University of Ottawa
MAT 1330A Midterm Exam

October 14, 2009. Duration: 80 minutes. Instructor: Frithjof Lutscher

Family Name:______________________

First Name:______________________

DGD 1 (Monday)   DGD 2 (Tuesday)   DGD 3 (Wednesday)   DGD 4 (Thursday)

Do **not** write you student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number:____________________, Total marks: ___________ out of 29

Problem	1	2	3	4	5	6
Marks						

Question 1. [4 points] Find the derivative of the following function:

$$f(x) = \frac{x^4 \ln(x)}{2x + 8e^x}$$

Question 2. [4 points] Over the course of a year, the city of Ottawa has its highest average monthly temperature of 26°C in August and its lowest monthly average of -8°C in February. Assume that temperature varies sinusoidally over a period of one year. Find the parameters in the standard cosine description , i.e.,

$$f(t) = A + B\cos(2\pi(t - \Phi)/T),$$

where t is in months, and $t = 0$ correspond to the month of January. Draw the graph of the function and identify the four parameters A, B, Φ, T in the graph. Give the names of the four parameters.

Question 3. [4 points] (a) Find the following limit without using a calculator

$$\lim_{x \to 2} \frac{|x-3|-1}{x^2-4}$$

(b) Does the following limit exist? If yes, give the limit, if not, justify your answer. If you need a calculator to work out the answer, give at least 4 values of x that you tried.

$$\lim_{x \to -2} \frac{|x-3|-1}{x^2-4}$$

Question 4. [4 points] Use the definition of the derivative to calculate the derivative of the function $f(x) = (14x - 10)^2 + 2009$.

Question 5. [9 points] The human metabolism will break down morphine in the blood stream to 70% of its original amount within a 4-hour period. A patient is given a dose of $d = 9$ mg of morphine every 4 hours. The DTDS for the amount of morphine in the patient's body is given by

$$M_{t+1} = 0.7M_t + d = 0.7M_t + 9,$$

where t is measured in 4-hour intervals and morphine is measured just after a new dose has been administered.

(a) [1 point] What is the updating function of the DTDS?

(b) [1 point] Find the fixed point of the DTDS if there is one.

(c) [2 points] Find the general solution formula for the DTDS, i.e., $M_t = ...$

(d) [2 points] Graph the updating function and draw the cobwebbing, starting from $M_0 = 5$ for at least 4 steps.

(e) [1 point] Is the fixed point stable or unstable?

(f) [2 points] The doctor wants to adjust the dose d so that the patient has 40 mg of morphine in the blood stream in the long run. What should the dose d be in that case?

Additional space for question 5

Question 6. [4 points] Consider the graph of a function f below.
Find the critical points of f. Determine the intervals where f' is positive and the intervals where f' is negative.