**SOLUTION**

1. In the 4-mesh, 5-node network shown in Fig.1, it is required to find the power P dissipated in the 3 Ω resistor connected between nodes 2 & 3.
   (a) You are expected to choose only between mesh- or node- analysis methods. State the more efficient method of the two that you would use to obtain the required result. Explain the reason for your choice. [**Do not use Source Transformation, Thevenin/Norton or Superposition !**].
   (b) Using the chosen method, solve the problem of finding P, clearly explaining each procedural step, before doing the associated calculation.
   (c) How would your answer to (a) above change, if the two ideal voltage sources were replaced with ideal current sources ?

( 2 + 6 + 2 marks)

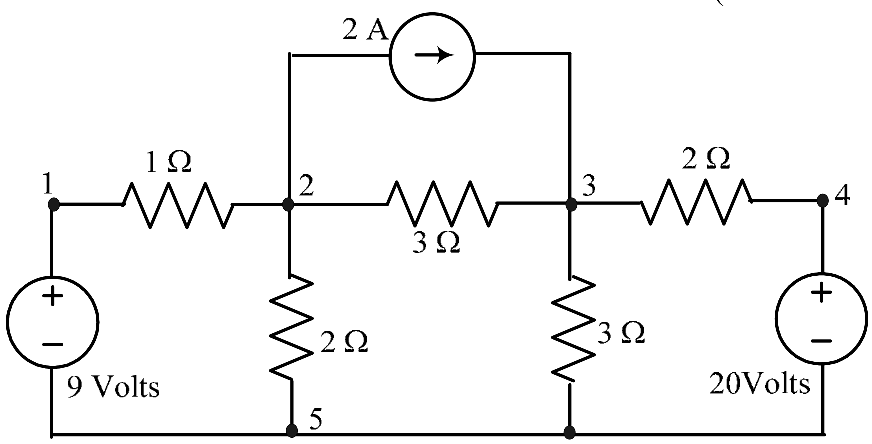



Figure 1

Solution:

(a) In order to find P , we require the voltage across the resistor ($V_{23}$ or $V_{32}$) or the current through it ( $I_{23}$ or $I_{32}$).Since there are 2 voltage constraints and 1 current constraint, Mesh Analysis will require 3 mesh equations (lower 3 meshes) , and Node Analysis will require 2 nodal equations (nodes 2 & 3 with 5 as ground.)

Nodal analysis is preferable since it requires the lesser number of equations.

(b) Using Node 5 as the ground , the nodal equations at nodes 2 and 3 are, respectively,:

$$\frac{V_2 - 9}{1} + \frac{V_2 - V_3}{3} + \frac{V_2}{2} + 2 = 0 \quad \text{or} \quad 11V_2 - 2V_3 = 42$$

and

$$\frac{V_3 - V_2}{3} + \frac{V_3}{3} + \frac{V_3 - 20}{2} - 2 = 0 \quad \text{or} \quad -2V_2 + 7V_3 = 72$$

Solving ,

**SOLUTION**

$$V_2 = \frac{(42)(7)-(72)(-2)}{(11)(7)-(-2)(-2)} = \frac{438}{73} = \underline{6\text{ volts}}$$

and $$V_3 = \frac{(11)(72)-(-2)(42)}{(11)(7)-(-2)(-2)} = \frac{876}{73} = \underline{12\text{ volts}}$$

From the above node voltages $V_{32} = V_3 - V_2 = 12 - 6 = 6$ volts

and $P = (V_{32})^2/R = (6)^2/3 = 36/3 = \underline{12\text{ Watts}}$

(c) If the 2 ideal voltage sources are replaced by ideal current sources, there will be 3 current constraints and mesh analysis will require only 1 mesh equation(for the central mesh), whereas nodal analysis (with 5 as ground) will require 2 equations at nodes 2 & 3.. Clearly, MA will then be the preferred method of analysis.

**SOLUTION**

2. For the op-amp circuit shown in Figure 2, (a) write the nodal equations at nodes 1 & 2, (b) determine voltage $v_o$ and currents $i_f$ and $i_o$ .

( 2 + 2 + 2 + 2 + 2 marks)

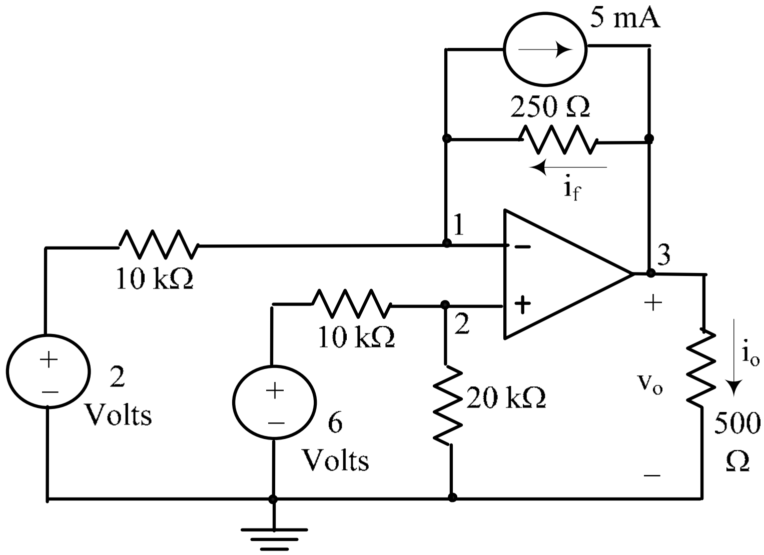



Figure 2

Solution: (a) With currents in **mA**,

At Node 1 , $\frac{v_1 - 2}{10} + \frac{v_1 - v_3}{0.25} + 5 = 0$, $v_3 = v_o$ and $v_1 = v_2$

At Node 2 , $\frac{v_2 - 6}{10} + \frac{v_2}{20} = 0$,

(b) From the Node 2 equation (or by voltage division),

$$v_1 = v_2 = 6\,[\,20/30\,] = 4 \text{ volts.}$$

Substitution for $v_1$ in the Node 1 equation yields:

$\frac{4-2}{10} + \frac{4 - v_o}{0.25} + 5 = 0$ or $2 + 40(4 - v_o) + 50 = 0$ or $v_o = \underline{5.3 \text{ volts}}$

$i_f = (v_o - 4)/0.25$ mA = $1.3/0.25 = \underline{5.2 \text{ mA}}$

$i_o = v_o/0.5$ mA = $5.3/0.5 = \underline{10.6 \text{ mA}}$

**SOLUTION**

3. The network to the left of terminals a-b , in Figure 3, is to be suddenly connected to the <u>initially uncharged</u> capacitor by means of the normally-open switch S .

   (a) With the switch S open, determine [**Hint: Consider using Mesh Analysis**]
   (i) the open-circuit voltage $v_{ab}(oc)$ across terminals a-b
   (ii) the current $i_{ab}(sc)$ that would flow between terminals a-b if these terminals are short circuited
   (iii) the Thevenin equivalent circuit or the Norton equivalent circuit with respect to terminals a-b
   (b) If the switch S is closed at time t = 0, determine the capacitor voltage v(t), t > 0.

( 2 + 2 + 2 + 4 marks)

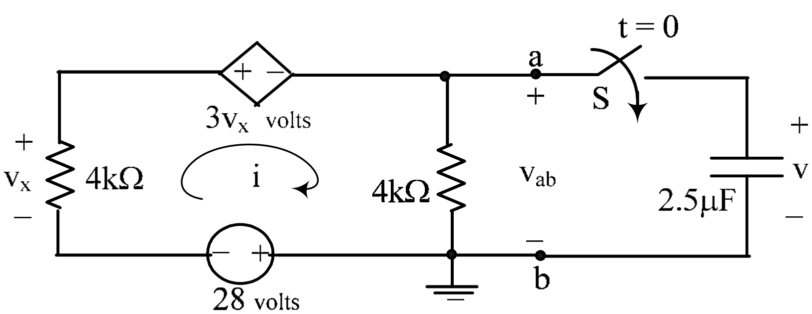
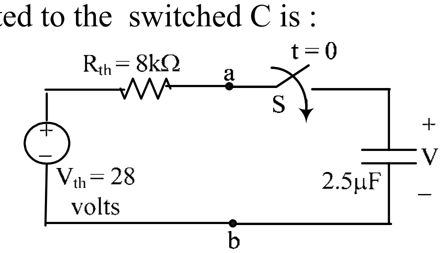



Figure 3

<u>Solution:</u> (a)

(i) Using current i (**mA**) , $v_x = -4i$ volts & $v_{ab}(oc) = v_{th} = 4i$ volts
KVL with a-b OC : $28 - (-4i) + 3(-4i) + 4i = 0$ or $i = 28/4 = 7$ mA ,
$v_{ab}(oc) = v_{th} = 4i$ = **<u>28 volts</u>**
(ii) KVL with a-b SC [ $i = i_{ab}(sc) = i_N$]:
$28 - (-4i) + 3(-4i) = 0$ or $i_{ab}(sc) = i_N = = 28/8 =$ **<u>3.5 mA</u>**
$R_{Th} = V_{ab}(oc)/ i_{ab}(sc) = 28/3.5m =$ **<u>8 kΩ</u>**

(iii) (The TEC connected to the switched C is :




(b) $\tau = (8000)(2.5\mu) = 20$ msec $V(t) = Ae^{-50t} + 28$ , $V(0) = 0 = A + 28$, or $A = -28$
Hence $V(t) = \underline{28 [1 - e^{-50t}]}$ volts

4. For the RLC circuit shown in Figure 4, L = 360 mH, C = 100μF
   (a) Obtain the governing differential equation for the voltage v(t).
   (b) Find the value of R which will result in a critically-damped natural response .
   (c) If R = 30 Ω obtain the natural response v(t), t > 0 if v(0)= 3 Volts and i(0) = 0.2 A. ( 4 + 2 + 4 marks)

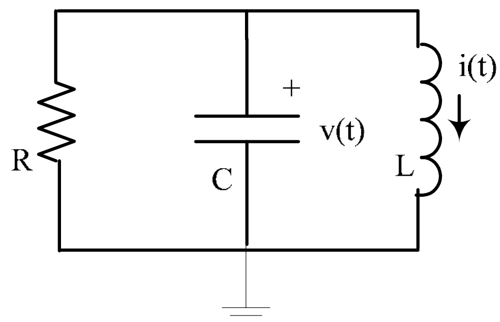



Figure 4

<u>Solution:</u> (a) KCL : $\frac{v}{R} + C\frac{dv}{dt} + i = 0$ , KVL: $L\frac{di}{dt} - v = 0$

To obtain the differential equation in v , obtain di/dt from the KCL equation and substitute into the KVL equation.

ie $L\left[-\frac{1}{R}\frac{dv}{dt} - C\frac{d^2v}{dt^2} - v\right] = 0$ or $\frac{d^2v}{dt^2} + \frac{1}{CR}\frac{dv}{dt} + \frac{1}{LC}v = 0$

$\alpha = \frac{1}{2CR}$ , $\omega_n^2 = \frac{1}{LC} = \frac{1}{36(10)^{-6}}$ (or $\omega_n$= 166.667 rad/s)

(b) For critical damping $\alpha = \omega_n$ or $R = \frac{1}{2}\sqrt{\frac{L}{C}}$ = 60/2 = <u>30 Ω</u>

(c) For R = 30Ω, $v(t) = [A_1 + A_2 t]e^{-166.7t} + 0$
$v(0) = 3 = [A_1 + 0]1 + 0$ or $A_1 = 3$

From the KCL equation $3/30 + 100(10)^{-6}\frac{dv}{dt}(0) + 0.2 = 0$

or $\frac{dv}{dt}(0) = -0.3/100(10)^{-6} = -3000$

Therefore $-166.667A_1 + A_2 = -3000$ or $A_2 = -2500$

$v(t) = \underline{[3 - 2500t]e^{-166.7\,t}}$ volts

**SOLUTION**

5. An AC circuit operating at a radian frequency $\omega = 4$ rad/sec is shown (in the time domain) in Figure 5.
   (a) Draw the phasor domain equivalent circuit and determine the phasor current $I_1$
   (b) Hence determine the voltage $v_b(t)$ in the form
   $$v_b(t)= V\cos(\omega t \pm \phi) \text{ , volts} \qquad (4 + 4 + 2 \text{ marks})$$

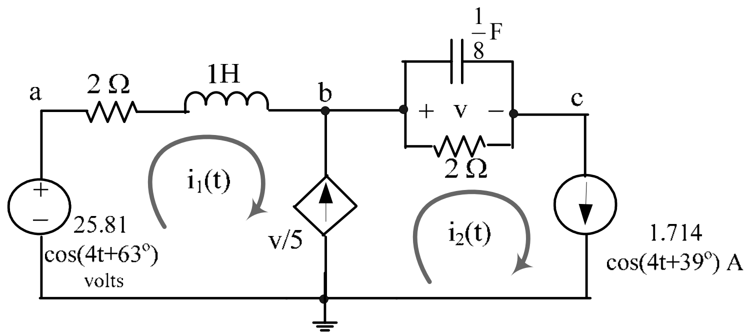
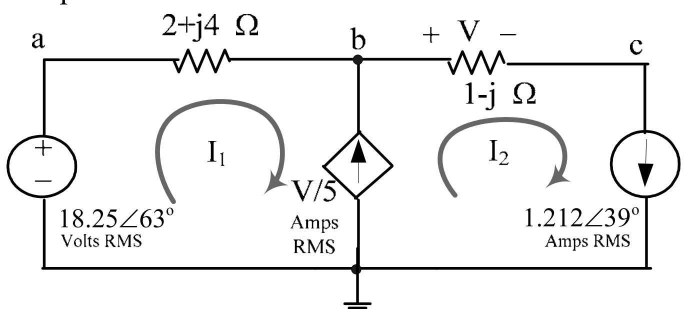



Figure 5

Solution: (a) $Z_{ab} = 2 + j(4)(1) = 2 + j4$ ohms $= 4.47\angle 63.43^\circ$
and $Z_{bc} = 2$ P $[-j/(4)(1/8)] = 2$ P $(-j2) = -j4/(2-j2) = -j2(1-j) = 1-j$ ohms
$= 1.414\angle -45^\circ$
The phasor equivalent circuit is shown below




Ohm's Law : $V = Z_{bc}I_2 = 1.414\angle -45^\circ(1.212\angle 39^\circ) = 1.714\angle -6^\circ$ volts (2.424pk)

And $I_2 - I_1 = V/5$

Hence $I_1 = I_2 - V/5 = 1.212\angle 39^\circ) - 0.343\angle -6^\circ$
$= 0.942 + j\,0.763 - 0.341 + j\,0.036 = 0.6 + j\,0.799$
$= 1\angle 53.1^\circ$ (1.414 pk)

(b) KVL: $V_b = 18.25\angle 63^\circ - (2+j4)I_1 = 18.25\angle 63^\circ - 4.47\angle 116.53^\circ$
$\approx 8.285 + j\,16.26 + 1.99 - j\,4 = 10.275 + j\,12.26$
$= 15.996\angle 50^\circ \approx \mathbf{16}\angle 50^\circ$ vRMS (22.62 pk)

Hence $v_b(t) = \underline{16\cos(4t + 50^\circ)}$ volts RMS
$= \underline{22.62\cos(4t + 50^\circ)}$ volts peak

**SOLUTION**

6. The circuit of Figure 6 shows a series RLC load connected to an AC source whose a voltage is 972 volt RMS at a radian frequency of 2765 radians/sec. Determine :
   (a) the power-factor of the load , also stating whether the PF is lagging or leading
   (b) the RMS current delivered by the source
   (c) the average power P and the reactive power Q in the load
   (d)A capacitor C is to be connected in parallel with the load (as shown, by closing the switch S) in order to change the overall load PF to 0.71 lagging. Determine the required value of C .

( 3 + 1 + 2 + 4 marks)

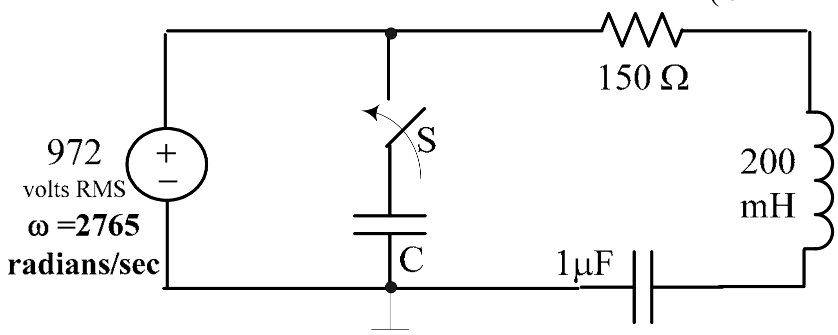



Figure 6

Solution: (a) The load impedance $Z_L = 150 + j\,(2765)(0.2) - j\,/\,2765(10)^{-6}$
$= 150 + j553 - j\,361.7$
$= 150 + j191.3 = 243.1\angle 51.9^o$

The load power factor is $\cos 51.9^o$ = 0.617 lagging

(lagging,since the impedance angle is positive)

(b) The RMS current delivered by the source = 972/243.1= 3.998 A ≈ 4 A

(c) Average power $P = I^2\,\text{Re}\,Z_L \approx 16\,(150) =$ 2.4 kW
Reactive power $Q = I^2\,\text{Im}\,Z_L \approx 16\,(191.3) \approx$ 3.06 kvar

(d) For a PF = 0.71 lagging , $\theta = \cos^{-1}(0.71) = 44.77^o$ and $\tan\theta = 0.9918$

$Q_{new} = P\tan\theta_{new} = 2400(0.9918) = 2380.4$ var

Ie $\Delta Q = 2380.4 - 3060 = -679.6 = -\omega C V^2$

or $C = 679.6/(2765)(972)^2 \approx$ **0.26 μF**