Question 1.

MET Research Group at the University of Ottawa is developing a new generation of clinical decision support systems called MET4 to be used in the Ottawa Hospital for supporting Oncology healthcare teams. MET research group has been concerned with the amount of time that it takes to complete development of the final product. A list of activities and their optimistic, pessimistic, and most likely completion times in days are given in the following table:

Activity	Days			Immediate Predecessors
	a	m	b	
A	3	6	8	-
B	2	4	4	-
C	1	2	3	-
D	6	7	8	C
E	2	4	6	B, D
F	6	10	14	A, E
G	1	2	4	A, E
H	3	6	9	F
I	10	11	12	G
J	14	16	20	C
K	2	8	10	H, I

a) Develop a project network for this problem.

b) Determine the expected completion of and variance for each activity.

c) MET research group would like to determine the total project completion time and the critical path for development of MET4 system. In addition, determine EST, EFT, LST, LFT, and slack for each activity.

d) What is the probability that MET research group will finish the project in 40 days or less?

e) Find the number of days within which MET research group is 97% sure the project will be completed.

Part of the solution is provided below:

Activity	*a*	*m*	*b*	Expected Time	Variance
A	3	6	8	5.83	0.69
B	2	4	4	3.67	0.11
C	1	2	3	2.00	0.11
D	6	7	8	7.00	0.11
E	2	4	6	4.00	0.44
F	6	10	14	10.00	1.78
G	1	2	4	2.17	0.25
H	3	6	9	6.00	1.00
I	10	11	12	11.00	0.11
J	14	16	20	16.33	1.00
K	2	8	10	7.33	1.78

Activity	ES	EF	LS	LF	Slack	Critical Path
A	0	5.83	7.17	13.00	7.17	No
B	0	3.67	5.33	9.00	5.33	No
C	0	2.00	0	2.00	0	Yes
D	2.00	9.00	2.00	9.00	0	Yes
E	9.00	13.00	9.00	13.00	0	Yes
F	13.00	23.00	13.00	23.00	0	Yes
G	13.00	15.17	15.83	18.00	2.83	No
H	23.00	29.00	23.00	29.00	0	Yes
I	15.17	26.17	18.00	29.00	2.83	No
J	2.00	18.33	20.00	36.33	18.00	No
K	29.00	36.33	29.00	36.33	0	Yes

Question 2.

A type of steel called IAG is manufactured by mixing raw materials composed of different Iron and Copper alloys. The prices for buying each raw material (in $/ton) are given in a table below:

	Iron alloys		Copper alloys		
Raw materials	Iron alloy 1	Iron alloy 2	Copper alloy 1	Copper alloy 2	Copper alloy 3
Price ($/ton)	115	128	132	109	114

The final product sells for $180 per ton. Raw materials have to be processed before mixing. Iron and Copper alloys require different production lines for processing. It is not possible to process more than 210 tons of any type of Iron alloys and no more than 260 tons of any type of Copper alloys on the processing lines. There is no loss of weight in the processing and the cost of processing should be ignored.

Production process imposes the following conditions:

- The IAG product may never be made up of more than three alloys
- If a given raw material is used for production, then it has to be at least at 30 tons level
- If either of "Iron alloy 1" or "Iron alloy 2" are used then "Copper alloy 2" must also be used

The objective is to determine the optimal composition of the raw materials for manufacturing of the IAG product that maximizes the total profit.

Formulate this problem algebraically using appropriate variables

Question 3.

A small paint company manufactures two types of paint, latex and enamel. In production, the company uses 10 hours of labor to produce 100 gallons of latex and 15 hours of labor to produce 100 gallons of enamel. The company has 40 hours of daily labor and 30 hours of overtime labor available each week. Furthermore, if enamel paint is produced, latex paint must also be produced. Each paint generates a profit at the rate of $1.00 per gallon. The company has the following objectives listed in decreasing priority:

- avoid the use of overtime
- achieve a weekly profit of $1000
- produce at least 700 gallons of enamel paint each week

Formulate the linear goal programming model for this problem.

Question 4.

Patrick family owns 410 acres of farmland in Ottawa area on which they grow corn and tobacco. Each acre of corn costs $105 to plant, cultivate, and harvest; each acre of tobacco costs $210. The Patricks have a budget of $52,500 for the next year. The Ontario regulations limit a number of acres of tobacco that can be planted to 100 acres. The profit from each acre of corn is $300; the profit from each acre of tobacco is $520. The Patricks want to know how many acres of each crop to plant in order to maximize their profit.

The LP formulation, answer report, and LP sensitivity report for this problem are shown below.

$$\text{Max } Z = 300x1 + 520\,x2$$
$$\text{S.t.}$$
$$x1 + x2 \le 410$$
$$105x1 + 210\,x2 \le 52{,}500$$
$$x2 \le 100$$
$$x1, x2 \ge 0$$

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Cost	$400.00	$142,800.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$5	Number of acre Corn	1.333	320.000	Contin
$C$5	Number of acre tobacco	0.000	90.000	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Total number of acres of land	410.0	$D$8<=$F$8	Binding	0
$D$9	Total budget available	52500.0	$D$9<=$F$9	Binding	0
$D$10	Number of acres of tobacco	90.0	$D$10<=$F$10	Not Binding	10

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Number of acre Corn	320	0	300	220	40
$C$5	Number of acre tobacco	90	0	520	80	220

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Total number of acres of land	410	80	410	90	10
$D$9	Total budget available	52500	2.095238095	52500	1050	9450
$D$10	Number of acres of tobacco	90	0	100	1E+30	10

The following questions refer to the changes made separately to the original LP formulation (i.e. each change is applied to original problem only). Please answer the following questions in the space provided.

a) How many acres of farmland will not be cultivated at the optimal solution? Do the Patricks use the entire 100-acre tobacco allotment?

b) If the Patricks can obtain an additional 100 acres of land, will the number of acres of corn and tobacco they plan to grow change?

c) If they decide not to cultivate a 50-acre section as part of a crop recovery program, how will it affect their crop plans?

d) The Patricks have an opportunity to lease some extra land from a neighbor. The neighbor is offering the land to them for $110 per acre. Should the Patricks lease the land at that price? what is the maximum price the Patricks should pay their neighbor for the land, and how much should they lease at that price?

e) The Patricks are considering taking out a loan to increase their budget. For each dollar they borrow, how much additional profit would they make? If they borrowed an additional $1,000, would the number of acres of corn and tobacco they plant change?