

Course	Number	Section(s)
Mathematics	209	ALL
Examination	Date	Pages
Final	April 2013	2
Instructors	Course Examiner	
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Special Instructions		
▷ Ruled booklets to be used.		
▷ Only approved calculators are allowed.		

MARKS

[12] 1. (a) Find  $\lim_{x \rightarrow -4} \frac{4x^2 + 7x - 36}{x^2 + 28x + 96}$

(b) Find  $\lim_{x \rightarrow 2} \frac{4 - \sqrt{6x + 4}}{3x^2 - 12}$

(c) Give an example of a function  $f$  defined for all real numbers which has the property that  $\lim_{x \rightarrow +\infty} f$  and  $\lim_{x \rightarrow -\infty} f$  are both equal to  $+\infty$ .

[8] 2. Find  $dg$  if  $g = x^{1.5}$ ,  $x = 4$ , and the change in the  $x$  is 0.1. What is the value of  $\Delta g$  for these values?

[12] 3. Find the derivatives of the following functions. YOU DO NOT HAVE TO SIMPLIFY.

(A)  $f(x) = \frac{7^3}{\sqrt{x^5}} - 70x^3 - e^x$ .

(B)  $g(x) = (3x^5 + \ln x - e^3 x)(5\sqrt{x} - \frac{8}{x} - 5x + 7)$ .

(C)  $h(x) = x^2 \ln(x) - \frac{5}{4x} - e^{(-x^3+6)}$ .

(D)  $m(x) = [x^5 - 3\ln(x^3 + 6)]^{10}$ .

[8] 4. Use implicit differentiation to find  $y' = dy/dx$

$$2x^5y^3 - 5x^3 + y^2 \ln x = 7y + 4x - 10$$

[10] 5. Boyle's law for enclosed gases states that if the temperature is kept constant, the pressure  $P$  and volume  $V$  of a gas are related by the equation  $VP = k$  where  $k$  is constant. If the volume is decreasing by 5 cubic inches per second, what is the rate of change of pressure when the volume is 1,000 cubic inches and the pressure is 40 pounds per square inch?

[10] 6. Use the price-demand equation  $p = 90 - 0.1x^2$  to find the values of  $p$  for which the demand is elastic and for which the demand is inelastic.

[12] 7. Graph the sales function  $N(x) = 4x^3 - 0.25x^4 + 500$  over the interval  $0 \leq x \leq 12$ . Determine when  $N$  is increasing, when it is decreasing. Does  $N$  have a maximum? If so, find it. Does  $N$  have a point of inflection? If so, find it.

[8] 8. Find the absolute maximum and absolute minimum for the function

$$f(x) = x^3 - 21x^2 + 135x - 170 \text{ on the interval } [2, 12].$$

[10] 9. Compute the following:

(A)  $\int e^{-2x} dx.$

(B)  $\int (3x^4 - 5x^7) dx .$

(C)  $\int \left( \frac{x^2}{(x^3-5)^7} \right) dx.$

(D)  $\int_2^3 \left( 4x - \frac{5}{x} + 3e^x \right) dx.$

[10] 10. Suppose that a country has Lorentz curve of the form  $f(x) = x^a$  and a Gini index of 0.286 . Find  $a$ .