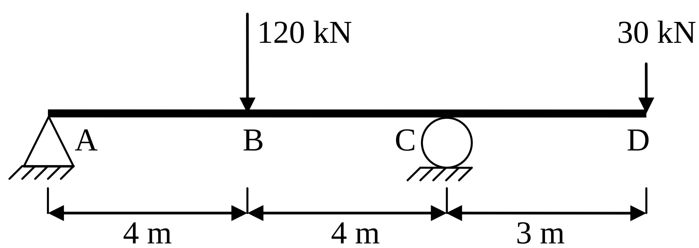
Carleton University
Department of Civil and Environmental Engineering

CIVE 3204 Introduction to Structural Design (Fall 2014)

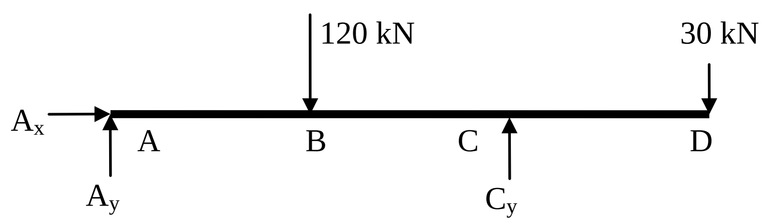
Assignment 1 Solution (out of 100)

Question 1: **(30 Points)**
Draw the free body diagram, shear and bending moment diagram for the following structures:

a)



FBD:



Step1 (solve for support reaction):

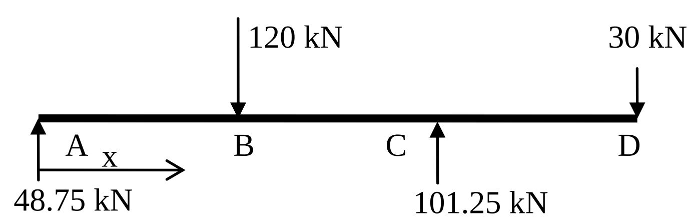
In the whole beam: $\Sigma M_A = 0$ → $8*C_y - 4*120 - 11*30 = 0$ → $C_y = 101.25$ kN

$\Sigma F_y = 0$ → $A_y - 120 - 30 + 101.25 = 0$ → $A_y = 48.75$ kN

$\Sigma F_x = 0$ → $A_x = 0$

Step 2 (Show all forces on the free body diagram):

FBD:

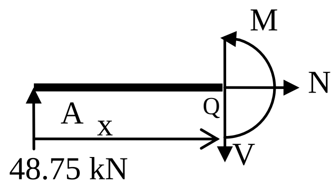


Step 3 (Solve for V=V(x) and M=M(x) by section cut method):

Part AB: **$0 \le x \le 4^m$**

$\Sigma F_x = 0 \rightarrow N = 0$
$\Sigma F_y = 0 \rightarrow V = 48.75$ (kN)
$\Sigma M_Q = 0 \rightarrow M = 48.75x$ (kN.m)

FBD:

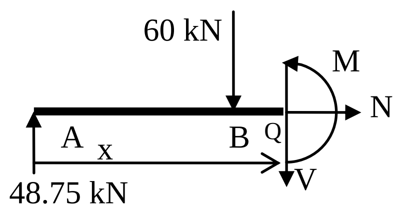


x = 0	**x = 4^m**
N = 0	N = 0
V = 48.75 (kN)	V = 48.75 (kN)
M = 0 (kN.m)	M= 195 (kN.m)

Part BC: **$4^m \le x \le 8^m$**

$\Sigma F_x = 0 \rightarrow N = 0$
$\Sigma F_y = 0 \rightarrow V = 48.75 - 120 = -71.25$ (kN)
$\Sigma M_Q = 0 \rightarrow M = 48.75x - 120(x-4)$ (kN.m)
$M = -71.25x + 480$ (kN.m)

FBD:

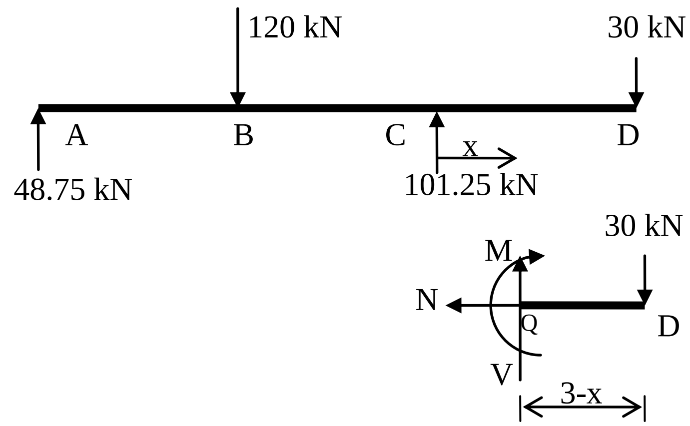


x = 4^m	**x = 8^m**
N = 0	N = 0
V = -71.25 (kN)	V = -71.25 (kN)
M = 195 (kN.m)	M= -90 (kN.m)

Part CD:

<u>Let's define a new x</u> and consider right hand side of the cut section: $0 \le x \le 3^m$

FBD:



Σ $F_x = 0$ → $N = 0$

Σ $F_y = 0$ → $V = 30$ (kN)

Σ $M_Q = 0$ → $M = -30(3-x)$ (kN.m)

$M = 30x-90$ (kN.m)

x = 0

N = 0

V = 30 (kN)

M = -90 (kN.m)

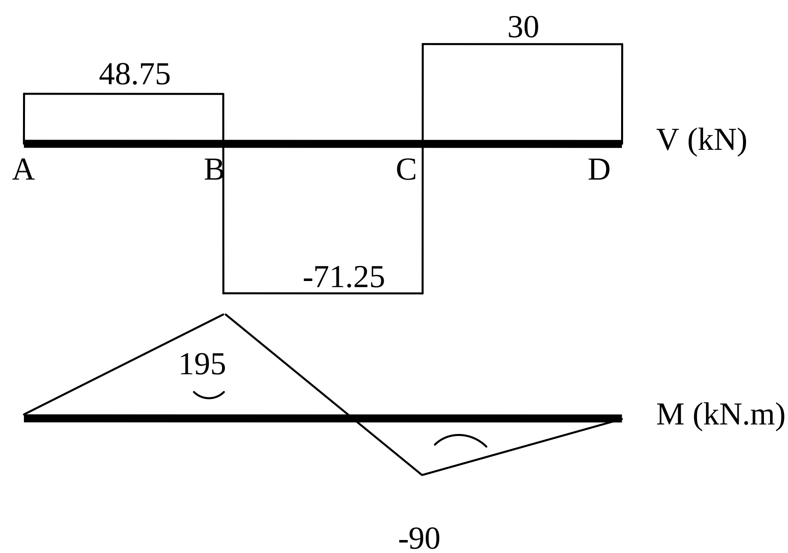
$x = 3^m$

N = 0

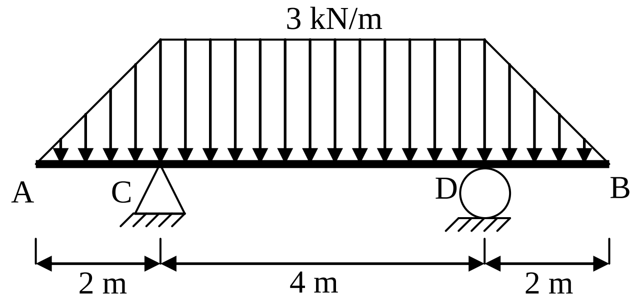
V = 30 (kN)

M= 0 (kN.m)

Step 4 (Draw shear and bending moment diagram):



b)

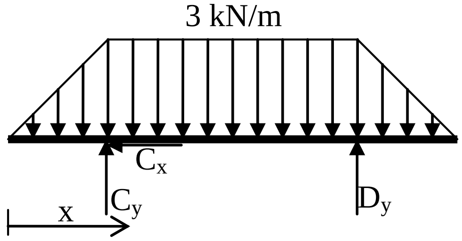


Step1 (solve for support reaction):

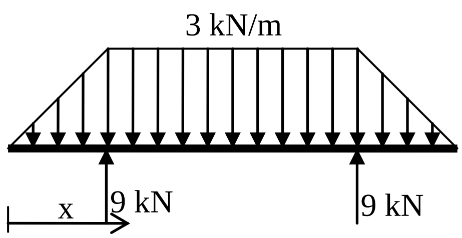
In the whole beam: $\Sigma F_x = 0$ → $C_x = 0$

Due to symmetry: $C_y = D_y$

$\Sigma F_y = 0$ → $2C_y = 3*(8+4)/2$ → $C_y = D_y = 9$ kN



Step 2 (Draw free body diagram with known values):



Step 3 (Solve for V=V(x) and M=M(x) by section cut method):

$0 \le x \le 2$ m

let q = q(x) : intensity of the distributed load at distance x from A

$q/4 = x/2$ → $q = 3x$

$\Sigma F_x = 0$ → $N = 0$

$\Sigma F_y = 0$ → $V = -(1.5x*x)/2 = -1.5x^2/2$ (kN)

$\Sigma M_Q = 0$ → $M = -[(1.5x^2/2)]*x/3$

$M = -1.5x^3/6$ (kN.m)

<u>x = 0</u>	<u>x = 2 m</u>
N = 0	N = 0
V = 0 (kN)	V = -3 (kN)
M = 0	M= -2 (kN.m)

$2 \text{ m} \leq x \leq 4 \text{ m}$

$\Sigma F_x = 0$ → N = 0

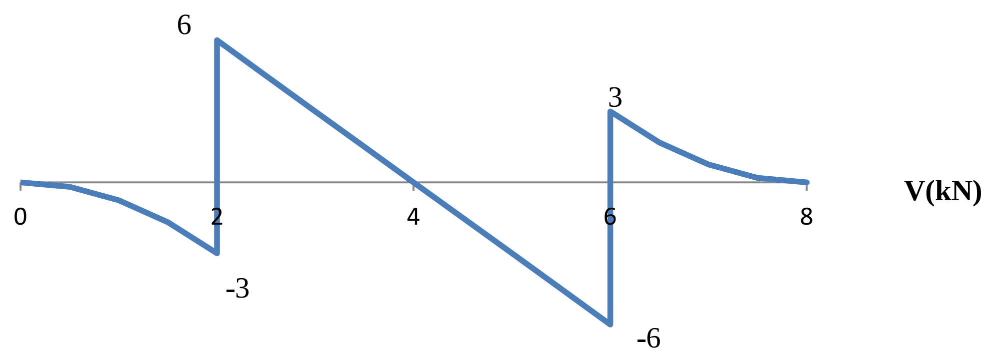
$\Sigma F_y = 0$ → $V = 9 - 3 - 3*(x-2) = 12 - 3x$ (kN)

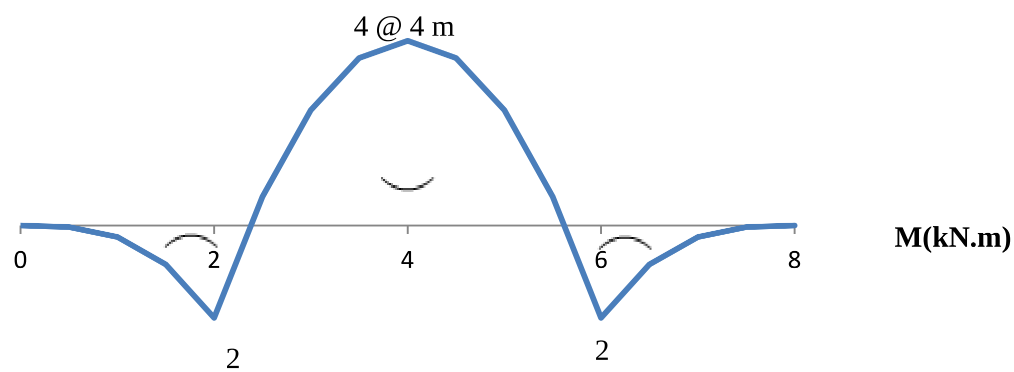
$\Sigma M_Q = 0$ → $M = 9*(x-2) - 3*(x-2*2/3) - 3*(x-2)^2/2$

$M = -20+12x - 1.5x^2$ (kN.m)

<u>x = 2 m</u>	<u>x = 4 m</u>
N = 0	N = 0
V = 6 (kN)	V = 0 (kN)
M = -2 (kN.m)	M= 4 (kN.m)

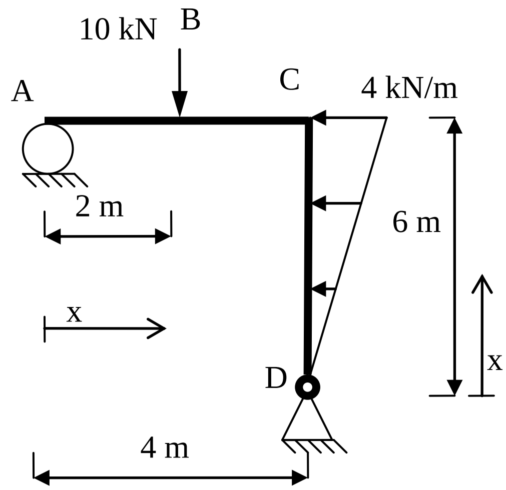
<u>Step 4 (Draw shear and bending moment diagram):</u>

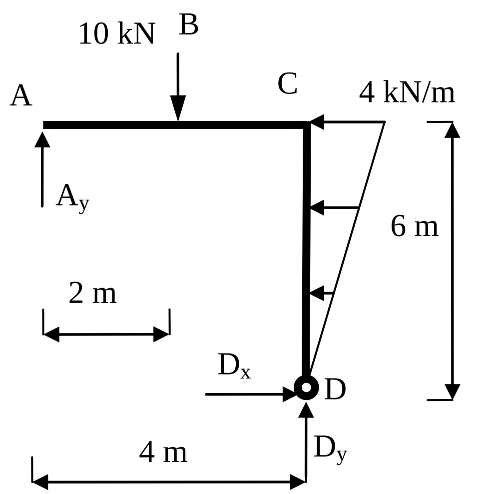




Question 2: **(30 Points)**

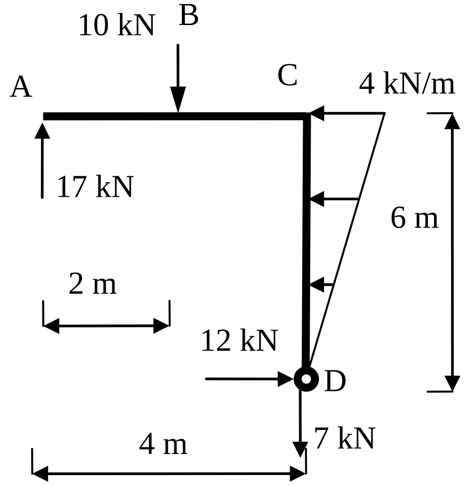
Draw free body diagrams, axial force, shear force, and bending moment diagrams for the following frame. Find the maximum axial load in the column and the maximum shear and bending moment in the beam and report locations where they occur.





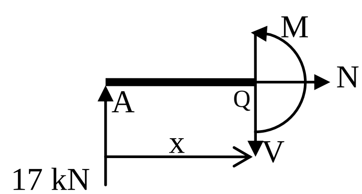
In the whole frame:	$\Sigma M_D = 0$	→ A_y *4 = 10*2+4*6/2*(6*2/3)	→ A_y = 17 kN
	$\Sigma F_y = 0$	→ D_y = -17+10	→ D_y = -7 kN
	$\Sigma F_x = 0$	→ D_x = 4*6/2	→ D_x = 12 kN

Free body diagram:



Part AB: $0 \leq x < 2^m$

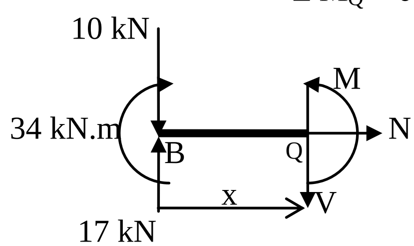
$\Sigma F_x = 0$ → $N = 0$
$\Sigma F_y = 0$ → $V = 17$ (kN)
$\Sigma M_Q = 0$ → $M = 17x$ (kN.m)



$\underline{x = 0}$	$\underline{x = 2^m}$
N = 0	N = 0
V = 17 (kN)	V = 17 (kN)
M = 0	M= -17*2 = 34 (kN.m)

Part BC: $2 \leq x \leq 4^m$

$\Sigma F_x = 0$ → $N = 0$ (kN)
$\Sigma F_y = 0$ → $V = 17-10$ (kN)
$\Sigma M_Q = 0$ → $M = 34+(17-10)(x-2)$



$\underline{x = 2}$	$\underline{x = 4^m}$
N = 0	N = 0
V = 7 (kN)	V = 7 (kN)
M = 34+0 = 34 (kN.m)	M= 34+(17-10)(4-2) = 48 (kN.m)

Part CA: $0 \le x \le 6$ m

Note: Consider x along longitudinal axis of member CD.

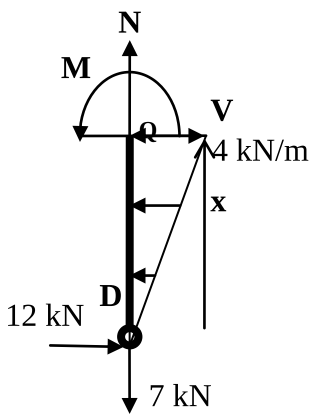
$\Sigma F_x = 0$ → N = 7 (kN)

$\Sigma F_y = 0$ → V = -12+(4*x/6)*x/2 kN

→ $V = -12+4x^2/12$

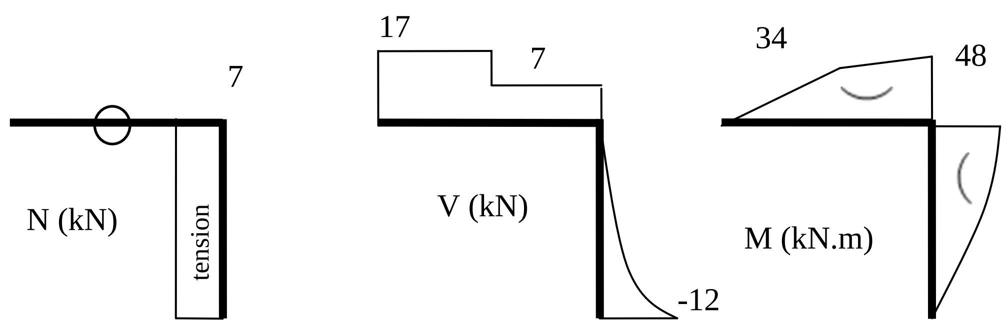
$\Sigma M_Q = 0$ → M = -12x+(4*x/6)*x/2*x/3 (kN.m)

→ $M = -12x+4x^3/36$



x = 0	x = 6^m
N = 7	N = 7
V = -12 (kN)	V = 0 (kN)
M = 0 (kNm)	M= $-12*6+4*6^{3/36}$ = -48 (kN.m)

Draw axial force, shear force, and bending moment diagrams:



max axial load in the column:	7 kN	tension along the column length
max shear in the beam:	17 kN	between points A and B
max bending moment in the beam:	48 kN.m	at point C

<u>Question 3:</u> **(40 Points)**

1.a1) Critical case for tension

Assume tension force as positive. Based on this assumption we should look for <u>maximum real number</u> to find the critical case for tension.

D = -180 kN
L = -90 kN
S = -55 kN
W = 120 kN
E = 135 kN
T = 15 kN

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
1	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
3	(1.25 or 0.9)D+1.5S	0.5L or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S
5	1.0D+1.0E	0.5L+0.25S

Since the dead load has compressive action and we are looking for max tension, we use 0.9 as the coefficient for the dead load for cases 2, 3 and 4:

Case	Principal loads	Companion loads	Total (kN)	
1	1.4(-180)	0	-252	
2	0.9(-180)+1.5(-90)	0.5(-55) or <u>0.4(120)</u>	-249	
3	0.9(-180)+1.5(-55)	0.5(-90) or <u>0.4(120)</u>	-196.5	
4	0.9(-180)+1.4(120)	0.5(-90) or <u>0.5(-55)</u>	-21.5	→governs
5	1.0(-180)+1.0(135)	0.5(-90)+0.25(-55)	-103.75	

As it can be seen in the above table, column AB is under compression in all cases and load combination 4 governs, since it gives the maximum real number. We need to include the differential settlement effect as well:

Maximum tension force = -21.5 + 1.25 (15) = -2.75 kN
(due to the negative sign, it acts as compression not tension)

Therefore, column AB will not undergo tension under all different possible load combinations, and **Minimum compression force = 2.75 kN**

1.a2) Critical case for compression

Assume tension force as positive. Based on this assumption we should look for minimum real number to find the critical case for compression.

D = -180 kN
L = -90 kN
S = -55 kN
W = -120 kN
E = -135 kN
T = 15 kN

The Ultimate Limit States (ULS) load combination cases are as follows:

Case	Principal loads	Companion loads
	1.4D	0
2	(1.25 or 0.9)D+1.5L	0.5S or 0.4W
3	(1.25 or 0.9)D+1.5S	0.5L or 0.4W
4	(1.25 or 0.9)D+1.4W	0.5L or 0.5S
5	1.0D+1.0E	0.5L+0.25S

Since the dead load has compressive action and we are looking for max compression, we use 1.25 as the coefficient for the dead load for cases 2, 3 and 4:

Case	Principal loads	Companion loads	Total (kN)	
1	1.4(-180)	0	-252	
2	1.25(-180)+1.5(-90)	0.5(-55) or 0.4(-120)	-408	
3	1.25(-180)+1.5(-55)	0.5(-90) or 0.4(-120)	-355.5	
4	1.25(-180)+1.4(-120)	0.5(-90) or 0.5(-55)	-438	→governs
5	1.0(-180)+1.0(-135)	0.5(-90)+0.25(-55)	-373.75	

We need to include the differential settlement effect as well:

Minimum axial force = -438 + 0.9 (15) = -424.5 kN

Maximum compression force = 423 kN

1.b) Required cross-sectional area
Since in all of the load combinations, column AB is only under compression, we need to design it only for compression.

F_y = 350 MPa = 350 N/mm^2

Compression capacity = 0.75 F_y A = 0.75*350 (N/mm^2) * A (mm^2) = 262.5A (N)

Compression demand = 424.5* 10^3 (N)

262.5A > 424.5* 10^3 → A > 1616.8 mm^2

Therefore, the minimum required area for cross-section is 1650 mm^2

Also, one may apply strength reduction factor φ= 0.9, which results in having the minimum required area for cross-section equal to 1800 mm^2.