MAT 2377 3X (Spring 2011)

Assignment 1 - Solutions

[3] 1. Define the events A="engine repair under warranty" and B="drive train under warranty". In the statement of the question, we are given $P(A) = 0.87$, $P(B) = 0.36$ and $P(A \cap B) = 0.29$.

(a) We want

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.87 + 0.36 - 0.29 = 0.94.$$

(b) We want

$$P(A') = 1 - P(A) = 1 - 0.87 = 0.13.$$

(c) We want

$$P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.29 = 0.71.$$

[5] 2. Define the events A="high conductivity" and B="high strength".

(a) We want

$$P(A \cap B) = \frac{N(A \cap B)}{N(S)} = \frac{74}{102} = 0.7255.$$

(b)

$$P(A' \cup B') = \frac{N(A' \cup B')}{N(S)} = \frac{16 + 4 + 8}{102} = \frac{28}{102} = 0.27451.$$

(c)

$$P(B'|A') = \frac{N(B' \cap A'}{N(A')} = \frac{4}{20} = 0.2.$$

(d) Since $N(A' \cap B') = 4 \neq 0$, then $A' \cap B' \neq \emptyset$. Thus, the events A' and B' are not mutually exclusive.

(e) Since $P(A' \cap B') = 4/102 = 0.03922$, but

$$P(A')\,P(B') = \left(\frac{20}{102}\right)\left(\frac{12}{102}\right) = 0.02307,$$

then $P(A' \cap B') \neq P(A')P(B')$. Therefore A' and B' are not independent (i.e. they are dependent).

[3] 3. Refer to Question 2. Suppose that we randomly select 3 of the 20 strands of wire with low conductivity.

(a) The probability that all three have high strength is

$$\frac{\binom{16}{3}\binom{4}{0}}{\binom{20}{3}} = 0.49123.$$

Alternative Solution : We could answer using the multiplication rule. Let A_i be the event the the ith selected strand has high strength. We want

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3) &= P(A_3|A_1 \cap A_2)P(A_1 \cap A_2) \\ &= P(A_3|A_1 \cap A_2)P(A_2|A_1)P(A_1) \\ &= (14/18)(15/19)(16/20) = 0.49123 \end{aligned}$$

(b) The probability that at most one will have high strength is the sum of the probability that none will have high strength and the probability that exactly one will have high strength, which is

$$\frac{\binom{16}{0}\binom{4}{4}}{\binom{20}{3}} + \frac{\binom{16}{1}\binom{4}{3}}{\binom{20}{3}} = 0.05702.$$

[3] 4.

Let A be the event that a person is accident prone and B the event that a new policy holder has an accident within a year of purchasing his/her policy. We are given the following probabilities : $P(B|A) = 0.35$ and $P(B|A') = 0.05$.

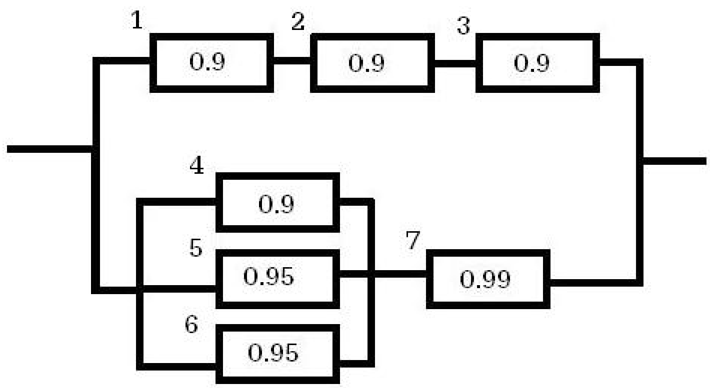
(a) We are given $P(A) = 0.3$. We want

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = (0.35)(0.3) + (0.05)(0.7) = 0.14.$$

(b) We want

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.35)(0.3)}{(0.14)} = 0.75.$$

[4] 5. We will number the components.



Define the events E_i="component i works". We will decompose the circuit into sub-circuits.

Consider the components 1, 2 and 3 which are assembled into series. We will denote this component as 8. So

$$P(E_8) = P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3) = (0.9)^3 = 0.729.$$

Consider the components 4, 5 and 6 which are assembled into parallel. We will denote this component as 9. So

$$\begin{aligned} P(E_9) &= P(E_4 \cup E_5 \cup E_6) \\ &= 1 - P(E_4')P(E_5')P(E_6') \\ &= 1 - (0.1)(0.05)(0.05) = 0.99975. \end{aligned}$$

Consider the components 9 and 7 which are assembled in series. We will denote this component as 10. So

$$P(E_{10}) = P(E_9 \cap E_7) = P(E_9)P(E_7) = (0.99975)(0.99) = 0.9897525.$$

The circuit is composed of component 10 and 8 which are assembled in parallel. Therefore, the probability that the circuit operates is

$$P(E_8 \cup E_{10}) = 1 - P(E_8')P(E_{10}') = 1 - (1 - 0.729)(1 - 0.9897525) = 0.9972.$$

[6] 6. Using the multiplication principle, we get

$$P(A) = \frac{N(A)}{N(S)} = \frac{1 \times 3 \times 5 \times 3 \times 5}{4 \times 3 \times 5 \times 3 \times 5} = \frac{1}{4} = 0.25,$$

$$P(B) = \frac{N(B)}{N(S)} = \frac{4 \times 3 \times 4 \times 3 \times 5}{4 \times 3 \times 5 \times 3 \times 5} = \frac{4}{5} = 0.8.$$

and

$$P(A \cap B) = \frac{1 \times 3 \times 4 \times 3 \times 5}{4 \times 3 \times 5 \times 3 \times 5} = \frac{4}{20} = 0.2.$$

(a) We want

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.8 - 0.2 = 0.85.$$

(b) We want

$$P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.2 = 0.8.$$

(c) We want

$$\begin{aligned} P(A \cup B') &= P(A) + P(B') - P(A \cap B') \\ &= P(A) + [1 - P(B)] + [P(A) - P(A \cap B)] \\ &= 0.25 + (1 - 0.8) + (0.25 - 0.2) = 0.5 \end{aligned}$$

Remark : The computation of $P(A)$, $P(B)$ and $P(A \cap B)$ get one point each. Instead of using the above counting techniques, it is reasonable to have assigned probabilities as follows :

$$P(A) = \frac{1}{4} \quad \text{(since red is 1 colour from 4);}$$

$$P(B) = \frac{4}{5} \quad \text{(since the smallest font size is 1 font size from 5).}$$

Also, you could have used independence to compute

$$P(A \cap B) = P(A)\,P(B) = \frac{4}{20} = 0.2.$$

However, since it was not stated that A and B are independent, to get the point for $P(A \cap B) = 0.2$ by using independence, you must have written a short statement like

"Since the colour and the font size are independently chosen, then A and B are independent events",

to justify the use of independence.

[/**24**]