MAT 2377 3X (Spring 2011)

Assignment 3 - solutions

1. Let X be the compressive strength of a sample of cement. So $X \sim N(6050, 100^2)$.

[1] (a) We want

$$P(X < 6200) = \Phi\left(\frac{6200 - 6050}{110}\right) = \Phi(1.36) = 0.9131.$$

[1] (b) We want

$$\begin{aligned} P(5800 < X < 6000) &= \Phi\left(\frac{6000 - 6050}{110}\right) - \Phi\left(\frac{5800 - 6050}{110}\right) \\ &= \Phi(-0.45) - \Phi(-2.27) \\ &= 0.3264 - 0.0116 = 0.3148. \end{aligned}$$

[1] (c) We want to solve

$$0.95 = P(X > x_0) = 1 - \Phi\left(\frac{x_0 - 6050}{110}\right).$$

Which implies

$$\Phi\left(\frac{x_0 - 6050}{110}\right) = 0.05.$$

Using Table III, we get

$$\frac{x_0 - 6050}{110} = -1.64 \quad \Rightarrow x_0 = -1.64(110) + 6050 = 5869.6.$$

[1] (d) We want to solve

$$0.10 = P(X > x_1) = 1 - \Phi\left(\frac{x_1 - 6050}{110}\right).$$

Which implies

$$\Phi\left(\frac{x_1 - 6050}{110}\right) = 0.90.$$

Using Table III, we get

$$\frac{x_1 - 6050}{110} = 1.28 \quad \Rightarrow x_1 = 1.28(110) + 6050 = 6190.8.$$

[1] 2. (a) X has a Poisson distribution with mean $\lambda = \alpha\, t = (10,000)(5) = 50,000$ hits.

[2] (b) Since λ is large, then the normal approximation to the Poisson distribution should be good. We get

$$\begin{aligned} P(X < 49{,}500) &= P(X \le 49{,}499) \\ &= P(X \le 49{,}499.5) \quad \text{(continuity correction)} \\ &\approx \Phi\left(\frac{49{,}499.5 - 50{,}000}{\sqrt{50{,}000}}\right) \quad \text{(normal approximation)} \\ &= \Phi(-2.24) = 0.0125. \end{aligned}$$

Remark :

– 0.5 point for the continuity correction, i.e. using 49,499.5

– 1 point for properly standardizing

– 0.5 point for the table lookup.

[1] (c) T has an Erlang distribution with $r = 100{,}000$ and $\lambda = 10{,}000$.

[2] (d) The mean of T is $E[T] = r/\lambda = 10$ days and its standard deviation is

$$\sigma_T = \sqrt{V[T]} = \sqrt{r/\lambda^2} = 31.6228 \quad \text{days.}$$

[1] (e) The event that we require more than 30 seconds to observe 3 hits is equivalent that there are at most 2 hits in 30 seconds. Let Y be the number of hits in 30 seconds (or equivalently in 30/86,400 days). Y has a Poisson distribution with mean $\lambda = \alpha\, t = (10{,}000)(30/86{,}400) = 3.4722$. We want

$$P(Y \le 2) = e^{-3.4722}\left[\frac{(3.4722)^0}{0!} + \frac{(3.4722)^1}{1!} + \frac{(3.4722)^2}{2!}\right] = 0.3260.$$

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