MAT 2377 3X (Spring 2011)

Assignment 4
Deadline : Thursday, July 12 in class.

Note : You must give complete details in your solutions. To receive points for the question, you must clearly justify your final answer.

Please answer the following 6 questions.

1. Consider the random sample X_1, X_2, X_3, X_4 from a population with mean μ and standard deviation σ. Consider the following two estimators for μ.

$$\hat{\mu}_1 = 2\,X_1 - X_4, \quad \hat{\mu}_2 = (5/8)\,X_1 + (1/8)X_2 + (1/8)X_3 + (1/8)X_4.$$

   (a) Are they unbiased estimators of μ? (Why?)

   (b) Which of the two estimators is best for estimating μ? (Why?)

2. Suppose that the pull-off force (in pounds) for connectors used in an automobile engine application has a population mean of 75 pounds and a standard deviation of 1.6 pounds. A random sample of $n = 45$ observations will be collected from this population. Approximate the probability that the mean pull-off force of the 45 observations will be between 74.25 and 75.75 pounds.

3. We will perform a preliminary study to describe the variability of a random variable X, the "cold start ignition time" of an automobile engine (in seconds). From the $n = 8$ observations, we compute the following sums :

$$\sum_{i=1}^{8} x_i = 19.22 \quad \text{and} \quad \sum_{i=1}^{8} x_i^2 = 48.114.$$

   (a) Compute the sample mean and the sample standard deviation.

   (b) Using the sample standard deviation as the true value of the population standard deviation σ, compute the required sample size n to be 95% confident that the error of the estimate of the population mean when using the sample mean is less than a tenth of a second (i.e. the error is less than 0.1).

4. A manufacturer of coil springs is interested in implementing a quality control system to monitor his production process. As part of this quality system, it is decided to record the number of nonconforming coil springs in each production batch of size 50. During 40 days of production, 40 batches. Here are numerical summaries of the data.

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 mean        sd 0% 25% 50% 75% 100%  n
9.325 4.485804  3   6   8  12   19 40
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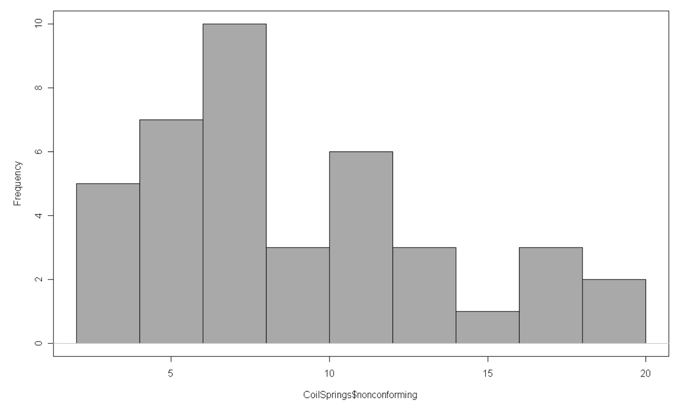
   (a) Use the mean and the median to describe the center of the sample.

(b) Use the range, the inter-quartile range and the standard deviation to describe the dispersion of the sample.

(c) Are there any outliers in the sample ?

(d) Below is a histogram of the number of nonconforming coil springs per batch. Describe the shape of the distribution.



(e) Let X be a random variable representing the number of nonconforming coil springs in a production batch of size 50. Its mean is denoted μ and its standard deviation is σ.

i. Give a point estimate for μ and σ.

ii. Compute the (estimated) standard error for the estimate of the mean.

iii. Construct a 95% confidence interval for μ.

iv. Without computing the 90% confidence interval for μ, would you expect the length of the interval to be smaller or larger than the 95% confidence interval ? Explain your answer in a few sentences.

5. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of $\mu = 12$ kilograms with a standard deviation of $\sigma = 0.5$ kilograms. The company wishes to test the hypothesis

$$H_0 : \ \mu = 12 \quad \text{against} \quad H_1 : \ \mu < 12,$$

with a random sample of $n = 15$ specimens. Assume that thread elongation is normally distribution with $\sigma = 0.5$ kilograms.

(a) Write down a critical region to test these hypotheses in terms of $\overline{x}$ at $\alpha = 5\%$, i.e. find c such that we reject H_0 if $\overline{x} < c$.

(b) Compute the probability of committing an error of type II if in reality $\mu = 11.5$.

6. A postmix beverage machine is adjusted to release a certain amount of syrup into a chamber where it is mixed with carbonated water. A random sample of 25 beverages was found to have a mean syrup content of 1.05 fluid ounces and a standard deviation of 0.15 fluid ounces. Assume that the syrup content of a beverage is normally distributed.

   (a) Give a point estimate for the mean syrup content in a beverage. Give also the (estimated) error of the estimate.

   (b) Construct a 95% confidence interval for the mean syrup content.

   (c) The goal is to have about one ounce of syrup per beverage. Use a critical region to test

   $$H_0: \ \mu = 1 \quad \text{against} \quad H_1: \ \mu \neq 1$$

   at a significance level of $\alpha = 5\%$.

   (d) Compute the p-value for the test in part 6c.