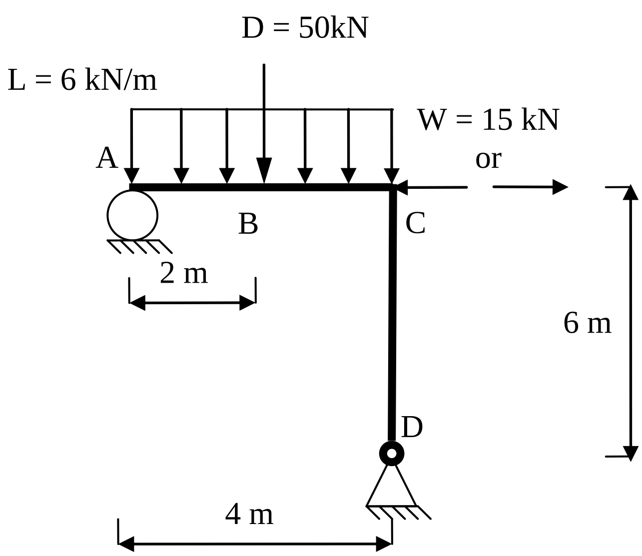
Carleton University
Department of Civil and Environmental Engineering

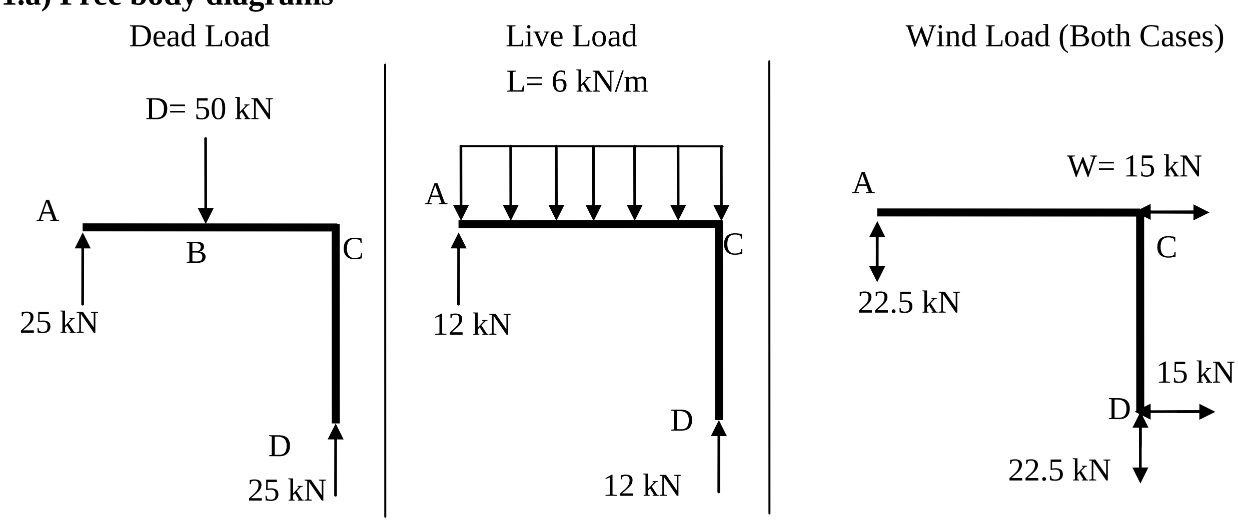
CIVE 3204 Introduction to Structural Design (Fall 2014)

Solution 2 **(out of 100)**

Question 1 **(50 points)**



1.a) Free body diagrams



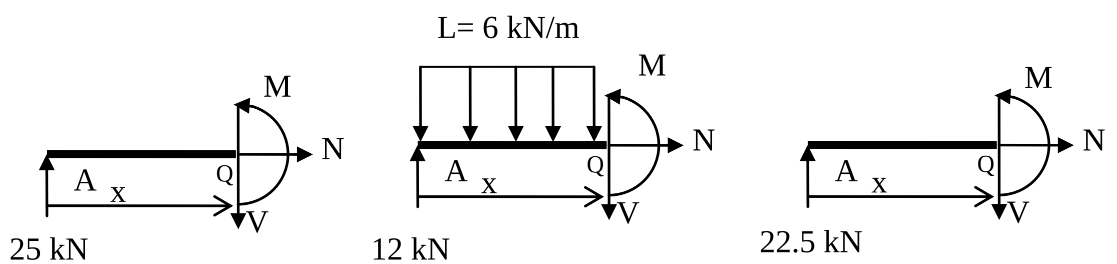
1.b) Factored shear at point B to use for design

Since point B is located under the dead load, the amount of shear at the left side is different from the right side. Due to possible opposite action of different load types for shear, we should consider positive and negative shear (based on a defined convention) similar to the tension and compression case in Question 3 from the previous assignment. This will lead to checking 4 different cases for factored shear at point B to use for design:

1.b.1) Left side of point B

Here, we cut part AB at a point slightly to the left side of point B, and show the internal forces at this section. We can solve for the unknown V_B under dead, live and wind load, separately. Alternatively, one can draw shear diagrams under each load case, separately, and then report the shear at the left side of point B. (Obviously, positive direction should be considered the same for each load case for all the members of the frame).

Dead Load Live Load Wind Load



$V_{B,Dead}$ = 25 kN
$M_{B,Dead}$ = 50 kN.m

$V_{B,Live}$ = 0 kN
$M_{B,Live}$ = 12 kN.m

$V_{B,Wind}$ = 22.5 kN
$M_{B,Wind}$ = 45 kN.m

For opposite wind direction: $V_{B,Wind}$ = -22.5 kN
$M_{B,Wind}$ = -45 kN.m

Now, we can do the ULS load combinations for positive and negative shear at the left side of point B:

1b.1.1) Positive shear at the left side of point B

Assuming positive direction as V_B shown in the above figure, we are looking for maximum real number.

D = $V_{B,Dead}$ = 25 kN
L = $V_{B,Live}$ = 0 kN
W= $V_{B,Wind}$ = 22.5 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN)
1	1.4(25)	0	35
2	1.25(25)+1.5(0)	0.5(0) or 0.4(22.5)	40.25
4	1.25(25)+1.4(22.5)	0.5(0) or 0.5(0)	62.75→governs

Positive factored shear at the left side of point B = 62.8 kN

1b.1.2) Negative shear at the left side of point B

Assuming positive direction as V_B shown in the above figure, we are looking for minimum real number.

$D = V_{B,Dead} = 25$ kN
$L = V_{B,Live} = 0$ kN
$W= V_{B,Wind} = -22.5$ kN
$S = 0$ → load case 3 will not govern
$E = 0$ → load case 5 will not govern

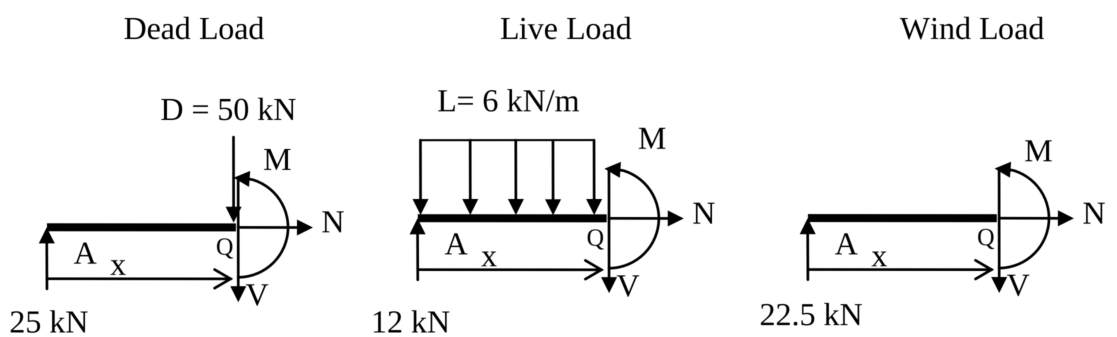
The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN)
1	1.4(25)	0	35
2	0.9(25)+1.5(0)	0.5(0) or <u>0.4(-22.5)</u>	13.5
4	0.9(25)+1.4(-22.5)	<u>0.5(0)</u> or 0.5(0)	-9 →governs

Negative factored shear at the left side of point B = -9 kN
Therefore, from 1b.1.1 and 1b.1.2: **Factored shear at the left side of point B = 62.8 kN**

<u>1b.2) Right side of point B</u>
Here, we cut part AB at a point slightly to the right side of point B, and show the internal forces at this section. We can solve for the unknown V_B under dead, live and wind load, separately.
<u>Alternatively</u>, one can draw shear diagrams under each load case, separately, and then report the shear at the right side of point B. (Obviously, positive direction should be considered the same for each load case for all the members of the frame).



$V_{B,Dead}$ = -25 kN
$M_{B,Dead}$ = 50 kN.m

$V_{B,Live}$ = 0 kN
$M_{B,Live}$ = 12 kN.m

$V_{B,Wind}$ = 22.5 kN
$M_{B,Wind}$ = 45 kN.m

For opposite wind direction: $V_{B,Wind}$ = -22.5 kN
$M_{B,Wind}$ = -45 kN.m

Now, we can do the ULS load combinations for positive and negative shear at the right side of point B:

1b.2.1) Positive shear at the right side of point B

Assuming positive direction as V_B shown in the above figure, we are looking for maximum real number.

D = $V_{B,Dead}$ = -25 kN
L = $V_{B,Live}$ = 0 kN
W= $V_{B,Wind}$ = 22.5 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN)
1	1.4(-25)	0	-35
2	0.9(-25)+1.5(0)	0.5(0) or 0.4(22.5)	-13.5
4	0.9(-25)+1.4(22.5)	0.5(0) or 0.5(0)	9 →governs

Positive factored shear at the right side of point B = 9 kN

1b.2.2) Negative shear at the right side of point B

Assuming positive direction as V_B shown in the above figure, we are looking for minimum real number.

D = $V_{B,Dead}$ = -25 kN
L = $V_{B,Live}$ = 0 kN
W= $V_{B,Wind}$ = -22.5 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN)
1	1.4(-25)	0	-35
2	1.25(-25)+1.5(0)	0.5(0) or 0.4(-22.5)	-40.25
4	1.25(-25)+1.4(-22.5)	0.5(0) or 0.5(0)	-62.75 → governs

Negative factored shear at the right side of point B = -62.8 kN

Therefore, from 1b.2.1 and 1b.2.2: **Factored shear at the right side of point B = -62.8 kN**

Therefore, from 1b.1 and 1b.2:
Factored shear at point B to use for design = max(62.8,- 62.8) = 62.8 kN

1.c) Factored bending moment at point B to use for design

Unlike shear at point B, bending moment has same amount at both sides of point B. However, positive and negative values should be checked separately.

1c.1) Negative bending moment at point B

Referring to section 2b.1 and assuming positive direction as M_B shown in the figures, we are looking for minimum real number.

D = $M_{B,Dead}$ = 50 kN.m
L = $M_{B,Live}$ = 12 kN.m
W= $M_{B,Wind}$ = -45 kN.m
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN.m)
1	1.4(50)	0	70
2	0.9(50)+1.5(12)	0.5(0) or 0.4(-45)	45
4	0.9(50)+1.4(-45)	0.5(12) or 0.5(0)	-18 →governs

Negative factored bending moment at point B for design = -18 kN.m.

1c.2) Positive bending moment at point B

Referring to section 1b.1 and assuming positive direction as M_B shown in the figures, we are looking for maximum real number.

$D = M_{B,Dead} = 50$ kN.m

$L = M_{B,Live} = 12$ kN.m

$W = M_{B,Wind} = 45$ kN.m

S = 0 → load case 3 will not govern

E = 0 → load case 5 will not govern

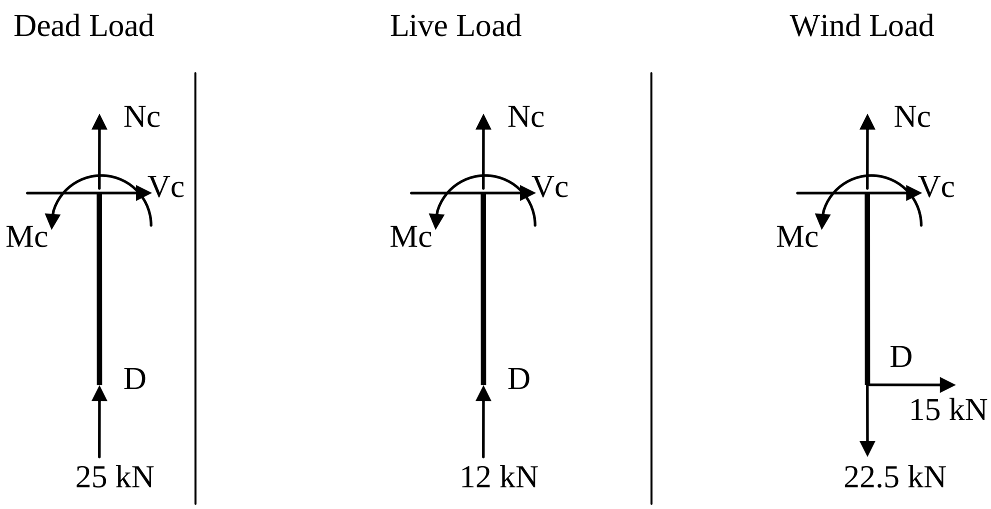
The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN.m)
1	1.4(50)	0	70
2	1.25(50)+1.5(12)	0.5(0) or 0.4(45)	98.5
4	1.25(50)+1.4(45)	0.5(12) or 0.5(0)	131.5→governs

Positive factored bending moment at point B for design = 132 kN.m

1.d) Factored shear in the column at point C to use for design



$N_{C,Dead}$ = -25 kN
$V_{C,Dead}$ = 0
$M_{C,Dead}$ = 0

$N_{C,Live}$ = -25 kN
$V_{C,Live}$ = 0
$M_{C,Live}$ = 0

$N_{C,Wind}$ = 22.5 kN
$V_{C,Wind}$ = -15 kN
$M_{C,Wind}$ = -90 kN

For opposite wind direction:
$N_{C,Wind}$ = -22.5 kN
$V_{B,Wind}$ = 15 kN
$M_{C,Wind}$ = 90 kN

<u>1d.1) Negative shear in the column at point C</u>

Assuming positive direction as V_C shown in the above figure, we are looking for <u>minimum real number</u>.

D = $V_{C,Dead}$ = 0
L = $V_{C,Live}$ = 0
W= $V_{C,Wind}$ = -15 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN)	
1	1.4(0)	0	0	
2	0+1.5(0)	0.5(0) or 0.4(-15)	-6	
4	0+1.4(-15)	0.5(0) or 0.5(0)	-21	→governs

Negative factored shear in the column at point C = -21 kN

1d.2) Positive shear in the column at point C

Since D=0 and L=0, we get a result similar to 2d.1 for positive shear in the column at point C.

Therefore, from 1d.1 and 1d.2:

Factored shear in the column at point C to use for design = 21 kN

1.e) Factored bending moment in the column at point C to use for design

1e.1) Negative bending moment in the column at point C

Assuming positive direction as M_C shown in the above figure, we are looking for minimum real number.

D = $M_{C,Dead}$ = 0

L = $M_{C,Live}$ = 0

W= $M_{C,Wind}$ = -90 kN.m

S = 0 → load case 3 will not govern

E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN.m)	
1	1.4(0)	0	0	
2	0+1.5(0)	0.5(0) or <u>0.4(-90)</u>	-36	
4	0+1.4(-90)	0.5(0) or 0.5(0)	-126	→ governs

Negative factored bending moment in the column at point C to use for design=-126 kN.m

<u>1e.2) Positive bending moment in the column at point C</u>
Since D=0 and L=0, we get a result similar to 2e.1 for positive bending moment in the column at point C, but with a positive sign.

Positive factored bending moment in the column at point C to use for design = 126 kN.m

1.f) Factored axial force in the column CD to use for design

<u>1f.1) Negative axial force in the column CD</u>

Assuming positive direction as N_C shown in the above figure, we are looking for <u>minimum real number</u>.

D = $N_{C,Dead}$ = -25 kN
L = $N_{C,Live}$ = -12 kN
W= $N_{C,Wind}$ = -22.5 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN.m)	
1	1.4(-25)	0	-35	
2	1.25(-25) +1.5(-12)	0.5(0) or 0.4(-22.5)	-58.25	
4	1.25 (-25) +1.4(-22.5)	0.5(-12) or 0.5(0)	-68.75	→governs

Negative factored axial force in the column CD to use for design=-68.8 kN

1f.2) Positive axial force in the column CD

Assuming positive direction as N_C shown in the above figure, we are looking for maximum real number.

D = $N_{C,Dead}$ = -25 kN
L = $N_{C,Live}$ = -12 kN
W= $N_{C,Wind}$ = 22.5 kN
S = 0 → load case 3 will not govern
E = 0 → load case 5 will not govern

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S

Case	Principal loads	Companion loads	Total (kN.m)	
1	1.4(-25)	0	-35	
2	0.9(-25) +1.5(-12)	0.5(0) or 0.4(22.5)	-31.7	
4	0.9(-25) +1.4(22.5)	0.5(-12) or 0.5(0)	8.3	→governs

Positive factored axial force in the column CD to use for design = 8.3 kN

Question 2 **(50 points)**

2.a) Beam

Note: In simply supported beams with symmetrical geometry and loading system, maximum shear occurs at support location, and maximum bending moment occurs at mid-span.

Dead load calculation:

Concrete floor slab: $0.14m * 2400 kg/m^3 * 9.81 m/s^2 * 10^{-3} = 3.296\ kN/m^2$

Finishing screed and vinyl tiles: $0.5\ kN/m^2$

Acoustic tiles: $0.2\ kN/m^2$

Partition allowance: $1.0\ kN/m^2$

Total: **$4.996\ kN/m^2$**

Beam W250x28 self-weight: 0.279 kN/m

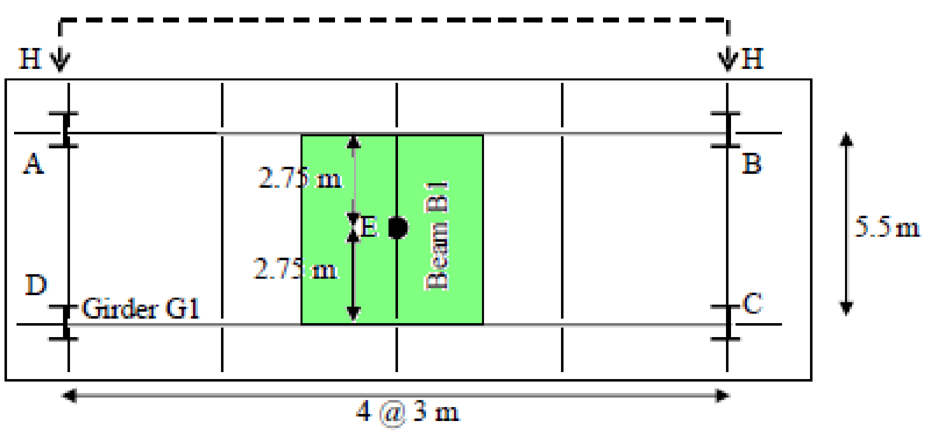
Girder W690x192 self-weight: 1.88 kN/m

Loads acting on beam B1:
Dead Load:

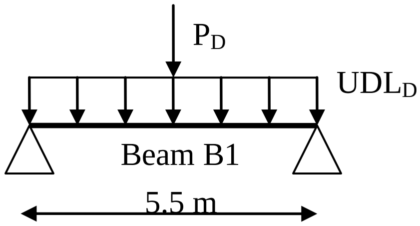
$UDL_D = 3*4.996 + 0.279 = 15.267\ kN/m$
$P_D = 55\ kN$ @ mid-span

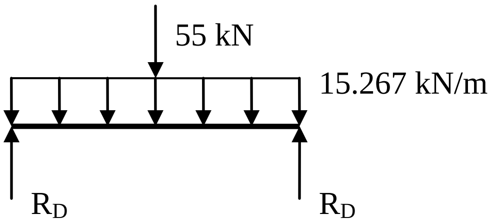
Live Load:

$UDL_L = 3*2.4 = 7.2\ kN/m$



Dead Load



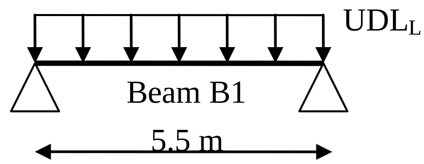


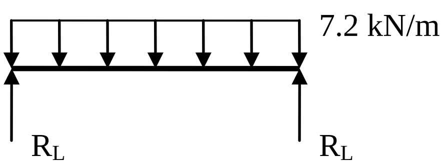
R_D = (55+15.267*5.5)/2 = 69.5 kN

$V_{max,\,D}$ = R_D = 69.5kN @ supports

$M_{max,\,D}$ = 69.5*2.75 -15.267*2.75*2.75/2
= 133.4 kN.m @ mid-span

Live Load





R_L = (7.2*5.5)/2 = 19.8 kN

$V_{max,\,L}$ = R_L = 19.8 kN @ supports

$M_{max,\,L}$ = 19.8*2.75 – 7.2*2.75*2.75/2
= 27.2 kN.m @ mid-span

Load Combination for <u>Beam B1</u>:

Load Case I: $V_{I,max}$= 1.4* $V_{max,\,D}$ = 1.4*69.5 = 97.3 kN

$M_{I,max}$= 1.4* $M_{max,\,D}$ = 1.4*133.4 = 186.8 kN.m

Load Case II: $V_{II,max}$ = 1.25*$V_{max,\,D}$ +1.5*$V_{max,\,L}$ = 1.25*69.5 +1.5*19.8
= 116.6 kN → Governs

$M_{II,max}$ = 1.25*$M_{max,\,D}$ +1.5*$M_{max,\,L}$ = 1.25*133.4 +1.5*27.2
= 207.6 kN.m → Governs

Note: There is no wind, snow, or earthquake load. Thus, load cases III, IV and V will not govern.

In <u>Beam B1</u>: **V_{max} = $V_{II,max}$ = 117 kN** → Maximum factored shear
M_{max} = $M_{II,max}$ = 208 kN.m → Maximum factored bending moment

Required Section Modulus for <u>Beam B1</u>:

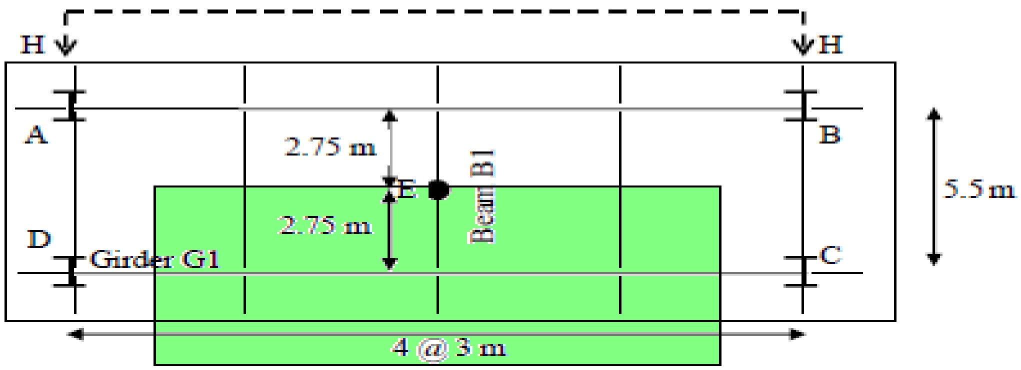
bending stress under factored bending moment ≤ reduced yield stress

$M_{max}/S \leq \varphi f_y \rightarrow S \geq M_{max}/(\varphi f_y)$

$\varphi = 0.9$; $f_y = 350$ Mpa = 350 N/mm^2

$S \geq 207.6 * 10^6 / (0.9*350) = 659.0 * 10^3$ mm^3 → $\mathbf{S \geq 659 * 10^3}$ **mm^3**

2.b) Girder

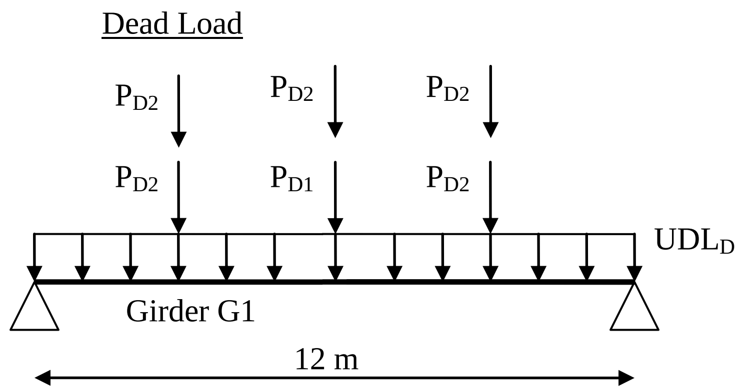


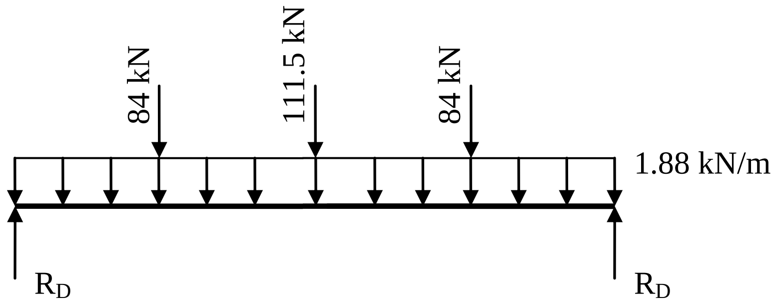
Loads acting on <u>Girder G1:</u>
Dead Load:

UDL_D = 1.88 kN/m
P_{D1} = Support Reaction from Beam B1&2 under Dead Load = R_D = 69.5 kN
P_{D2} = Support Reaction from other beams under Dead Load
= R_D – 55/2 = 42 kN

Live Load:

UDL_L = 0
P_L = Support Reaction from each beam under Live Load = R_L = 19.8 kN



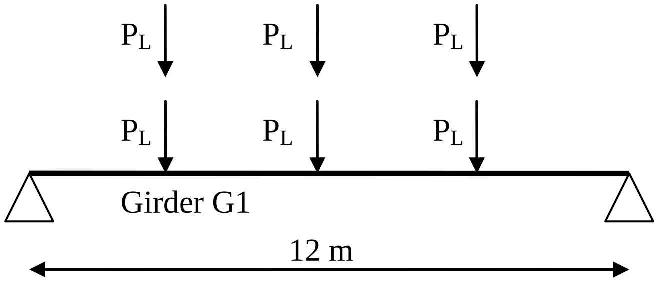


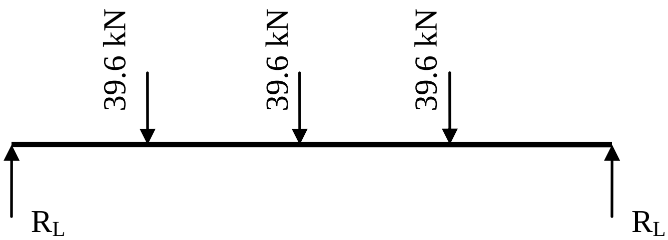
R_D = (1.88*12+84*2+111.5)/2 = 151 kN

$V_{max, D}$ = R_D = 151 kN   @ supports

$M_{max, D}$ = R_D*12/2 – 1.88*$(12/2)^2$ / 2 – 84*(12/2 – 3)
= 620.2 kN.m   @ mid-span

Live Load





$R_L = (39.6*3)/2 = 59.4$ kN

$V_{max, L} = R_L = 59.4$ kN   @ supports

$M_{max, L} = R_L*12/2 - 39.6*(12/2 - 3)$
$= 237.6$ kN.m   @ mid-span

Load Combination for <u>Girder G1</u>:

Load Case I: $V_{I,max} = 1.4* V_{max, D} = 1.4*151 = 211.4$ kN

$M_{I,max} = 1.4* M_{max, D} = 1.4*620.2 = 868.28$ kN.m

Load Case II: $V_{II,max} = 1.25*V_{max, D} + 1.5*V_{max, L} = 1.25*151 + 1.5*59.4$
$= 277.85$kN → Governs

$M_{II,max} = 1.25*M_{max, D} + 1.5*M_{max, L} = 1.25*620.2 + 1.5*237.6$
$= 1131.65$ kN.m → Governs

In <u>Girder G1</u>: **$V_{max} = V_{II,max} = 278$ kN** → Maximum factored shear
$M_{max} = M_{II,max} = 1.13$ MN.m → Maximum factored bending moment

Required Section Modulus for <u>Girder G1</u>:

bending stress under factored bending moment ≤ reduced yield stress

$M_{max}/S \leq \varphi f_y \rightarrow S \geq M_{max}/(\varphi f_y)$

$\varphi = 0.9$; $f_y = 350$ Mpa $= 350$ N/mm^2

$S \geq 1131.65 * 10^6 / (0.9*350) = 3593 * 10^3$ mm^3 → **$S \geq 3593 * 10^3$ mm^3**

2.c) Deflection Check for <u>beam B1</u> (SLS):

$\Delta_L = 5WL^4/(384EI)$ $\Delta_L \leq L/360$ → $5WL^4/(384EI) \leq L/360$ →

$I \geq 5*360WL^3/(384E)$

$W = UDL_L = 7.2$ kN/m $= 7.2 * 10^{-3}$ kN/mm
$L = 5.5$ m $= 5500$ mm
$E = 200$ Gpa $= 200$ kN/mm^2

$I \geq 28.0 * 10^6$ mm^4