Solutions to Test 1 - MATH 1107C - Winter 2010

Version 1

PART I: Multiple choice questions (4 points each)

Choose and circle only one answer. No partial marks here.
No justification is required.

1. The equation of the plane through $P(1, -2, 3)$ that is parallel to the plane with equation $5x + 2y - 3z = 10$ is:

(a) $x - 2y + 3z = 10$ **(b)** $5x + 2y - 3z = -8$
(c) $x - 2y + 3z = -8$ **(d)** $5x + 2y - 3z = 6$

Solution: **(b)**

2. Let $\overrightarrow{v_1} = [1\ \ -1\ 2]^T$ and $\overrightarrow{v_2} = [0\ 2\ 3]^T$. The cross product $\overrightarrow{v_1} \times \overrightarrow{v_2}$ is:

(a) $[-7\ \ -3\ 2]^T$ **(b)** $[7\ 3\ \ -2]^T$ **(c)** $[1\ 3\ 2]^T$ **(d)** $[-1\ \ -3\ \ -2]^T$

Solution: **(a)**

3. If z is a complex number satisfying the equation $z \cdot (1 + i) = |\sqrt{3} + i| \cdot i$, then

(a) $z = 1 + i$ **(b)** $z = 1 - i$ **(c)** $z = -1 + i$ **(d)** $z = -1 - i$

Solution: **(a)**

4. Which of the following matrices are in reduced row echelon form (RREF)?

$$A = \begin{bmatrix} 1 & 4 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 4 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

(a) C only **(b)** D only **(c)** A and B only **(d)** C and D only

Solution: **(d)**

PART II: Long answer questions
Show all your work.

5. [**10 points**] Find the shortest distance from the point $P(3, 1, -2)$ to the line given by the equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Solution: The line has direction vector $\overrightarrow{d} = [0\ 1\ -1]^T$ and passes through $P_0(1, -2, 1)$. Let $\overrightarrow{u} = \overrightarrow{P_0P} = [2\ 3\ -3]^T$. Then

$$\overrightarrow{u_1} = \text{proj}_{\overrightarrow{d}}\overrightarrow{u} = \frac{\overrightarrow{u} \cdot \overrightarrow{d}}{||\overrightarrow{d}||^2}\overrightarrow{d} = \frac{6}{2}[0\ 1\ -1]^T = [0\ 3\ -3]^T.$$

Hence the shortest distance is given by $||\overrightarrow{u} - \overrightarrow{u_1}|| = ||[2\ 0\ 0]^T|| = 2$.

6. Let $z = 1 - \sqrt{3}i$.

(a) [**5 points**] Express z in polar form using the principal argument $(-\pi < \theta \leq \pi)$.

(b) [**5 points**] Express z^9 in the form $a + bi$.

Solution: **(a)** $r = |z| = \sqrt{1+3} = 2$, $\cos\theta = 1/2$, and $\sin\theta = -\sqrt{3}/2$. Thus $\theta = -\pi/3$. The polar form is $z = 2e^{-\pi i/3}$.

(b) $z^9 = (2e^{-\pi i/3})^9 = 2^9 e^{-3\pi i} = -2^9 = -512$

7. [**9 points**] Find all solutions (if any) of the following system of linear equations:

$$\begin{cases} x_1 + 3x_2 + x_3 + 2x_4 &= 2 \\ x_1 + 3x_2 - 2x_3 - 10x_4 &= 8 \\ 2x_1 + 6x_2 - x_3 - 8x_4 &= 10 \end{cases}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 2 \\ 1 & 3 & -2 & -10 & 8 \\ 2 & 6 & -1 & -8 & 10 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 2 \\ 0 & 0 & -3 & -12 & 6 \\ 0 & 0 & -3 & -12 & 6 \end{array}\right]$$

$$\xrightarrow{R_2 \leftarrow (-\frac{1}{3})R_2} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & -3 & -12 & 6 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + 3R_2]{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{cccc|c} 1 & 3 & 0 & -2 & 4 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The general solution is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4-3s+2t \\ s \\ -2-4t \\ t \end{bmatrix}$$

where s and t are arbitrary.