Solutions to Test 1 - MATH 1107C - Winter 2010

Version 2

PART I: Multiple choice questions (4 points each)
Choose and circle only one answer. No partial marks here.
No justification is required.

1. The equation of the plane through $P(1, 4, -1)$ that is parallel to the plane with equation $3x - 2y + z = 8$ is:

(a) $x + 4y - z = 8$ **(b)** $3x - 2y + z = 2$
(c) $x + 4y - z = -6$ **(d)** $3x - 2y + z = -6$

Solution: **(d)**

2. Let $\overrightarrow{v_1} = [1\ 2\ 0]^T$ and $\overrightarrow{v_2} = [2\ 1\ -1]^T$. The cross product $\overrightarrow{v_1} \times \overrightarrow{v_2}$ is:

(a) $[2\ 1\ -5]^T$ **(b)** $[2\ -1\ 3]^T$ **(c)** $[-2\ 1\ -3]^T$ **(d)** $[-2\ -1\ 5]^T$

Solution: **(c)**

3. If z is a complex number satisfying the equation $z \cdot (1 - i) = |\sqrt{3} - i| \cdot i$, then

(a) $z = 1 + i$ **(b)** $z = 1 - i$ **(c)** $z = -1 + i$ **(d)** $z = -1 - i$

Solution: **(c)**

4. Which of the following matrices are in reduced row echelon form (RREF)?

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 5 & 4 & -8 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

(a) A and B only **(b)** C and D only **(c)** B only **(d)** D only

Solution: **(a)**

PART II: Long answer questions
Show all your work.

5. **[10 points]** Find the shortest distance from the point $P(2,4,-5)$ to the line given by the equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}.$$

Solution: The line has direction vector $\overrightarrow{d} = [2\ 0\ -1]^T$ and passes through $P_0(1,-1,3)$. Let $\overrightarrow{u} = \overrightarrow{P_0P} = [1\ 5\ -8]^T$. Then

$$\overrightarrow{u_1} = \text{proj}_{\overrightarrow{d}}\,\overrightarrow{u} = \frac{\overrightarrow{u}\cdot\overrightarrow{d}}{||\overrightarrow{d}||^2}\overrightarrow{d} = \frac{10}{5}[2\ 0\ -1]^T = [4\ 0\ -2]^T.$$

Hence the shortest distance is given by $||\overrightarrow{u} - \overrightarrow{u_1}|| = ||[-3\ 5\ -6]^T|| = \sqrt{70}$.

6. Let $z = \sqrt{3} - i$.

(a) **[5 points]** Express z in polar form using the principal argument $(-\pi < \theta \leq \pi)$.

(b) **[5 points]** Express z^{12} in the form $a + bi$.

Solution: **(a)** $r = |z| = \sqrt{3+1} = 2$, $\cos\theta = \sqrt{3}/2$, and $\sin\theta = -1/2$. Thus $\theta = -\pi/6$. The polar form is $z = 2e^{-\pi i/6}$.

(b) $z^{12} = (2e^{-\pi i/6})^{12} = 2^{12}e^{-2\pi i} = 2^{12} = 4096$

7. **[9 points]** Find all solutions (if any) of the following system of linear equations:

$$\begin{cases} x_1 + 2x_2 + x_3 + 2x_4 &= -1 \\ x_1 + 2x_2 - x_3 - 4x_4 &= 9 \\ 2x_1 + 4x_2 - 2x_4 &= 8 \end{cases}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & -1 \\ 1 & 2 & -1 & -4 & 9 \\ 2 & 4 & 0 & -2 & 8 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & -1 \\ 0 & 0 & -2 & -6 & 10 \\ 0 & 0 & -2 & -6 & 10 \end{array}\right]$$

$$\xrightarrow{R_2 \leftarrow (-\frac{1}{2})R_2} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & -1 \\ 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & -2 & -6 & 10 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + 2R_2]{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 4 \\ 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The general solution is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4-2s+t \\ s \\ -5-3t \\ t \end{bmatrix}$$

where s and t are arbitrary.