50}]$
$= \frac{50}{2}[4 - 584]$
$= -14500$

(b) $\sum_{h=0}^{h=19} 3(0.4)^h = 3 + 3 \times (0.4) + 3(0.4)^2 + \cdots$
$a = 1, r = 0.4 \Rightarrow S'_{20} = \frac{a(r^{20} - 1)}{r} = \boxed{\frac{3[1 - 0.00000001]}{1 - 0.4}}$
$= 4.99999945 \approx 5$

4 (A) Interest on 2500 at 6%, each month
$I = Pr = 2500 \times 0.06 = \$150$
Semi-annual interest $= \$150$/every 2 months.

(B) The sinking fund must have \$2500 at the 4 years in the account. This includes interest on all deposits. Thus
$FV = PMT \frac{(1+i)^n - 1}{i}$
Here $FV = \$2500$, $i = \frac{0.08}{1}$ 0.08
$\therefore PMT = \frac{FV \times i}{(1+i)^n - 1}$; Here $FV = \$2500$, $i = 0.08$, $m = 1$, $n = 1 \times 4 = 4$
$PMT = \frac{2500 \times 0.08}{(1.08)^4 - 1}$
$PMT = \$554.80$ per year

(B) Table

End year	Total amount
1	\$554.862011
2	599.19 + 554.80
3	1246.30 + 554.80
4	1945.19 + 554.80
	\$2499.99 ≈ 2500

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5 $24{,}000 \times 0.20 = \$48{,}000$ Down
Loan $= \$24000 - 48000 = \$192{,}000$
This is the present value of loan, so
$PV = PMT\left[\frac{1 - (1+i)^{-n}}{i}\right] \Rightarrow PMT = \frac{PV \times i}{1 - (1+i)^{-n}}$

(A) Here $i = \frac{0.072}{12} = 0.006$, $n = 12 \times 30 = 360$
$PMT = \frac{192000 \times 0.006}{1 - (1.006)^{-360}} = \$1303.27$/month

(B) Unpaid balance after 96th payment
$PV = PMT\left(\frac{1 - (1+i)^{-(360-96)}}{i}\right)$
$1303.27336\left[\frac{1 - (1.006)^{-264}}{0.006}\right]$
$= \$172439.11$

(C) Present value at the end of 7 years or after 84th payment,
$PV = 1303.27336\left[\frac{1 - (1.006)^{-276}}{0.006}\right]$
$= \$175540.49$
Loan payment in year 8 $= \$175540.49$
$175540.49 - 172439.11 = \$3101.38$
Total interest paid in year 8th
$= 1303.27 \times 12 - 3101.38$
$= \$12537.86$

55

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	208/1	All
Examination	**Date**	**Pages**
Final	June 2008	3
Instructors		**Course Examiner**
A. Atoyan, J. Ruddy, U. Tiwari		D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** The marketing research department for a company that manufactures and sells notebook computers established the following price-demand, revenue and cost functions:

$$p(x) = 2,000 - 60x$$
$$R(x) = x(2,000 - 60x)$$
$$C(x) = 4,000 + 500x$$

where $p(x)$ is the wholesale price in dollars at which x thousand computers can be sold, and $C(x)$ and $R(x)$ are in thousands of dollars. Both functions have domain $1 \leq x \leq 25$.

(A) Find the output that will produce the maximum revenue. What is the maximum revenue to the nearest thousand dollars? $X = 16,167$, $R = \$16,666667$

(B) What is the wholesale price per computer (to the nearest dollar) that produces the maximum revenue? $p = \$1000$

(C) For what outputs will a loss occur? Will a profit occur?
For Profit $2.696 < x < 24.7303$. Thousan

[10] **2.** Solve for x in the following equations:

(A) $3^{3x-x^2} = \dfrac{1}{81}$ $\quad x = 4, -1$

(B) $(81)^{2x} = (9)^{x^2-12}$ $\quad x = -2, 6$

(C) $\log_b x = 3\log_b 2 + 0.5\log_b 25 - \log_b 20$ $\quad x = 2$

(D) $\log_3 x + \log_3(x-3) = \log_3 10$ $\quad x = 5$

(E) $\log_{10}(x+6) - \log_{10}(x-3) = 1$ $\quad x = 4$

[10] 3. For $f(x) = 18 - 3x$ and $g(x) = 4^{x-4}$ find the following:

(A) $\sum_{k=0}^{24} f(k) = f(0) + f(1) + f(2) + \cdots + f(24)$. $= -450$ Ans

(B) $\sum_{h=0}^{19} g(h) = g(0) + g(1) + g(2) + \cdots + g(19)$. $= \frac{4^{16} - 4^{-4}}{3}$

[10] 4. A company estimates that it will have to replace a piece of equipment at a cost of $800,000 in 5 years. To have this money available in 5 years, a sinking fund is established by making equal monthly payments into an account paying 6.6% compounded monthly.

(A) How much should each payment be? $11290.42

(B) How much interest is earned during the last year? $46238.08

[10] 5. A student receives a student loan for $8,000 at 5.5% interest compounded monthly to help her finish the last 1.5 years of college. Starting 1 year after finishing college, the student must amortize the loan in the next 5 years by making equal monthly payments.

(A) What will the payments be? $165.92

(B) What total interest will the student pay? $3428.62

[10] 6. Solve by using Gauss-Jordan Elimination:

Ans $x_1 = 1, x_2 = -1, x_3 = 0$

$$3x_1 + x_2 - 2x_3 = 2$$

$$2x_1 - 4x_2 + 2x_3 = 6$$

$$2x_1 - x_2 - 3x_3 = 3$$

<u>No other method of solving these systems of equations will be accepted!</u>

[10] 7. An economy is based on three sectors, agriculture, energy, and manufacturing. Production of a dollar's worth of agriculture requires an input of \$0.20 from the agriculture sector and \$0.40 from the energy sector. Production of a dollar's worth of energy requires an input of \$0.20 from the energy sector and \$0.40 from the manufacturing sector. Production of a dollar's worth of manufacturing requires an input of \$0.10 from the agriculture sector, \$0.10 from the energy sector, and \$0.30 from the manufacturing sector.

(A) Write the technological matrix M for this economy. $M = \frac{1}{10}\begin{bmatrix} 2 & 0 & 1 \\ 4 & 2 & 1 \\ 0 & 4 & 3 \end{bmatrix}$

(B) If a final demand of \$40 billion for agriculture, \$20 billion for energy, and \$60 billion for manufacturing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

Ag. $x = \$72$ Bill, Energy $y = \$248$ Bill
Manufacturing $z = 128$ Billion

[10] 8. Extremize $P(x, y) = 30x + 10y$ subject to

$$2x + 2y \geq 4,\ 6x + 4y \leq 36,\ 2x + y \leq 10,\ x \geq 0,\ y \geq 0.$$

Max $P = 120$ at $x = 2, y = 6$
Min $P = 10$ at $x = 0, y = 1$

[10] 9. A software development department consists of 6 women and 4 men.

(A) How many ways can they select a chief programmer, a backup programmer, and a programming librarian? $P_{10,3}$

(B) How many ways can they select a team of 3 programmers to work on a particular project? $C_{10,3} = 120$

(C) If the positions in part(A) are selected by lottery, what is the probability that women are selected for all 3 positions? $\frac{1}{6}$

[10] 10. Ann and Barbara are playing a tennis match. The first player to win 2 sets wins the match. For any given set, the probability that Ann wins that set is $\frac{2}{3}$. Find the probability that

(A) Ann wins the match. $\frac{20}{27}$

(B) 3 sets are played. $\frac{4}{9}$

(C) The player who wins the first set goes on to win the match. $\frac{16}{27}$

FINAL EXAM: JUNE 208 (Soln)

1 $R(x) = -60x^2 + 2000x$ [Parabola]

Vertex V: $x = -\frac{b}{2a} = \frac{50}{3}$, $y = \frac{4ac-b^2}{4a} =$

$= 16,666.667$

(a) $x = 16.667$ thousand, Rev $= \$16,666667$

(b) Price per computer $p = \$1000$

(c) $C(x) = R(x)$: $6x^2 - 1500x - 4000 = 0$

Solve $3x^2 - 750x - 2000 = 0$

$x = 2.696$ thousand or less

or 247.303 thousand or more

2 (a) $3^{3x-x^2} = 3^{-4} \Rightarrow -4 = 3x - x^2$,

$x^2 - 3x + 4 = 0$

$x = 4, -1$ ANS.

(b) $(9)^{2(2x)} = 9^{x^2-12} \Rightarrow x^2 - 12 = 4x$

$x = -2, 6$

(c) $\log_b x = \log_b \frac{8.5}{20} \Rightarrow x = 2$

(d) $\log_3 x(x-3) = \log_3 10 \Rightarrow x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$ $\cancel{x = -2}$ reject 5 ANS

(e) $\log \frac{x+6}{x-3} = \log_{10} 10 \Rightarrow \frac{x+6}{x-3} = 10$

$\boxed{x = 4 \quad \text{Ans}}$

3 (A) A. Series: $n = 25$, $a_1 = 18$, $a_{25} = -54$

$S_{25} = \frac{25}{2}[a_1 + a_{25}] = \frac{25}{2}[-36] = -450$

B: G. Series, $n = 20$, $a_1 = 4^{-4}$, $r = 4$

$S_{20} = \frac{a(r^n - 1)}{r-1} = \frac{\frac{1}{4^4}[4^{20} - 1]}{4-1}$

$= \frac{4^{16} - \frac{1}{4^4}}{3} = 1431655765$

4 (A) $PMT = \frac{PV(i)}{(1+i)^n - 1} = \frac{800000 \times .0055}{(1.0055)^{60} - 1}$

$= \$11290.42.$

(B) Total interest in year 5

$= \$(800,000 -$ PV at the end of 4 yrs $- 12 \times 11,290.42)$

$= 800000 - \frac{11290.42[1.0055^{60} - 1]}{0.0055}$

$- 135485.01$

$= \$46238.$

5 Principal + Interest after 1.5 yr.

$A = 8000(1 + 0.0045833)^{18}$

$= \$8686.35$ Loan + Int = PV

$PV = PMT\left[\frac{1 - (1+i)^{-n}}{i}\right]$

(A) $PMT = \frac{PV(i)}{1 - (1+i)^{-n}} = \frac{8686.35 \times 0.0045}{1 - (1.004583)^{-60}}$

$= \$165.92$ per month.

(B) Total Interest on loan.

$= FV - 8000$

$= 165.92\left[\frac{(1.004583)^{60} - 1}{0.0045833}\right] - 8000$

$= 11428.62 - 8000$

$= \$3428.62.$

6 Augmented matrix of the system

[Divide eqn 2 by 2 and switch with 1]

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 3 & 1 & -2 & 2 \\ 2 & -1 & -3 & 3 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 7 & -5 & -7 \\ 0 & 3 & -5 & -3 \end{array}\right] \xrightarrow{R_2 - 2R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 5 & -1 \\ 0 & 3 & -5 & -3 \end{array}\right] \xrightarrow[R_3 - 3R_2]{R_1 + 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 11 & 1 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & -20 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{20}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 11 & 1 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_2 - 5R_2]{R_1 - 11R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Ans $x_1 = 1$, $x_2 = -1$, $x_3 = 0$

7 (A)

$$\begin{array}{c} \\ A \\ E \\ M \end{array} \begin{array}{c} \begin{array}{ccc} A & E & M \end{array} \\ \begin{bmatrix} 0.20 & 0 & 0.10 \\ 0.40 & 0.20 & 0.10 \\ 0 & 0.40 & 0.30 \end{bmatrix} \end{array} \Rightarrow M = \frac{1}{10}\begin{bmatrix} 2 & 0 & 1 \\ 4 & 2 & 1 \\ 0 & 4 & 3 \end{bmatrix}$$

(B) Let x, y, and z be the total Production of Agri, Energy, and Manufacturing respectively. Thus

$x = 0.2x + 0.1z + 40$ Bill.

$y = 0.4x + 0.2y + 0.1z + 20$

$z = \quad +0.4y + 0.3z + 60$

⑦ $\underline{\underline{C}}$ Solve the system in (B) above using Gauss-Jordan Elimination method. Rewrite System in B above

$$\left.\begin{array}{l} x - 0.2x - 0.1z = 40 \\ -0.4x + y - 0.2y - 0.1z = 20 \\ -0.4y + z - 0.3z = 60 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 0.8x + 0y - 0.1z = 40 \\ -0.4x + 0.8y - 0.1z = 20 \\ 0x - 0.4y + 0.7z = 60 \end{array}\right\} \text{Solve This}$$

Consider the augmented matrix of the system an reduce it to RREF.

$$\left[\begin{array}{ccc|c} 8 & 0 & -1 & 400 \\ -4 & 8 & -1 & 200 \\ 0 & -4 & 7 & 600 \end{array}\right] \xrightarrow{R_1+2R_2} \left[\begin{array}{ccc|c} 0 & 16 & -3 & 800 \\ -4 & 8 & -1 & 200 \\ 0 & -4 & 7 & 600 \end{array}\right] \xrightarrow[R_2+2R_2]{R_1+4R_3} \left[\begin{array}{ccc|c} 0 & 0 & 25 & 3200 \\ -4 & 0 & +13 & 1400 \\ 0 & -4 & 7 & 600 \end{array}\right]$$

$$\xrightarrow{\frac{1}{25}R_1,\ -\frac{1}{4}R_2,\ -\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 128 \\ 1 & 0 & -\frac{13}{4} & 56 \\ 0 & 1 & -\frac{7}{4} & 24 \end{array}\right] \xrightarrow[R_3+\frac{7}{4}R_1]{R_2+\frac{13}{4}R_1} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 128 \\ 1 & 0 & 0 & 472 \\ 0 & 1 & 0 & 248 \end{array}\right] \Rightarrow \begin{array}{l} x = 472 \text{ Bill} \\ y = 248 \text{ Bil} \\ z = 128 \text{ Bill} \end{array}$$

Ans.

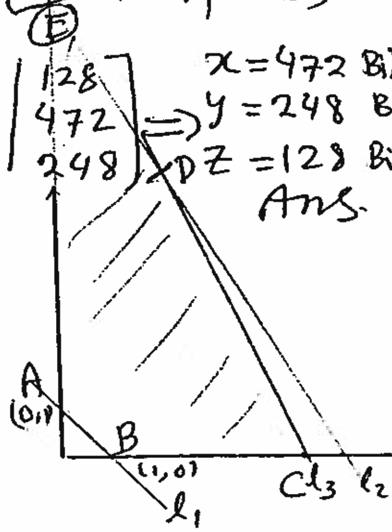
$\underline{\underline{8}}$ Intersection of l_2: $3x+2y=18$, l_3: $2x+y=10$ $\Rightarrow$ $x=2$, $y=6$

Evaluate $P = 30x + 10y$. At A, B, C, D & E

P is min at $x=0, y=1$ Min $P=10$

P is max at $x=2, y=6$ Max $P=120$

D (2,6)
E (0,9)
C (5,0)
A (0,1)
B (1,0)

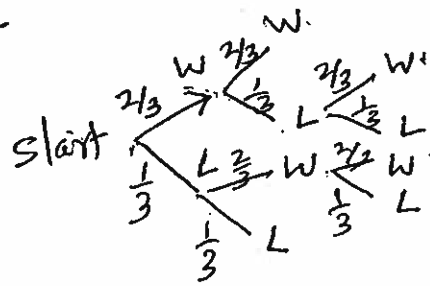


$\underline{\underline{9}}$ (A) $P_{10,3} = 10 \times 9 \times 8 = 720$

(B) $C_{10,3} = \dfrac{10!}{7!\,3!} = 120$

(C) $Pro(3W) = \dfrac{P_{6,3}}{P_{10,3}} = \dfrac{C_{6,3}}{C_{10,3}} = \dfrac{20}{120} = \dfrac{1}{6}$

$\underline{\underline{10}}$



(A) $P(\text{Ann wins}) = \frac{2}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3} + \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$

$= \dfrac{12+4+4}{27} = \dfrac{20}{27}$.

(B) $P(\text{3 sets are played}) = \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}$.

$\frac{1}{3}\times\frac{2}{3}*\frac{2}{3} + \frac{1}{3}\times\frac{2}{3}\times\frac{1}{3} = \frac{4}{9}$

or $= \frac{2}{3}\times\frac{1}{3} + \frac{1}{3}\times\frac{2}{3} = \frac{4}{9}$.

(C) $P(WW + WLW) = \frac{2}{3}*\frac{2}{3} + \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3} = \frac{16}{27}$.

[Solution : JUNE 208 FINAL EXAM]

6394930

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**		**Section(s)**
Mathematics	208/4		All
Examination	**Date**	**Time**	**Pages**
Midterm	February 2013	1 Hour 30 minutes	2

Instructors	**Course Examiner**
F. Romanelli, K. Benchekroun, L. Dube, U. Tiwari, V. Chandee, V. Kalvin	D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[4+3+3] **1.** Given the quadratic function $f(x) = 1.2 - 0.96x - 0.12x^2$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form of f.

(C) Find the vertex and the maximum or minimum.

[$2\frac{1}{2} * 4$] **2.** Solve for x in the following equations:

(A) $4^{2x^2+2x} = 8$

(B) $\ln(x-3) + \ln(x-2) = \ln(2x+24)$

(C) $e^{-2x^2+x-5} = \left(\frac{1}{e}\right)^{x^2-3x+20}$

(D) $2^{\log_7 x} = 2^3$

PLEASE TURN OVER

$-1.92 + 3.84$

[5+5] **3.**

(A) Find the sum of the first twenty terms of an arithmetic sequence

$$5, 4.5, 4, 3.5, 3, 2.5, \ldots\ldots\ldots$$

2, 1.5, 1, 0.5, 0, -0.5, -1, -1.5, -2, -2.5, -3, -3

(B) If the first and 4th terms of a geometric sequence are $\frac{1}{64}$ and $\frac{-1}{8}$, respectively, find the 8th term of the sequence.

-2

-4.

[5+5] **4.**

(A) Which is the better investment and why: 8% compounded quarterly or 8.3% compounded annually?

(B) What is the annual nominal rate compounded quarterly for a bond that has an Annual Percentage Yield of 5.8%?

[5+5] **5.** Raul Vasquez, a 25-year old professional, puts \$750 in a retirement fund at the end of each quarter untill he reaches age 60. The account pays 8% interest compounded quarterly.

(A) How much will be in the account when he is 60?

(B) If Raul makes no further deposits after age 60, how much will he have for retirement at age 65?

[5+5] **6.** The Rechtien family buys a house for \$140,000 with a down payment of \$30,000. The family take out a 30 year mortgage for \$110,000 at 6.6% compounded monthly.

(A) Find the amount of the monthly payment needed to amortize this loan.

(B) Find the the part of the first payment that is interest and the part that is applied to reducing the debt.

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All
Examination	**Date**	**Time**	**Pages**
Alternate Midterm	October 2013	1 Hour 30 minutes	2

Instructors	Course Examiner
A. Kokotov, F. Romanelli, H. Greenspan, M. Padamadan, R. Rodriguez, U. Tiwari, V. Kalvin	D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[4+3+3] **1.** A charter fishing company buys a new boat for $224,000 and assumes that it will have a trade in value of $115,200 after 16 years.

(A) Find a linear equation for the depreciated value V of the boat t years after it was purchased.

(B) What is the depreciated value of the boat after 10 years?

(C) When will the depreciated value fall below $100,000?

[$2\frac{1}{2} * 4$] **2.** Solve for x in the following equations:

(A) $27^{2x-4} = \left(\frac{1}{81}\right)^{x-7}$ $x = 4$

(B) $\log_6(x+4) + \log_6(x+9) = 2\log_5 10 - \log_5 4$ $= \log_5 100 - \log_5 4$ $= \log_5 25$ $= 2$

(C) $e^{-2x^2+x-5} = \left(\frac{1}{e}\right)^{x^2-3x+20}$ $x = 3, -5$

(D) $\log_5(x^2+x+19) = 2$

$x^2 + x + 19 = 25$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = 2, -3$

PLEASE TURN OVER

$a_{25} = a + 24d$ $a + 24d = -17$ $a + 12d = 31$ $12d = -48$ $d = -4$

[5+5] **3.** $a = 79,\ d = -4$

$a_{60} = -315$ (A) If the 13th and 25th terms of an arithmetic sequence are 31 and −17 respectively, find the sum of the first 60th term of the sequence. $a_{60} = -79 + 39(4)$ $= -79 -$

(B) Find the sum of the first twelve terms of the geometric sequence

$S_{12} = \frac{100[(1.08)^{12} - 1]}{0.08}$ $100, 100(1.08), 100(1.08)^2, \ldots 100(1.08)^3$ $S_{12} = 1897.71$

108, 116.64

[10] **4.** A person borrows \$3,600 and agrees to repay the loan in monthly installments over a period of 3 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 3 years?

$\rightarrow 100 + 36 \rightarrow 100$ ~~\$666~~ 10 18 (1+36)
$136 \rightarrow 135$

[5+5] **5.** American Express's online banking division offered a money market account with an annual percentage yield(APY) of 5.65%.

(A) If interest is compounded monthly, what is the equivalent annual nominal rate? 5.5087% APY = $= A\left(1 + \frac{r}{m}\right)^{mt}$

$i = .00459635$ (B) If a company wishes to have \$1,000,000 in this account after 8 years, what equal deposit should be made each month? $e^r - 1$

PMT = ~~\$12903.55~~ \$8314.33

[4+3+3] **6.** On December 31, 1990, a house was purchased with the buyer taking out a 30 year, \$112,475 mortgage at 9% interest, compounded monthly. The mortgage payments are made at the end of each month.

(A) Calculate the amount of the monthly payment. \$905

(B) Calculate the unpaid balance of the loan on December 31, 2016. \$36367.14

(C) How much of the principal will be paid off during the year 2016? \$7229.66

$APY = \left(1 + \frac{r}{12}\right)^{12} - 1$ $\left(1 + \frac{r}{12}\right)^{12} = (1 + 0.0565)^{12}$

$PMT = \frac{1,000,000 \times 0.00459}{(1.00459)^{96} - 1}$

$\frac{r}{12} = (1.00565)^{1/12} - 1$

$r = 12\left[(1.00565)^{1/12} - 1\right]$

$= 0.00550876$

$= 5.5087$

$r = 5.51\%$

31 dec 1990 31 Jan 1991 28 Feb 1991

13 payments.

Math 208 : Midterm Winter 13. Solution

1. (A) $12x^2+96x-120=0$

$x^2+8x-10=0$

$x=\frac{-8\pm\sqrt{64+40}}{2}=\frac{-8\pm10.2}{2}$

$x=-9.1,\ 1.1$

(B) $f(x)=-0.12x^2-0.96x+1.2$

$a=-0.12,\ b=-.96,\ c=1.2$

vertex: $\left(-\frac{b}{2a},\ \frac{4ac-b^2}{4a}\right)=(-4,\ 3.12)$

Max $=3.12$

(B) Form:

$f(x)=-0.12(x^2+8x+16)+1.92+1.2$

$f(x)=-0.12(x+4)^2+3.12$

Max $=3.12$

2. (A) $2^{4x^2+4x}=2^3\Rightarrow 4x^2+4x=3$

$4x^2+4x-3=0,\ x=\frac{-4\pm\sqrt{16+48}}{8}$

$x=\frac{-4\pm8}{8}=\frac{1}{2},-\frac{3}{2}$

(B) $\ln(x-3)(x-2)=\ln(2x+24)$

$(x-3)(x-2)=2(x+12)$

$x^2-5x+6-2x-24=0$

$x^2-7x-18=0$

$(x-9)(x+2)=0$

$x=9,\ \cancel{-2}\Rightarrow x=9$

(C) $e^{-2x^2+x-5}=e^{-x^2+3x-20}$

$-2x^2+x-5=-x^2+3x-20$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5,3$

(D) $\log_7 x=3\Rightarrow x=7^3=343$

3 (A) $a_1=5,\ d=-0.5$

$S_{20}=\frac{20}{2}\left[5\times2+21(-0.5)\right]$

$=10\left[10-10.5\right]=-5$

(B) $a_1=\frac{1}{2^6},\ a_4=a_1r^3=-\frac{1}{2^3}$

$\frac{a_1r^3}{a_1}=r^3=\frac{-\frac{1}{2^3}}{\frac{1}{2^6}}=-\frac{2^6}{2^3}=-8$

$r=-2$

$a_8=a_1r^7=\frac{1}{2^6}(-2^7)=-2.$

4 (A) APY for 8% compounded quarterly $=(1+.02)^4-1=8.24\%$

So 8.3% compounded annually is better investment

(B) $APY=\left(1+\frac{r}{4}\right)^4-1,\quad APY=0.058$

so $\left(1+\frac{r}{4}\right)^4=1+0.058.$

$\left(1+\frac{r}{4}\right)^4=1.058$

$1+\frac{r}{4}=\sqrt[4]{1.058}=1.0141948$

$r=0.0568$

$r=5.68\%$

5 (A) $FV=\frac{750\left[(1.02)^{140}-1\right]}{0.02}=\$562367.47$

(B) $A=P(1+i)^n=(562367.47)(1.02)^{20}$

$=\$835648.48$

6 $PV=\frac{PMT\left[1-(1+i)^{-n}\right]}{i}$

(A) $PMT=\frac{(PV)(i)}{1-(1+i)^{-360}}$

$=\frac{(110000)\times(0.0055)}{1-(1.0055)^{-360}}$

$=\$702.53$

(B) Interest on \$110,000 in one month's period $=110000\times0.0055$

$=\$605.00$

Principal payment $=702.53-605.00$

$=\$97.53.$

Solutions: Math 208 Mid-term Exam Fall 2012

1 (A) Linear model form:

$V = mt + C$ When $t = 0$, $V = 224000$

$\therefore V = mt + 224000$, when $t = 16$

$V = 115200$ Thus $m = -6800$

Hence $V = -6800t + 224000$

(B) When $t = 10$, $V = \$156,000$

(C) When $V < 100,000$ first?

$100,000 < -6800t + 224000$

$-124,000 < -6800t$

$t > \frac{124000}{6800} = 18.24$ yrs

2. $2^{3(x^2-3)} = 2^{x+4} \Leftrightarrow 3x^2 - 9 = x + 4$ or

i) $3x^2 - x - 13 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+156}}{6}$

$x = 2.255, 1.922$

ii) $\ln\left(\frac{x+2}{x-1}\right) = 2\ln 2 \Rightarrow \frac{x+2}{x-1} = 4$

$4x - 4 = x + 2 \Rightarrow x = 3$

3) $4x^2 - 17x = -8 - 5x$

$4x^2 - 12x + 8 = 0 \Rightarrow x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0 \Rightarrow x = 1, 2$

$x^2 - x + 6 = 3^2 \Rightarrow x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{13}}{2} = \frac{1 \pm 3.6}{2} = 2.30, -1.30$

(A) A. sequence $a_n = a_1 + (n-1)d$

$a_1 = -5, a_{15} = 23, \quad a_{15} = -5 + 14d$

$23 + 5 = 14d \Rightarrow d = 2$

$a_n = a_1 + (n-1)d = -5 + (n-1)2 = 2n - 7$

$a_n = 2n - 7$, $\boxed{a_{73} = 139}$

(B) $a_1 = 4, \quad a_{10} = 40$

$\therefore a_n = a_1 r^{n-1} = 4r^{n-1}$

$a_{10} = 4r^9 \Rightarrow r^9 = 10$

Now $a_{46} = 4r^{45} = 4(r^9)^5$

$a_{46} = 4(10)^5 = 400,000$

(4) Total interest in 270 days.

$I = Prt = 3500 \times 0.1 \times \frac{270}{360}$

$I = \$262.50$

Third party pays \$3550 in 60 days

and receives $\$262.50 - \$50 = \$212.50$

in 210 days on an investment of \$3500

$I = Prt \Rightarrow 212.50 = 3550 \times r \times \frac{210}{360}$

$r = \frac{212.50 \times 360}{3550 \times 210} = 0.102615$

$= 10.26\%$

5 $FV = \frac{PMT[(1+i)^n - 1]}{i}$

Here $i = \frac{.06}{12} = 0.005$, $m = 12$

FV at the end of year 1

$FV = \frac{100[(1.005)^{12} - 1]}{0.005} = \$1233.56$

Interest in yr 1 $= 1233.56 - 1200$

$= \$33.56$

At the end of year 2, $n = 24$

$PV = \frac{100[(1.005)^{24} - 1]}{0.005} = \$2543.20$

Interest in yr 2 $= \$2543.20 - 1233.56 - 1200$

$= \$109.64$

$FV_{yr\,3} = \frac{100[(1.005)^{36} - 1]}{0.005} = \$3933.61$

Interest in year 3 $= \$3933.61 - 2543.20 - 1200$

$= \$190.41$

6 $i = 0.0015$

$PV = \frac{PMT[1 - (1+i)^{-n}]}{i}$ $\quad n = 52 \times 3 = 156$

$PMT = \frac{PV \times i}{1 - (1+i)^{-n}} = \frac{7000 \times 0.0015}{1 - (1.0015)^{-156}} = \$50.36$

Interest paid $= 50.36 \times 156 - 7000 = \$856.1$

Remaining balance after two yrs

$PV = \frac{PMT[1 - (1+i)^{-51}]}{i} = \frac{50.36[1 - (1.0015)^{-51}]}{0.0015}$

$= \$2517.38$