

CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 2004 A, B, C, D
Fall 2013

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 2004 A, B, C, D.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Dr. R. Cova (Sect. B, C), Dr. E. Xhua (Sect. D)

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- Please verify that you are in possession of a SCANTRON FORM
- Please fill in your COURSE CODE (e.g., MATH 2004) and COURSE SECTION (e.g., A, B, C, D), as per above list of instructors; YOUR NAME and YOUR STUDENT NUMBER where required on the Scantron form AND on this examination.
- **The entire examination consists of 5 pages and two parts, A and B, and is marked out of a total of 80, that is, 40% of your final mark. Part A consists of 12 multiple choice questions each worth 3 marks. Please fill in only one answer on your SCANTRON sheets with a pencil as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question). Part B consists of 6 traditional type questions (with explanations required), for a total of 44 marks for that section. **Both Part A and B must be submitted along with the scantron sheet, so do not detach nor unstaple this examination. Do not submit any of your rough work. All supporting arguments in Part B must appear in the appropriate location.****

Print Name :

Student Number:

Section (either A, B, C, or D):

- **A1.** Find the slope of the tangent line to $r = 3 \sin \theta$ at $\theta = \frac{\pi}{2}$.
(a) 0 (b) $\sqrt{3}$ (c) 2 (d) 3
- **A2.** Let θ be the angle between the two planes $x + 2y - 2z - 3 = 0$ and $2x + 2y - z - 5 = 0$. Find $\cos \theta$.
(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{8}{9}$ (d) $\frac{9}{8}$
- **A3.** Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(-3, 0, 1)$, and $R(2, 4, 5)$.
(a) $6\sqrt{2}$ (b) $6\sqrt{3}$ (c) 12 (d) $3\sqrt{2}$

- **A4.** Given the line $L : \mathbf{r} = (1, 2, 3) + t(3, 2, -1)$. Let Π be the plane through the point $(1, 1, 3)$ and perpendicular to the line L . Which of the following is a point on the plane Π ?
 (a) $(0, 0, -1)$ (b) $(0, 0, -2)$ (c) $(0, 0, -3)$ (d) $(0, 0, -4)$

- **A5.** Let $w(x, y) = x^2 + y + 3xy^4$, where $x = \sin 2t, y = \cos t$. Use the Chain Rule to find the value of $\frac{\partial w}{\partial t}$ at $t = 0$:
 (a) 0 (b) 6 (c) 3 (d) 2

- **A6.** A vector giving the direction in which $f(x, y, z) = \frac{x}{y} + \frac{y}{z}$ increases most rapidly at the point $(1, -1, 2)$ is given by
 (a) $\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ (b) $-\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$ (c) $2\mathbf{i} - \frac{1}{4}\mathbf{j} - \frac{1}{2}\mathbf{k}$ (d) $-\mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{1}{4}\mathbf{k}$

- **A7.** Find all the critical points of $h(x, y) = y^2 - 2y - y\sqrt{x} + \frac{x}{2}$ where x, y are real, $x \geq 0$.
 (a) $(4, 2), (0, 0)$ (b) $(0, 0)$ only (c) $(2, 4), (0, 0)$ (d) $(4, 2)$

- **A8.** Evaluate the double integral $\iint_{\mathcal{R}} e^{-x} \sin y \, dx \, dy$, where $\mathcal{R} = \{(x, y) \mid 0 \leq x \leq \ln 2, 0 \leq y \leq \pi/2\}$.
 (a) $1 - e$ (b) $\ln 2 - 1$ (c) 0 (d) $\frac{1}{2}$

- **A9.** Evaluate the triple integral $\iiint_{\mathcal{T}} xyz \, dz \, dy \, dx$ where \mathcal{T} is the region
 $\mathcal{T} = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 1, \sqrt{x^2 + y^2} \leq z \leq 2\}$.
 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{2}$

- **A10.** Which one of the following integrals is equal to the double integral $\iint_{\mathcal{R}} 2xy \, dA$, under the change of variables $x = 3u, y = 4v$ where \mathcal{R} is the elliptical region $16x^2 + 9y^2 \leq 144$.
 (a) $\iint_{\mathcal{S}} 12uv \, dA$ (b) $\iint_{\mathcal{S}} 288uv \, dA$, (c) $\iint_{\mathcal{S}} 8(u^2 + v^2) \, dA$ (d) $\iint_{\mathcal{S}} 12u^2v^2 \, dA$
 where $\mathcal{S} = \{(u, v) : u^2 + v^2 \leq 1\}$.

- **A11.** Evaluate the line integral $\int_{\mathcal{C}} (6xy^2 - y^3) \, dx + (6x^2y - 3xy^2) \, dy$ where \mathcal{C} is the straight line from the point $(1, 2)$ to $(3, 4)$.
 (a) 236 (b) 0 (c) 188 (d) 48

- **A12.** Let $\mathbf{F}(x, y, z) = 7x\mathbf{i} - z\mathbf{k}$. Evaluate the surface integral $\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n} \, dS$ where \mathcal{S} is the surface of the sphere $x^2 + y^2 + z^2 = 4$ and \mathbf{n} is an outer normal unit vector to \mathcal{S} .
 (a) 24π (b) 16π (c) 0 (d) 64π

End of Part A

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PART B: Do All Six (6) Questions for a total of 44 marks out of a maximum of 80.

Do not detach nor unstaple this examination. Missing sheets will void any credit for those questions.

- **B1.** [8 marks] Using the method of Lagrange multipliers determine the local extrema and saddle points (if any) of $f(x, y, z) = x + 2y - 2z$ subject to the constraint $x^2 + 2y^2 + 4z^2 = 1$.

- **B2.** [7 marks] By changing to polar coordinates evaluate the integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2) dx dy$.

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- **B3.** [7 marks] The parametric curve C in the plane is given by $x = 2t^2$, $y = 1 + t^3$, where $0 \leq t \leq 1$. Find the length of the curve C .

- **B4.** [7 marks] Find the area of the region enclosed by the cardioid $r = 1 + \sin \theta$ and the circle $r = 3 \sin \theta$.

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- **B5.** [7 marks] Let $\mathbf{F}(x, y, z) = \frac{1}{yz} \mathbf{i} - \frac{x}{y^2z} \mathbf{j} - \frac{x}{yz^2} \mathbf{k}$.

a) Compute $\text{curl } \mathbf{F}$.

b) Find a scalar field $f(x, y, z)$ such that $\nabla f = \mathbf{F}$. If no such scalar field exists, explain why.

- **B6.** [8 marks] Let $\mathbf{F}(x, y, z) = -5y \mathbf{i} + 4x \mathbf{j} + z \mathbf{k}$ and let \mathcal{S} be the surface bounded by the circle $x^2 + y^2 = 4$ and $z = 1$. Evaluate the surface integral $\iint_{\mathcal{S}} \text{curl } \mathbf{F} \cdot \mathbf{n} \, dS$ using any method. Here \mathbf{n} is an outer normal unit vector to \mathcal{S} .