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Solution to Assignment 2, Fall 2014
Probability and Statistics for Engineers.

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2.5. We must choose c so that $\sum_x f(x) = 1$. This means

(a)

$$1 = c((0^2+4)+(1^2+4)+(2^2+4)+(3^2+4)) = c(30)$$

and so $c = \frac{1}{30}$.

(a)

$$1 = c(\frac{2!}{0!2!}\frac{3!}{3!0!} + \frac{2!}{1!1!}\frac{3!}{2!1!} + \frac{2!}{2!0!}\frac{3!}{1!2!}) = c(1+6+3) = 10c$$

and so $c = \frac{1}{10}$.

2.10.

(a)

$$\mathbb{P}[T=5] = F(5) - F(4) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}.$$

(b)

$$\mathbb{P}[T>3] = 1 - \mathbb{P}[T \leq 3] = 1 - F(3) = \frac{1}{2}.$$

(c)

$$\mathbb{P}[1.4 < T < 6] = F(6) - F(1.4) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

(d)

$$\mathbb{P}[T \leq 5 | T \geq 2] = \frac{\mathbb{P}[\{T \leq 5\} \cap \{T \geq 2\}]}{\mathbb{P}[T \leq 2]} = \frac{2}{3}$$

2.15.

(a) We must choose k so that $\int f(x)dx = 1$. This means

$$1 = \int_0^1 k\sqrt{x}dx = k\frac{2}{3}x^{1.5}|_0^1 = \frac{2k}{3}$$

and so $k = \frac{3}{2}$.

(b) Recall $F(x) = \int_0^x f(y)dy$, so

$$F(x) = \int_0^x \frac{3}{2}\sqrt{y}dy = y^{1.5}.$$

We conclude

$$\mathbb{P}[0.3 < X < 0.6] = F(0.6) - F(0.3) = 0.6^{1.5} - 0.3^{1.5} \approx 0.3.$$

2.22.

(a) We must choose k so that $\int f(x)dx = 1$. This means

$$1 = \int_{-1}^1 k(3 - x^2)dx = k(6 - \frac{2}{3}) = \frac{16k}{3}$$

and so $k = \frac{3}{16}$.

(b)

$$\mathbb{P}[X \leq \frac{1}{2}] = \int_{-1}^{0.5} \frac{3}{16}(3 - x^2)dx = \frac{99}{128}$$

.

(c)

$$\mathbb{P}[|X| > 0.8] = 1 - \mathbb{P}[|X| \leq 0.8] = \int_{-0.8}^{0.8} \frac{3}{16}(3 - x^2)dx = \frac{3}{16}(\frac{72}{75} - \frac{32}{75}) = \frac{1}{10}$$

.

2.54.

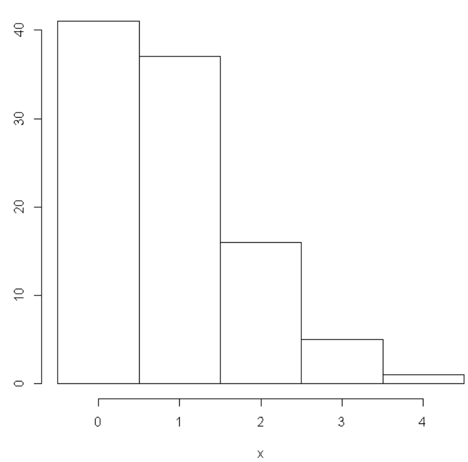
Let X be the amount of money won during a game. If the game is fair, the cost should be equal to $\mathbb{E}[X]$, which is

$$\mathbb{E}[X] = 0\mathbb{P}[X = 0] + 3\mathbb{P}[X = 3] + 5\mathbb{P}[X = 5] = 0 + 3\frac{8}{52} + 5\frac{8}{52} = \frac{16}{13}$$

.

2.75

(a) See :



(b) We have

$$\mathbb{E}[X] = (0)(0.41) + (1)(0.37) + (2)(0.16) + (3)(0.05) + (4)(0.01) = 0.88$$

.

(c) We have

$$\mathbb{E}[X^2] = (0)^2(0.41) + (1)^2(0.37) + (2)^2(0.16) + (3)^2(0.05) + (4)^2(0.01) = 1.62$$

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2.109

(a) For $x > 0$,

$$f(x) = F'(x) = \frac{1}{50}e^{-\frac{x}{50}}$$

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(b) Write X for the lifespan of a component, in hours.

$$\mathbb{P}[X > 70] = 1 - \mathbb{P}[X \leq 70] = 1 - (1 - e^{-\frac{70}{50}}) = 0.247$$

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2.129

(a) We note that X is Binomial with parameters 3 and 0.15. Thus :

$$f(0) = (0.85)^3,\ f(1) = 3(0.85)^2(0.15),\ f(2) = 3(0.85)(0.15)^2,\ f(3) = (0.15)^3$$

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(b) Since X is Binomial with parameters 3 and 0.15, $\mathbb{E}[X] = 3(0.15) = 0.45$.

(c) Since X is Binomial with parameters 3 and 0.15, $Var[X] = 3(0.15)(0.85) = 0.3825$.

(d) The system is successful unless $X = 3$. Thus,

$$\mathbb{P}[\text{success}] = 1 - \mathbb{P}[X = 3] = 1 - (0.15)^3 = 0.997$$

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(e) The system fails if it is not successful. Thus, by part (d), this probability is 0.003.

(f) Three components are sufficient.