University of Ottawa
MAT 1330 A, B, C, E Midterm Exam
October 16, 2010. Duration: 80 minutes.
Instructors: Aziz Khanchi, Frithjof Lutscher, Robert Smith?, Angelika Welte

Family Name:_____

First Name:_____

DGD 1 DGD 2 DGD 3 DGD 4

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Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
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- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Please do not detach the pages.
- Good Luck!

Student number:_____, Total marks: _____ out of 30

Problem	1	2	3	4	5	6
Marks						

Question 1. [6 points] (a) Compute the derivative of the function $g(x) = \dfrac{1+x^2}{\sqrt{x}+x^{-1}}$. [Do not simplify your result.]

$g'(x) =$ $\dfrac{2x\left(\sqrt{x}+\frac{1}{x}\right) - (1+x^2)\left(\frac{1}{2\sqrt{x}} - \frac{1}{x^2}\right)}{\left(\sqrt{x}+\frac{1}{x}\right)^2}$

(b) Let $f(x) = \dfrac{x^2-4}{|x-2|}$.

$\lim_{x\to 2^+} f(x) =$ $\lim_{x\to 2^+} \dfrac{(x-2)(x+2)}{x-2} = 4$

$\lim_{x\to 2^-} f(x) =$ $\lim_{x\to 2^-} \dfrac{(x+2)(x-2)}{-(x-2)} = -4$

Is f continuous at $x = 2$? Answer No

Question 2. [4 points] Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{1}{2x-3}.$$

$$f'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{2(x+h)-3} - \frac{1}{2x-3}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2x-3-(2x+2h-3)}{(2x+2h-3)(2x-3)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-2h}{(2x+2h-3)(2x-3)}\right]$$

$$= \lim_{h\to 0} \frac{-2}{(2x+2h-3)(2x-3)} = \frac{-2}{(2x-3)^2}$$

Question 3. [6 points] The angle of the sun above the horizon at 12 noon in Ottawa has its highest value in June with 68.1 degrees and its lowest value in December with 21.3 degrees. Assume that the height above the horizon can be written in standard cosine form.

(a) Find the values of the parameters A, B, Φ, T in the standard cosine description , i.e.,

$$f(t) = A + B\cos(2\pi(t - \Phi)/T),$$

where t is in months, and $t = 0$ corresponds to the month of January.

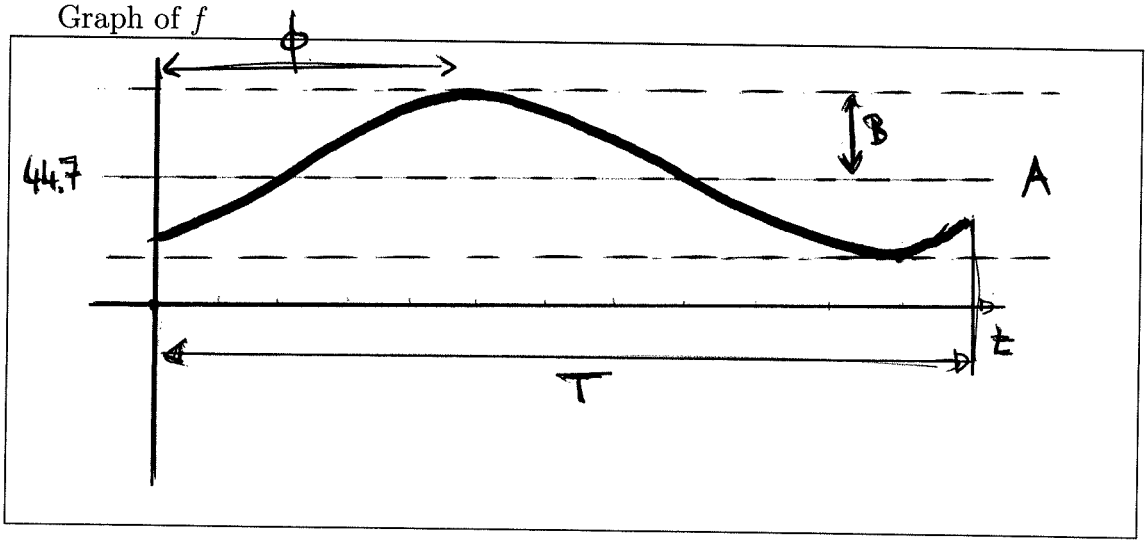
$f(t) =$ $44.7 + 23.4 \cos\left(\frac{2\pi}{12}(t-5)\right)$

(b) Give the names of the four parameters A, B, Φ, T.

A: average
B: amplitude
ϕ: phase
T: period

(c) Draw the graph of the function and identify the four parameters A, B, Φ, T in the graph.

Graph of f



Question 4. [2 points] Is the following function continuous at $x = 1$? Justify your answer in a short sentence.

$$f(x) = \cos(2x) + \frac{3x^2 - 5x}{x^2 - 2}$$

Answer: The function is a sum of a continuous function and a rational function. The rational function is continuous where its denominator is not zero. Hence the function is continuous at $x = 1$.

Question 5. [4 points]
(a) Find the critical point(s) of the function

$$f(x) = x^3 e^{-x}$$

$$f'(x) = (3 - x)x^2 e^{-x}$$

Answer: Critical points are $x_0 = 0$, $x_1 = 3$

(b) Find the intervals where the function is increasing and where it is decreasing.

Increasing: $x < 3$

Decreasing: $x > 3$

Question 6. [8 points] Consider the discrete-time dynamical system (DTDS)

$$M_{t+1} = -0.8M_t + 6$$

(a) [1 point] Find the updating function of the DTDS. $f(x) = -0.8x + 6$

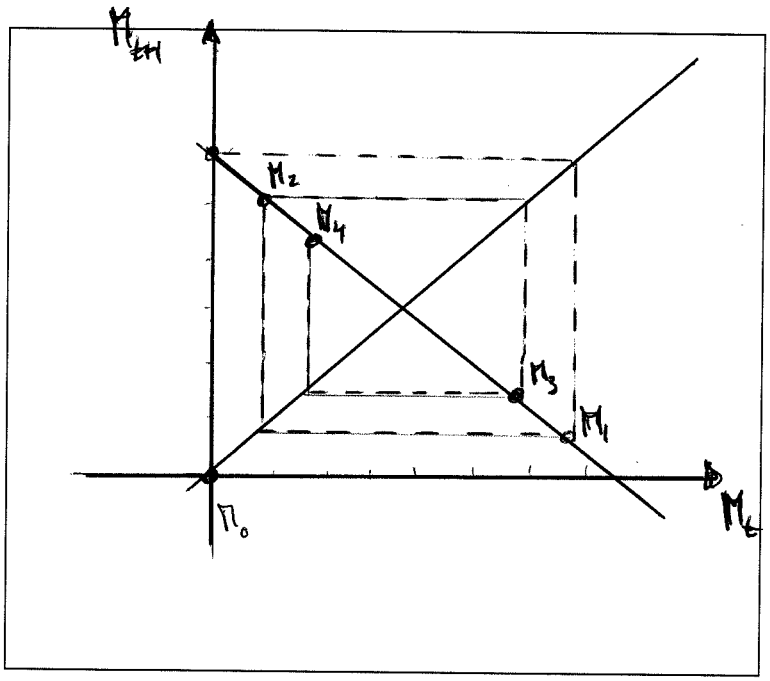
(b) [1 point] Find the equilibrium point of the DTDS. $x^* = \frac{10}{3}$

(c) [2 points] Give the solution formula for the DTDS with general initial condition M_0:

$M_t =$ $(-0.8)^t M_0 + 6\,\frac{1-(-0.8)^t}{1.8}$

(d) [1 point] Calculate M_{10} if $M_0 = 0$. $M_{10} =$ 2.975

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) [1 point] Is the equilibrium point stable or unstable? Stable

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Marks						

Question 1. [6 points] (a) Compute the derivative of the function $g(x) = \dfrac{1+x^{-3}}{\sqrt{x}+x^2}$. [Do not simplify your result.]

$g'(x) =$ $\dfrac{\frac{-3}{x^4}\left(\sqrt{x}+x^2\right) - \left(1+x^{-3}\right)\left(\frac{1}{2\sqrt{x}}+2x\right)}{\left(\sqrt{x}+x^2\right)^2}$

(b) Let $f(x) = \dfrac{|x-3|}{x^2-9}$.

$\lim_{x\to 3+} f(x) =$ $\lim_{x\to 3^+} \dfrac{x-3}{(x-3)(x+3)} = \dfrac{1}{6}$

$\lim_{x\to 3-} f(x) =$ $\lim_{x\to 3^-} \dfrac{-(x-3)}{(x-3)(x+3)} = -\dfrac{1}{6}$

Is f continuous at $x = 3$? Answer No

Question 2. [4 points] Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{1}{3x - 1}.$$

Question 3. [6 points] The angle of the sun above the horizon at 12 noon in Edmonton has its highest value in June with 59.1 degrees and its lowest value in December with 12.7 degrees. Assume that the height above the horizon can be written in standard cosine form.

(a) Find the values of the parameters A, B, Φ, T in the standard cosine description , i.e.,

$$f(t) = A + B\cos(2\pi(t - \Phi)/T),$$

where t is in months, and $t = 0$ corresponds to the month of January.

$f(t) =$ $35.9 + 23.2 \cos\left(\frac{2\pi}{12}(t-5)\right)$

(b) Give the names of the four parameters A, B, Φ, T.

(c) Draw the graph of the function and identify the four parameters A, B, Φ, T in the graph.

Graph of f

Question 4. [2 points] Is the following function continuous at $x = 5$? Justify your answer in a short sentence.

$$f(x) = \sin(2x) + \frac{3x^2 - 7x}{x^3 - 1}$$

Answer:

Question 5. [4 points]
(a) Find the critical point(s) of the function

$$f(x) = x^4 e^{-x}$$

$f'(x) = (4-x)x^3 e^{-x}$

Answer: $x = 0, \quad x = 4$

(b) Find the intervals where the function is increasing and where it is decreasing.

Increasing: $0 < x < 4$

Decreasing: $x < 0, \quad x > 4$

Question 6. [8 points] Consider the discrete-time dynamical system (DTDS)

$$M_{t+1} = -0.8M_t + 4$$

(a) [1 point] Find the updating function of the DTDS.

(b) [1 point] Find the equilibrium point of the DTDS. $\bar{X} = \frac{20}{9}$

(c) [2 points] Give the solution formula for the DTDS with general initial condition M_0:

$M_t =$

(d) [1 point] Calculate M_{10} if $M_0 = 0$. $M_{10} =$ 1.98

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.

(f) [1 point] Is the equilibrium point stable or unstable?

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$g'(x) =$ $\dfrac{\frac{1}{2\sqrt{x}}\left(x^2+\frac{1}{x}\right) - \left(1+\sqrt{x}\right)\left(2x-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x}\right)^2}$

(b) Let $f(x) = \dfrac{x^2-9}{|x-3|}$.

$\lim_{x\to 3+} f(x) =$ $\lim_{x\to 3^+} \dfrac{(x-3)(x+3)}{x-3} = 6$

$\lim_{x\to 3-} f(x) =$ $\lim_{x\to 3^-} \dfrac{(x-3)(x+3)}{-(x-3)} = -6$

Is f continuous at $x = 3$? Answer No

Question 2. [4 points] Use the definition of the derivative to calculate the derivative of the function

$$f(x) = \frac{1}{2x - 2}.$$

Question 3. [6 points] The angle of the sun above the horizon at 12 noon in Vancouver has its highest value in June with 64.0 degrees and its lowest value in December with 17.3 degrees. Assume that the height above the horizon can be written in standard cosine form.

(a) Find the values of the parameters A, B, Φ, T in the standard cosine description , i.e.,

$$f(t) = A + B\cos(2\pi(t - \Phi)/T),$$

where t is in months, and $t = 0$ corresponds to the month of January.

$f(t) =$ $40.65 + 23.35 \cos\left(\frac{2\pi}{12}(t-5)\right)$

(b) Give the names of the four parameters A, B, Φ, T.

(c) Draw the graph of the function and identify the four parameters A, B, Φ, T in the graph.

• Graph of f

Question 4. [2 points] Is the following function continuous at $x = 3$? Justify your answer in a short sentence.

$$f(x) = \cos(5x) + \frac{2x^3 - 5x}{x^2 - 7}$$

Answer:

Question 5. [4 points]
(a) Find the critical point(s) of the function

$$f(x) = x^5 e^{-x}$$

$f'(x) = (5-x)x^4 e^{-x}$

Answer: $x = 0$ and $x = 5$

(b) Find the intervals where the function is increasing and where it is decreasing.

Increasing: $x < 5$

Decreasing: $x > 5$

Question 6. [8 points] Consider the discrete-time dynamical system (DTDS)

$$M_{t+1} = -0.8M_t + 8$$

(a) [1 point] Find the updating function of the DTDS.

(b) [1 point] Find the equilibrium point of the DTDS. $\bar{x} = \frac{40}{9} \approx 4.44$

(c) [2 points] Give the solution formula for the DTDS with general initial condition M_0:

$M_t =$

(d) [1 point] Calculate M_{10} if $M_0 = 0$. $M_{10} =$ 3.967

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.

(f) [1 point] Is the equilibrium point stable or unstable?