Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Solution to Assignment 2, Fall 2014
Probability and Statistics for Engineers.

585, av. King-Edward
Ottawa (Ontario) K1N 6N5 Canada

585 King Edward Avenue
Ottawa, Ontario K1N 6N5 Canada

(613) 562-5864 • Téléc./Fax (613) 562-5776
Courriel/Email: uomaths@science.uottawa.ca

3.12

Probability of 2 or more of 4 engines operating when $p = 0.6$ is

$$P(X \geq 2) = 1 - P(X \leq 1) = 0.8208,$$

where $X \sim Binomuial(4, 0.6)$ and the probability of 1 or more of 2 engines operating when $p = 0.6$ is

$$P(X \geq 1) = 1 - P(X = 0) = 0.8400$$

where $X \sim Binomial(2, 0.6)$. Therefore a two engine airplane has a higher chance for a successful flight.

3.20. From the binomial table for $n = 20$;

(a) $p = 0.20, P(X \geq x) \leq 0.5$ and $P(X < x) > 0.5$ yields $x = 5$.

(b) $p = 0.80, P(Y \geq y) \geq 0.8$ and $P(Y < y) < 0.2$ yields $y = 15$.

3.26. Let X be the number of defective items. Then for (a)

$$P(X = 0) = \frac{\binom{3}{0}\binom{22}{3}}{\binom{22}{3}} \approx 0.67$$

Similarly for (b)

$$P(X = 1) = \frac{\binom{1}{1}\binom{24}{2}}{\binom{25}{3}} = 0.12$$

3.34.

From the negative binomial distribution, we get :

$$\binom{7}{1}(1/6)^2(5/6)^6 = 0.0651.$$

3.52. (a)

$$P(X \leq 1|\lambda t = 2) = 0.4060.$$

and (b) $\mu = \lambda t = (2)(5) = 10$ and

$$P(X \leq 4|\lambda t = 10) = 0.0293.$$

3.63 (a) $z = (15 - 18)/2.5 = -1.2$;

$$P(X < 15) = P(Z < -1.2) = 0.1151.$$

(b) $z = -0.76$, $k = (2.5)(-0.76) + 18 = 16.1$.

(c) $z = 0.91$, $k = (2.5)(0.91) + 18 = 20.275$.

(d) $z_1 = (17 - 18)/2.5 = -0.4$, $z_2 = (21 - 18)/2.5 = 1.2$;

$$P(17 < X < 21) = P(-0.4 < Z < 1.2) = 0.8849 - 0.3446 = 0.5403.$$

Problem 2.

$P(\text{Max}(X, Y, Z) > 1) = 1 - P(\text{Max}(X, Y, Z) \leq 1) = P(X \leq 1)P(Y \leq 1)P(Z \leq 1)$

$$= \left(\int_0^1 \frac{1}{\alpha}\exp(-x/\alpha)dx\right)^3 = (1 - \exp(-1/\alpha))^3 = 0.05.$$

Solve for α. We get

$$1 - \exp(-1/\alpha) = 0.3684031 \quad \Rightarrow \quad \exp(-1/\alpha) = 0.6315969.$$

Therefore

$$-1/\alpha = \ln 0.6315969 = -0.4595039.$$

This gives $\alpha = 2.17626$.