General

$\rho = m/V$ specific density $= \vartheta = V/m$ 1 bar = 100 000 Pa

$T(K) = T(°C) + 273\ K$

Ideal Gas

$pV = mRT$ $\vartheta = \frac{RT}{p}$ $\rho = \frac{p}{RT}$ useful arrengement: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

~~Gases~~

Energy for all systems

Change in internal energy: $(U_2 - U_1) = m(C_v(T_2 - T_1))$

" " " " per kg: $(u_2 - u_1) = C_v(T_2 - T_1)$

" " enthalpy : $(H_2 - H_1) = m\,C_p(T_2 - T_1)$

" " " per kg: $h_2 - h_1 = C_p(T_2 - T_1)$

Gas constant $R = C_p - C_v$

$\gamma = C_p / C_v$

For Polytropic process

$P_1 V_1^n = P_2 V_2^n$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{n-1}{n}\right)} = \left(\frac{V_1}{V_2}\right)^{n-1}$

For adiabatic Process (Q=0)

$P_1 V_1^\gamma = P_2 V_2^\gamma$, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

For constant volume heating: $Q = U_2 - U_1 = m C_v (T_2 - T_1)$

For constant pressure heating: $Q = H_2 - H_1 = m C_p (T_2 - T_1)$

For Mixed Saturation

$u = x u_g + (1-x) u_f$

$h = x h_g + (1-x) h_f$

$v = x v_g + (1-x) v_f$

Non-Flow Problems

$Q - W = U_2 - U_1$

Per unit mass: $q - w = u_2 - u_1$

$W = \int p\,dV$

When	Work
$V = C$	$W = 0$
$P = C$	$W = p(V_2/V_1) = W$
$pV = C$	$P_1 V_1 \ln(V_2/V_1) = W$
$pV^n = C$	$W = (P_1 V_1 - P_2 V_2)/n-1$
$pV^\gamma = C$	$W = (P_1 V_1 - P_2 V_2) / \gamma - 1$

Steady Flow Problems

$$Q - W = \dot{m}_{out}\left(h_1 + \tfrac{1}{2}V^2 + gz\right)_{out} - \dot{m}_{in}\left(h_2 + \tfrac{1}{2}V^2 + gz\right)_{in}$$

Entropy

Closed system entropy balance

$$\underbrace{S_2 - S_1}_{\text{entropy change}} = \underbrace{\int_1^2 \left(\frac{\delta Q}{T}\right)}_{\text{entropy transfer}} + \underbrace{\sigma}_{\text{entropy balance}}$$

Closed system entropy rate balance

$$S_2 - S_1 = \sum \frac{\dot{Q}}{T} + \sigma$$

Control Volumen entropy rate balance

$$\Delta S = \sum \frac{\dot{Q}}{T} + \sum \dot{m}_i s_i - \sum \dot{m}_o s_o + \dot{\sigma}$$, steady state when $\Delta S = 0$

Isentropic Turbine efficiency

$$\eta = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}, \quad \boxed{s_1 = s_{2s}}$$

Isentropic compressor efficiency

$$\eta = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Change in entropy

$$s(T_2, V_2) - s(T_1, V_1) = \int_{T_1}^{T_2} C_v(T)\frac{dT}{T} + R\ln\frac{V_2}{V_1}$$

If C_v = constant, $s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

When $s_1 = s_2$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}, \quad \frac{V_2}{V_1} = \frac{V_{r2}}{V_{r1}}$$

When $s_1 = s_2$, constant heat ratio k

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

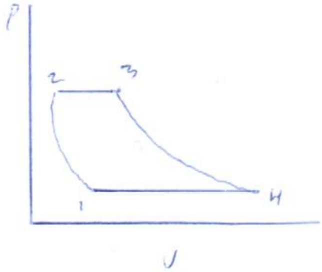
$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{k}$$

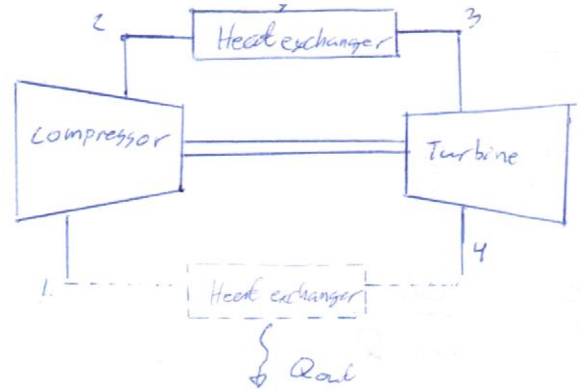
use for turbines & compressors

Heat exchanger

$$\eta_{HX} = \frac{T_2 - T_1}{T_x - T_1}$$

Brayton Cycle





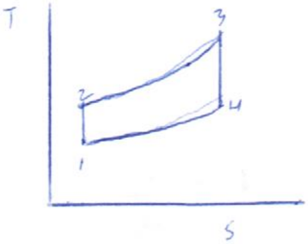
$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$s_1 = s_2$

$s_3 = s_4$

$P_2 = P_3$

$P_4 = P_1$



${}_1W_2 = h_2 - h_1$

${}_2q_3 = h_3 - h_2$

${}_3W_4 = h_3 - h_4$

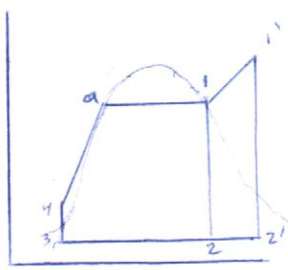
${}_4q_1 = h_4 - h_1$

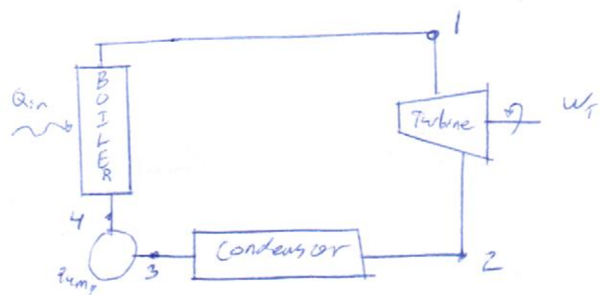
$$\eta_{cycle} = \frac{{}_3W_4 - {}_1W_2}{{}_2q_3} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$\eta_{cycle} = 1 - \frac{1}{(P_2/P_1)^{\frac{k-1}{k}}} = 1 - \frac{1}{r^{\left(\frac{k-1}{k}\right)}}, \quad r = P_2/P_1$$

$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1} \qquad \eta_{turb} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

Rankin





$$\eta = \frac{W_{out}}{Q_{in}} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$

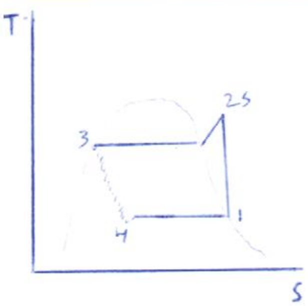
$$\text{Back work ratio} = BWR = \frac{h_4 - h_3}{h_1 - h_2}$$

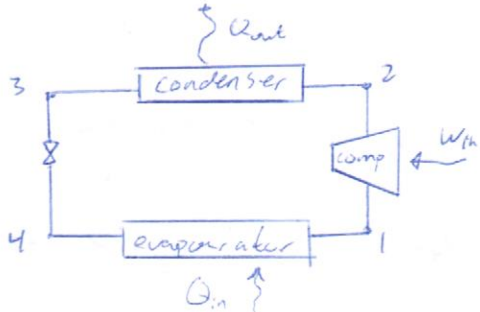
$W_t = h_1 - h_2$ $\quad W_p = h_4 - h_3$

$q_{out} = h_2 - h_3$ $\quad q_{in} = h_1 - h_4$

$P_2 = P_3$ $\quad P_4 = P_1$ $\quad W_{pump} = v(P_2 - P_1)$

Rankin VC Refridgeration





$$COP_{ref} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$COP_{hp} = \frac{h_2 - h_3}{h_2 - h_1}$$

$P_2 = P_3$ $\quad {}_1q_2 = 0$ $\quad {}_1W_2 = h_2 - h_1$

$h_4 = h_3$ $\quad {}_2q_3 = h_2 - h_3$ $\quad {}_2W_3 = 0$

${}_3q_4 = 0$ $\quad {}_3W_4 = 0$

${}_4q_1 = h_1 - h_4$ $\quad {}_4W_1 = 0$

Refridgeration Cycle

Coefficient of Performance

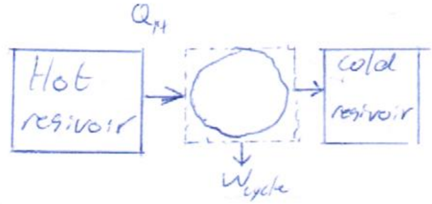
Refridgeration : $COP = \frac{Q_c}{W_{cycle}} = \frac{Q_c}{Q_H - Q_c}$

Heat Pump : $COP = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_c}$

Power Cycle

$E_2 - E_1 = {}_1Q_2 - {}_1W_2 = 0$

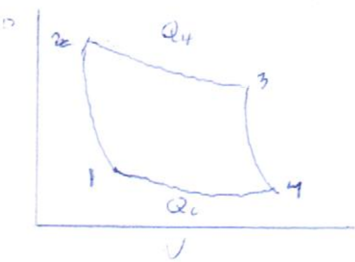
$W_{cycle} = Q_H - Q_c$



$\eta_{max} = 1 - \frac{T_c}{T_H}$

$\eta = \frac{W_{cycle}}{Q_H} = \frac{Q_H - Q_c}{Q_H} = 1 - \frac{Q_c}{Q_H}$

Carnot Cycle



1→2 - adiabatic compression until gas reaches T_h

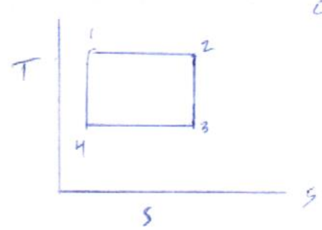
2→3 - isothermal expansion recieving Q_H $(T_2 = T_3 = T_H)$

3→4 - adiabatic expansion until gas reaches T_c

4→1 - Isothermal compression discharging Q_c $(T_4 = T_1 = T_c)$

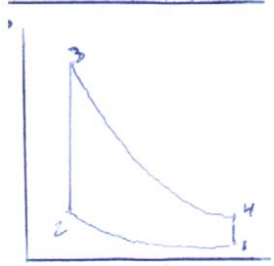
$\eta = 1 - \frac{T_c}{T_H} = \frac{W_{cycle}}{Q_H}$ } carnot power cycle

carnot refrigeration cycle { $\eta = \beta_{carnot} = \frac{T_c}{T_H - T_c} = \frac{Q_H}{W_{cycle}}$



carnot heat pump { $\gamma_{carnot} = \frac{T_H}{T_H - T_c}$

Otto Cycle



$s_1 = s_2$

$s_3 = s_4$

${}_1Q_2 = 0$

${}_3Q_4 = 0$

${}_1W_2 = u_2 - u_1$

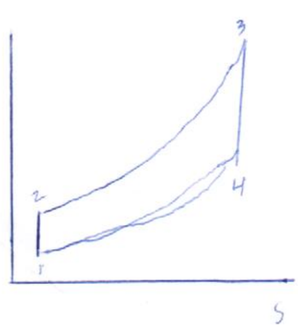
${}_2q_3 = u_3 - u_2$

${}_3W_4 = u_3 - u_4$

${}_4q_1 = u_4 - u_1$

$\eta = \frac{W_{34} - W_{12}}{q_{23}}$

$= \frac{(u_3 - u_2) - (u_4 - u_1)}{u_3 - u_2}$



$U_2 = U_3$

$U_3 = U_4$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$

~~$\eta = 1 - r^{1-k}$~~

~~$r = V_1/V_2$~~

$\eta = 1 - \frac{1}{r^{k-1}}, \quad r = V_1/V_2$

Heat Transfer: Conduction

$\dot{Q} = -kA \frac{dT}{dx} \Rightarrow \dot{Q}_2 = kA \frac{(T_1 - T_2)}{x_1}$

x = distance through material, sometimes shown as t

A = XC area

Convection

$\dot{Q}_{conv} = hA(T_s - T_\infty)$, h = convection heat transfer coefficient

Nusselt = $Nu_x = \frac{hx}{k}$ $\qquad Nu_x = 0.332\, Re^{1/2}\, Pr^{1/3}$

Reynolds = $Re_x = \frac{\rho U_\infty x}{\mu}$

Prandtl: $Pr = \frac{\nu}{\alpha} = \frac{(\mu/\rho)}{(k/\rho C_p)}$

x = distance from edge of plane
ρ = density
U = fluid viscosity (m/s)
μ = dynamic viscosity (kg/m·s)
ν = kinematic viscosity (m^2/s)
$T_f = 0.5(T_w + T_\infty)$, used to get values

Radiation Blackbody

$\dot{Q}_{rad} = \sigma A T_s^4$, where $\sigma = 5.6704 \times 10^{-8}\ W/m^2K^4$, T_s = surface temp
$= 0.1714\ Btu/hr \cdot ft^2 \cdot R^4$

~~$\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident}$, α = Absorptivity~~

Grey Body Radiation

$Q_{rad} = \varepsilon \sigma A (T_s^4 - T_{surround}^4)$, ε = emmisivity, $0 < \varepsilon < 1$, black body $\varepsilon = 1$

$\dot{Q}_{adsorbed} = \alpha Q_{incident}$ $\qquad \alpha$ = absorptivity

~~Combined Transfer~~

$Q_{incident} = A_{surr} \sigma T_{surr}^4$ $\qquad$ For small surface $Q_{adsorbed} = \varepsilon A_s (\sigma T_{surr}^4)$

$Q_{adsorbed} = \alpha A_s \frac{Q_{incident}}{A_{surround}} = \cancel{\varepsilon A_s}\, \alpha A_s (\sigma T_{surr}^4)$

$Q_{net} = \varepsilon \sigma A_1 (T_1^4 - T_2^4)$

$Q_{total} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4) + hA_s(T_s - T_\infty)$

Combined Transfer

Transfer methods at boundaries are equal