

Practice Questions 2

1-) Identify which kind of preferences the following utility functions represent. For each of the following utility functions, derive the demand functions for X and Y as functions of prices and income. Explain whether these goods are normal goods. Explain whether two goods are complements or substitutes.

- (i) (8 points) $U(X, Y) = X^{\frac{1}{2}} + Y$
(ii) (8 points) $U(X, Y) = \text{Min}(X/2, Y/3)$
(iii) (8 points) $U(X, Y) = \ln X + \ln Y$
(iv) (8 points) $U(X, Y) = X^{\frac{1}{2}}Y$

2-) a-) Suppose that Mrs. Rice will always trade (or substitute) 2 cups of tea (X) for three cups of coffee (Y). Write the utility function according to Mrs. Rice's preferences. What kinds of preferences are represented by this utility function? What is marginal rate of substitution? Draw an indifference map according to these preferences.

b-) Find demand functions for X and Y. Draw the demand curve for Y. Consider initially that Income (M) = \$1200, $P_X = \$10$ and $P_Y = \$10$. What is the initial optimal consumption levels of X and Y.

c-) Suppose that P_x rises from \$10 to \$12. Find the substitution effect, income effect and total effect on good Y. Explain the steps carefully and support your result with an explanation and a graph.

d-) Suppose instead now that P_x rises from \$10 to \$20. Find the substitution effect, income effect and total effect on good Y. Explain the steps carefully support your result with an explanation and a graph.

3-) a-) Suppose that Mrs. Star will always consume X and Y in fixed proportion: 1 unit of X is always consumed with 2 units of Y. What kinds of preferences are represented by this utility function? What is marginal rate of substitution? Draw an indifference map according to these preferences.

b-) What are the demand functions for X and Y. Draw the demand curve for Y.

c-) Consider initially that Income (M) = \$1200, $P_X = \$10$ and $P_Y = \$10$. What is the initial optimal consumption levels of X and Y.

d-) Suppose that P_x rises from \$10 to \$15. Find the substitution effect, income effect and total effect on good Y. Explain the steps carefully and support your result with an explanation and a graph.

4-) Suppose that the utility function is characterized as: $U(x_1, x_2) = x_1 + \ln x_2$

a-) Find the optimal demands for good 1 and good 2 as a function of prices and income.

b-) According to the above preferences, draw the Income Offer Curve that shows optimal bundle for different income levels. Then, find the corresponding Engel Curve for x_2 . Explain why you obtain these particular shapes.

c-) Consider initially that Income (I) = \$120, $P_1 = \$12$ and $P_2 = \$2$. Find the initial consumption levels for good 1 and good 2.

d-) Suppose that P_2 rises to \$6. Find the substitution effect, income effect and total effect on good 2. Explain the steps carefully and support your result with an explanation and a graph.

e-) Suppose now that Income (I) = \$1200, $P_1 = \$12$, $P_2 = \$2$ and government imposes \$2 sales tax on good 2 (consumer has to pay additional \$2). Find the equilibrium consumption of both goods after the sales tax. Then consider that government imposes an income tax that generates the same tax revenue as under the sales tax. Find the new equilibrium consumption under the income tax. Show these equilibrium points on a diagram. Which tax method (sales tax or income tax) does the consumer prefer? Why? Explain.

5-) Consider the following production functions:

(i) $Q = aL + bK\sqrt{L}$.

(ii) $Q = aL + bK$

(iii) $Q = L + \sqrt{K}$

(iv) $Q = \min\{aL, bK\}$

Find the MRTS for the given production functions. Do we obtain diminishing MRTS? What can you say about the returns to scale of the above production functions?

6-) Consider the following production function $Q = KL$

a-) For the above production function, find the Returns to Scale.

b-) Using the above production function, find the labor demand and capital demand as functions of output (Q), price of labor (w) and price of capital (r). Does the Law of Demand hold for each input? Are these inputs normal or inferior inputs in the production process?

c-) Find the cost function. Using both cost function verify whether Shephard's Lemma holds. Using cost function, verify that you end up with the same Returns to Scale findings as in part a.

e-) Suppose that a firm wants to produce 12 units of output and $w=1$, $r=1$. Find long run total cost. Suppose now that wage goes up to 2. Find the new long run total cost for each of the production functions. Does firm substitute capital for labor? What is the percentage of cost saving for each of the production functions relative to the case where firm is not able to substitute?

f-) Suppose now that the firm operates in short run and capital (K) is fixed at 16. Find the short run labor demand. Find the Short run total cost, total fixed cost, total variable cost, marginal cost, average variable cost, average fixed cost and average total cost when $w=1$, $r=1$. Is the short run total cost larger than long-run total cost? Why or why not?