

University of Ottawa - Economics
ECO3150C – Probability and Statistics for Economists
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Midterm Exam

Date: October 27, 2014

Time allotted: 17:30 to 18:50 (80 minutes)

Note: Please leave this sheet in the exam booklet.

Student Name: _____

Student number: _____

Exercise 1 (3 pts)

Let E_1 and E_2 be mutually exclusive and collectively exhaustive events, *i.e.*: $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$, where S is the sample space (set of all possible outcomes). Assume that $P(E_1) = 2P(E_2)$. Let A be a subset of S such that $P(A|E_1) = \frac{3}{4}$ et $P(A|E_2) = \frac{1}{2}$.

- a) (1.0 pt) Find $P(E_1)$, $P(E_2)$.
- b) (1.0 pt) Find $P(A)$.
- c) (1.0 pt) Find $P(E_1|A)$ et $P(E_2|A)$.

Exercise 2 (10 pts)

Let X and Y be two random variables with joint probability distribution described by the following table:

		X		
		0	1	2
Y	2	0.12	a	0.20
	4	0.05	0.13	0.11
	6	b	0.10	c

where a , b et c are real numbers.

- a) (1 pt) Find the value of $a + b + c$.
- b) (1.5 pts) You are given the following additional information:

$$P(X = 0) = 0.32$$

$$P(Y = 6) = 0.33$$

Use that information to find the values of a , b and c .

- c) (1.5 pt) Give the marginal probability distributions of X and Y .
- d) (2.0 pt) Find the population means and variances of X and Y .
- e) (1.0 pt) Compute the mean and variance of X conditional on $Y = 4$.
- f) (1.0 pt) Compute the mean and variance of Y conditional on $X = 2$.
- g) (1.0 pt) Compute the covariance of X and Y .
- h) (1.0 pt) Compute the linear correlation coefficient of X and Y .

Exercise 3 (7 pts)

$n = 100$ students are drawn randomly and independently and asked whether they smoke cigarette or not. Let $X_i = 1$ if Student $\#i$ smokes cigarette and $X_i = 0$ otherwise, for $i = 1, 2, \dots, n$. Suppose the true proportion of smokers amongst students is an unknown number p whilst the number of smokers observed in the sample is $Y = \sum_{i=1}^n X_i = 33$.

- a) (1.0 pt) What is the distribution of Y ?
- b) (1.0 pt) Find the population mean and variance of Y as a function of n and p .
- d) (1.0 pt) What is the best guess of the true proportion of smokers p based on the observed sample? Let \hat{p} denote this estimate.
- e) (1.0 pt) Find the population mean and variance of \hat{p} .
- c) (1.0 pt) What is the asymptotic distribution of \hat{p} ? (Hint: Use the Central Limit Theorem)
- f) (0.5 pt) Use the asymptotic distribution to find $P(\hat{p} < 0.3)$.
- g) (0.5 pt) Use the asymptotic distribution to find $P(\hat{p} < 0.27)$.
- h) (1.0 pt) Use the asymptotic distribution to find $P(0.27 < \hat{p} < 0.33)$.