Solutions

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2014

TEST 3

Only the STUDIO 56 calculator permitted, 1 or more blank sheets permitted for roughs (not to be attached here)

Print Name : ______________________

Student Number: ______________________

Tutorial Section (A1, A4, ...): ______________________

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[3 marks]** Evaluate $\int\int_{\mathcal{R}}(6-2y)\,dA$. where $\mathcal{R}=\{(x,y): 0\le x\le 4,\ 0\le y\le 2\}$
   (a) 6, (b) 16, (c) 12, (d) 32, (e) none of these.

2. **[3 marks]** Evaluate $\int\int_{\mathcal{R}}(4-x^2)\,dA$. where $\mathcal{R}=\{(x,y): 0\le x\le 2,\ 0\le y\le x\}$
   (a) 4, (b) 0, (c) 4/3, (d) 8, (e) none of these.

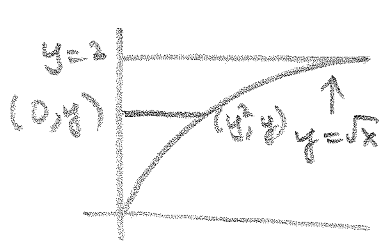
3. **[3 marks]** Use Fubini's theorem to interchange the order of integration in the double integral $\displaystyle\int\int_{\mathcal{R}} \frac{\sin x}{x}\,dx\,dy$ where $\mathcal{R}=\{(x,y): y\le x\le 1,\ 0\le y\le 1\}$. What does the new integral look like?
   (a) $\int_0^1\int_0^2 \frac{\sin x}{x}\,dy\,dx$, (b) $\int_1^2\int_0^x \frac{\sin x}{x}\,dy\,dx$, (c) $\int_0^1\int_0^x \frac{\sin x}{x}\,dy\,dx$, (d) $\int_0^2\int_0^y \frac{\sin x}{x}\,dy\,dx$, (e) none of these.

4. **[3 marks]** Evaluate the double integral $\int\int_{\mathcal{R}} xy\,dA$ where $\mathcal{R}$ is the region in the first quadrant bounded by the line $x=0$, $y=x$ and the circle $x^2+y^2=4$.
   (a) -1, (b) 1, (c) 0, (d) π, (e) none of these

5. **[3 marks]** Let T denote the solid region that consists of that part of the surface $z=x$ in three dimensions that lies directly above the region $x^2+y^2=1$ on the $xy-$plane. Then $\displaystyle\int\int_T z\,dA$ is equal to the volume of T.
   (a) True, (b) False,

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[10 marks]** a) Use Fubini's theorem to evaluate the double integral $\displaystyle\int_0^4\int_{\sqrt{x}}^2 \sin y^3\,dy\,dx. = \iint_{R} \sin y^3\,dA = I$



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where $R=\{(x,y): \sqrt{x}\le y\le 2,\ 0\le x\le 4\}$ is described using vertical slices here (see left). Using horizontal slices (as req'd by Fubini) we see that

$$R=\{(x,y): \underbrace{0\le x\le y^2}_{1+1},\ \underbrace{0\le y\le 2}_{1+1}\}.$$

$$\therefore\ I=\int_0^2\int_0^{y^2}\sin y^3\,dx\,dy=\int_0^2\left[x\sin y^3\right]_{x=0}^{x=y^2} \leftarrow 1$$

$$=\int_0^2 y^2\sin y^3\,dy=\underbrace{-\frac{1}{3}\cos(y^3)\Big|_{y=0}^{y=2}}_{2}$$

$$=\frac{1-\cos 8}{3}.$$

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7. [**marks]** Evaluate the iterated integral $\underbrace{\iiint_T 2\sqrt{y}\, e^{-x^2}\, dz\, dx\, dy}_{I}$ where $T = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 4, 0 \leq z \leq x\}$.

$$I = \int_0^4 \int_0^1 \int_0^x 2\sqrt{y}\, e^{-x^2}\, dz\, dx\, dy. \qquad 1$$

$$= \int_0^4 \int_0^1 \left[2z\sqrt{y}\, e^{-x^2} \right]_{z=0}^{z=x} dx\, dy \qquad 1/2$$

$$= \int_0^4 \int_0^1 2\sqrt{y}\, x\, e^{-x^2}\, dx\, dy \qquad 1$$

$$= \int_0^4 \left[-\sqrt{y}\, e^{-x^2} \right]_{x=0}^{x=1} dy. \qquad 1/2$$

$$= \int_0^4 \left(1 - \frac{1}{e}\right) \sqrt{y}\, dy \qquad 1/2$$

$$= \left(1 - \frac{1}{e}\right) \left(\frac{2}{3} y^{3/2}\right) \Big|_0^4 \qquad 1/2$$

$$= \frac{16\,(e-1)}{3e} \qquad 1$$