Concordia University

Department of Mathematics and Statistics

Course	Number	Section	
MATH	208	All	
Examination	**Date**	**Time**	**Page**
Mid Term FEB. 2013.		1 Hour 30 minutes	2
Instructor	**Course Examiner**		**Marks**
D. Dryanov, L. Dube & U. Tiwari	D. Sen		60
Special Instructions:	**Answer ALL questions.**		

Formulae:

$$\mathbf{A = P(1+i)^n,\ A = Pe^{rt},\ FV = PMT\frac{(1+i)^n - 1}{i},\ PV = PMT\frac{1-(1+i)^{-n}}{i}}$$

[10] Q 1. A small company manufactures picnic tables. The weekly fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total daily cost of producing x picnic tables. How many picnic tables can be produced for a total weekly cost of \$4,800?

[10] Q 2. Solve for x in the following equations:

(a) $4^{5x-x^2} = 4^{-6}$

(b) $16^{x-1} = 4^{1+x}$

(c) $3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20 = \log_b x$

(d) $\log_a x + \log_a(x+1) = \log_a 6$

[10] Q 3. A person borrows \$3,600 and agrees to repay the loan in monthly installments over a period of 3 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 3 years?

[10] Q 4. Parents have set up a sinking fund in order to have \$140,000 in 15 years for their children's college education. How much should be paid semiannually into an account paying 6.8% compounded semiannually?

[10] Q 4. It is about a sinking fund, so we should find PMT from the formula

$$\mathrm{FV} = \mathrm{PMT}\,\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}}.$$

Thus we have

$$\mathrm{PMT} = \mathrm{FV}\,\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}}-1},$$

where FV = \$140,000, $\mathrm{i} = \frac{6.8\%}{2} = 0.034$, and

$$n = 2(\text{number of payments per year}) \cdot 15(\text{ years}) = 30.$$

Therefore

$$\mathrm{PMT} = \$140{,}000\frac{0.034}{(1+0.034)^{30}-1} \simeq \$2{,}756.92.$$

[10] Q 5. (a) As we know, $\mathrm{APY} = \left(1+\frac{r}{m}\right)^m - 1$, where APY = 0.0565, $m = 12$ (compounded monthly means 12 times per year), and r is the equivalent annual interest rate. Hence we need to solve

$$0.0565 = \left(1+\frac{r}{12}\right)^{12} - 1$$

for r. We have

$$1.0565 = \left(1+\frac{r}{12}\right)^{12},$$

$$(1.0565)^{1/12} = 1+\frac{r}{12},$$

$$r = 12(1.0565)^{1/12} - 12 \simeq 0.055 = 5.5\%.$$

(b) It is a sinking fund problem. From the formula

$$\mathrm{FV} = \mathrm{PMT}\,\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}}$$

we find that

$$\mathrm{PMT} = \mathrm{FV}\,\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}}-1}.$$

Here FV = \$1,000,000, $\mathrm{i} = \frac{r}{12} \simeq 0.0046$, and

$$n = 8(\text{years}) \cdot 12(\text{payments per year}) = 96.$$

Thus we have

$$\mathrm{PMT} = \$1{,}000{,}000\frac{0.0046}{(1+0.0046)^{96}-1} \simeq \$8{,}308.96.$$

$$\log_b 2 = \log_b x,$$
$$x = 2.$$

Check:
$$3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20 = \log_b 2.$$

As it was found before, the left hand side of the latter equality is equal to $\log_b 2$. Thus $x = 2$ satisfies the initial equation.
Answer: $x \in \{2\}$.

(d)
$$\log_a x + \log_a(x+1) = \log_a 6, \quad a > 0,$$
$$\log_a\big(x(x+1)\big) = \log_a 6,$$
$$x(x+1) = 6,$$
$$x^2 + x - 6 = 0,$$
$$(x+3)(x-2) = 0,$$
$$x = -3;\ x = 2.$$

Check: $x = -3$ does not satisfy the initial equation as $\log_a(-3)$ has no sense as well as $\log_a(-3+1)$.
Next, $x = 2$ gives $\log_a 2 + \log_a 3 = \log_a 6$, which is true (since $\log_a 2 + \log_a 3 = \log_a(2 \cdot 3)$).
Answer: $x \in \{2\}$.

[10] Q 3. In this problem the periodic payments are unequal, so it is not about an ordinary annuity!!! Here every payment is a sum of \$100 (to reduce the balance) and the interest (which is the balance multiplied by the interest rate i = 1% per period). The balance before the first payment is \$3,600. Therefore the interest for the first month is I_1 = \$3,600 · i = \$3,600 · 0.01 = \$36 (and the first payment is \$100 + I_1 = \$136). The balance for the second month is B_2 = \$3,600 − \$100 and, in general, the balance for the n-th month is B_n = \$3,600 − (n − 1)\$100, $n \leq 36$. Therefore the interest for the second month is i · B_2 and, in general, the interest for the n-th month is

$$I_n = \mathrm{i} \cdot \mathrm{B_n} = 0.01 \cdot (\$3{,}600 - (\mathrm{n}-1)\$100) = \$37 - \mathrm{n};$$

and the n-th payment is \$100 + I_n, $n \leq 36$, so you can see that the payments are unequal. The total cost of the loan is the total interest plus \$3,600. The total interest is the sum

$$\sum_{n=1}^{36} I_n = I_1 + I_2 + \cdots + I_{35} + I_{36} = \$36 + \$35 + \cdots + \$2 + \$1.$$

Observe that this is the sum of first 36 elements of an arithmetic sequence with the common difference d = −\$1, a_1 = \$36, and a_{36} = \$1. Thus $\sum_{n=1}^{36} I_n = (a_1 + a_{36}) \cdot 36/2$ = \$37 · 18 = \$666. The total cost of the loan is the total interest \$666 plus \$3,600, which is \$4,266.

[10] Q 6. (a) It is about an amortization. From

$$\mathrm{PV} = \mathrm{PMT}\,\frac{1-(1+\mathrm{i})^{-n}}{\mathrm{i}}$$

we find that

$$\mathrm{PMT} = \mathrm{PV}\,\frac{\mathrm{i}}{1-(1+\mathrm{i})^{-n}}.$$

Here PV = \$800, i = 1.5% = 0.015 per period (per month), and $n = 18$. Thus we get

$$\mathrm{PMT} = \$800\frac{0.015}{1-(1+0.015)^{-18}} \simeq \$51.05.$$

(b) The total interest paid is

$$\mathrm{PMT}\cdot 18 - \$800 \simeq \$51.05 \cdot 18 - \$800 = \$118.90.$$