COMM 290 Final Review

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Quantitative Decision Making

- Linear Program
  - Formulation
  - Optimization
  - Sensitivity Analysis

- Probability
  - Basic Rules
  - Joint Probability, Conditional Probability, Bayes Theorem

- Decision Analysis
  - Decision Tree
  - Value of Information

- Random Variables
  - Expected value and variance
  - Independent and dependent variables

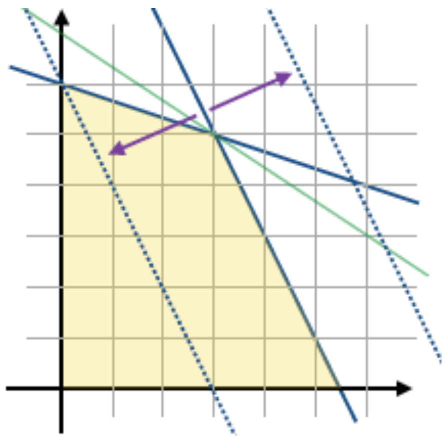
Linear Program

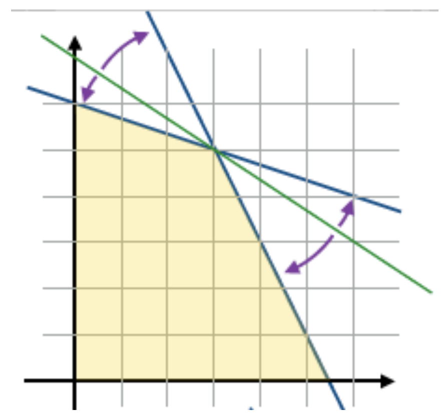
- Formulation
    - Binding Constraint
        - LHS = RHS
        - amount used = amount available
        - shadow price =/= 0
    - Non-binding Constraint
        - LHS =/= RHS
        - amount used =/= amount available
        - Shadow price = 0
    - Iso-profit line
        - graph the objective function
        - the slope of the objective function is -C1/C2

- Optimization
    - Assume input data are known precisely
    - use constraints to figure out what is feasible
    - then, use iso-profit line to fins the best corner point of the feasible region.
    - the point is the optimal solution.
    - the target cell value is the value of the objective function at the optimal solution.

- Sensitivity Analysis
    - Sensitivity analysis on an **objective coefficient**.
        - slope of one binding constraint <= slope of objective function <= slope of other objective constraint
        - Use C1 or C2 in place of actual value.
        - Find the range over which the coefficient can change.
        - Allowable increase and allowable decrease are how much the coefficient can change while maintaining the same optimal solution
    - Sensitivity analysis on RHS.
        - Allowable increase/decrease of a **binding constraint**
            - Move the constraint, with the same slope, to where the other binding constraint intersects with another constraint.

        - Allowable increase/ decrease is how much the original RHS can change by reach the new RHS
        - Within the allowable range, **shadow price remain the same**
    - Allowable increase/decrease of a <u>**non-binding constraint**</u>
        - Move the constraint away from the optimal solution, it can go infinitely far in the direction. so the change is **infinity**.
        - Move the constraint toward the optimal solution, it can move until it touches the optimal solution.
        - **Shadow price is zero**, but as soon as the non-binding constraint ouches the optimal solution, i becomes binding and it will no longer have a zero shadow price.



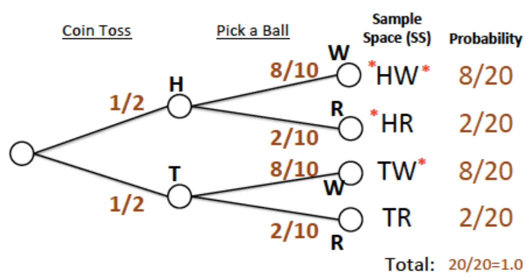


- <u>**Shadow Price**</u>
    - the shadow price indicates the change in the optimal value of the objective function when the right-hand side of some constraint changes by a given amount.
    - take a binding constraint and +1 to RHS
    - Find new optimal solution
    - calculate new profit
    - shadow price = new profit - old profit
    - the shadow price of non-binding constraint is zero.
- <u>Sensitivity Report</u>
    - 1E+30 means infinite
    - 5.2E-16 means 0
- <u>Multiple Optima</u>
    - if anyone of the allowable increase or decrease on the objective function is zero, then there are multiple optima.

- a RHS value is changed <u>beyond</u> the allowable range
    - shadow price will decrease and even be zero.w
- Infeasible models
    - there is no feasible region
- Unbounded models
    - the feasible rein continues forever in the same direction as improving objective function (i.e., you move iso-profit line and just keep going.

Probability

- Basic Rules
    - Probability Terminology
        - Sample space: set of all possible outcomes
        - Event: a possible outcome of an experiment
        - Probability: between 0 and 1
        - Independence: A and B are independent events if the outcome of A does not affect the outcome of B
        - Mutually exclusive: A and B are mutually exclusive events if they cannot happen simultaneously
    - Probability Tree



    -
    - Probability Formula "OR"
        - Mutually exclusive: P(A or B) = P(A) + P(B)
        - Generally: P(A or B) = P(A) + P(B) - P(A and B)
    - Probability Formula "AND"
        - Independence: P(A and B) = P(AB) = P(A)*P(B)
        - Generally: P(A and B) = P(AB) = P(A)*P(B|A)

- Conditional and Joint Probability, Bayes Theorem
    - Conditional equals joint over marginal

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

    - Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Decision Analysis

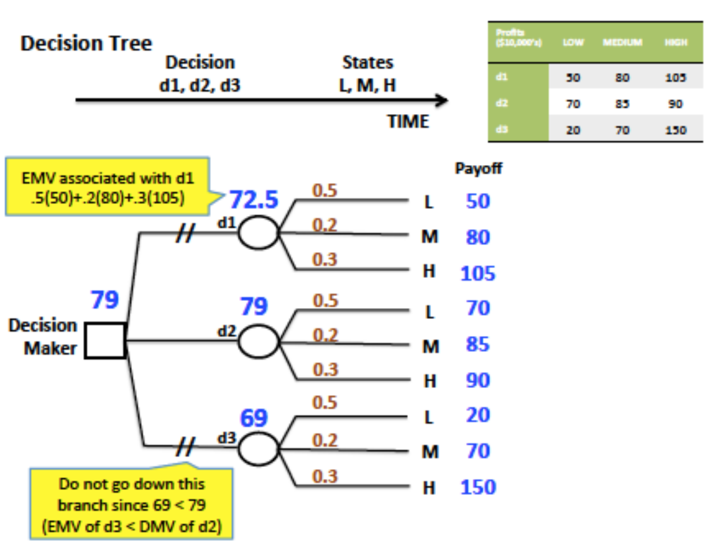
- Approaches to decision making (without probability)
  - Conservative Approach: For each decision, attach the **worst outcome**, pick the decision with the best attached
  - Optimistic Approach: For each decision, attach the **best outcome**, Pick the decision with the best attached
  - Regret Approach: For each decision, attach the **highest possible regret**, pick the decision with the smallest attached regret

Profits ($10,000's)	LOW	MEDIUM	HIGH	CONSER-VATIVE	OPTIMISTIC	REGRET
d1	50	80	105	50	105	150–105 = 45*
d2	70	85	90	70*	90	150-90 = 60
d3	20	70	150	20	150 *	70-20 = 50
d4	20	60	140	20	140	70-20 = 50

- Approach to decision making with probability (EMV)
  - Payoff Matrix

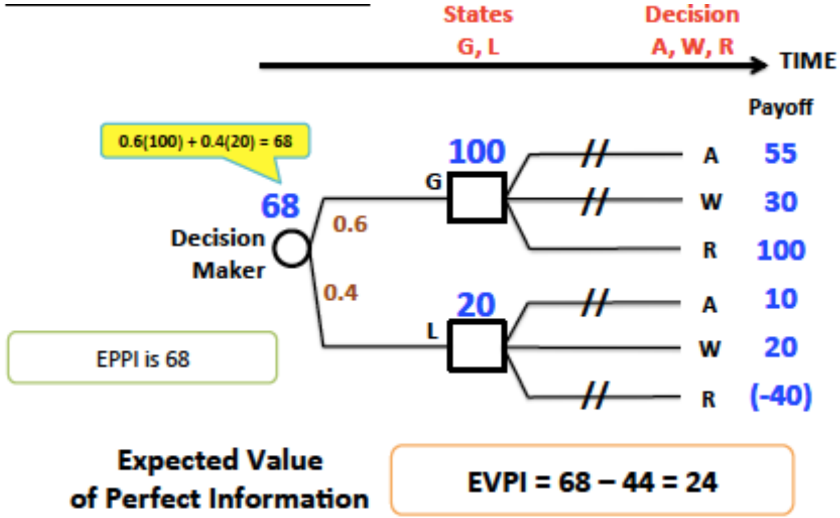
Probability	0.5	0.2	0.3	
Profits ($10,000's)	LOW	MEDIUM	HIGH	Expected Monetary Value (EMV)
d1	50	80	105	EMV(d1) = 0.5(50) + 0.2(80) + 0.3(105) = 72.5
d2	70	85	90	EMV(d2) = 0.5(70) + 0.2(85) + 0.3(90) = 79*
d3	20	70	150	EMV(d3) = 0.5(20) + 0.2(70) + 0.3(150) = 69
d4	20	60	140	EMV(d4) = 0.5(20) + 0.2(60) + 0.3(140) = 64

  - Decision Tree

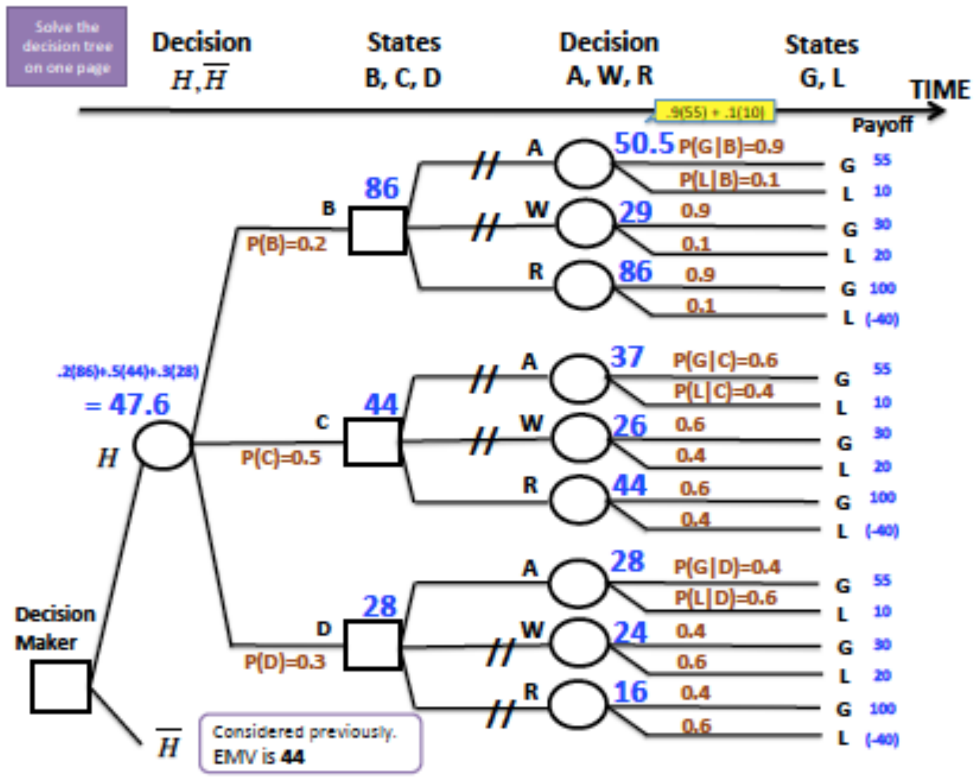


- **solving the tree**: work backwards by folding back the tree, for a "state" node, compute the expected value; for a "decision" node, choose the best alternative.

- Value of Information
  - Expected value of perfect information (EVPI)
    - EPPI: expected profit/payoff with perfect information
    - EMV: expected profit with uncertainty
    - EVPI: expected value of perfect information
    - EVPI = EPPI - EMV
    - Efficiency = EVPI/EPPI*100%



    - Sensitivity analysis
  - Expected value of perfect information (EVSI)
    - EPSI: expected profit/payoff with sample information
    - EVSI: expected value of sample information
    - EVSI = EPSI - EMV
    - Efficiency = EVSI/EVPI*100%



Solve the decision tree on one page
Decision
$H, \overline{H}$
States
B, C, D
Decision
A, W, R
States
G, L
TIME
.9(55) + .1(10)
Payoff
Decision Maker
H
.2(86)+.5(44)+.3(28)
= 47.6
B
P(B)=0.2
86
A
50.5
P(G|B)=0.9
P(L|B)=0.1
G 55
L 10
W
29
0.9
0.1
G 30
L 20
R
86
0.9
0.1
G 100
L (-40)
C
P(C)=0.5
44
A
37
P(G|C)=0.6
P(L|C)=0.4
G 55
L 10
W
26
0.6
0.4
G 30
L 20
R
44
0.6
0.4
G 100
L (-40)
D
P(D)=0.3
28
A
28
P(G|D)=0.4
P(L|D)=0.6
G 55
L 10
W
24
0.4
0.6
G 30
L 20
R
16
0.4
0.6
G 100
L (-40)
$\overline{H}$
Considered previously.
EMV is 44

Random Variables

- Expected Value & Variance
  - Random Variables
    - A random variable (RV) is a numerical description of the outcomes of an experiment.
    - the value is random
    - each value has an associated probability
    - types: discrete & continuous
  - Expected Value (Discrete Random Variable)
    - expected value of X

$$E(X) = \sum_{\text{all possible } x} x \cdot p(x)$$

    - E(Y) where Y=f(x)

$$E(Y) = \sum y \cdot p(y) = \sum f(x) \cdot p(x)$$

$$E(\alpha X) = \alpha \cdot E(X)$$

$$E(A+B) = E(A) + E(B)$$

  - Variance (Discrete Random Variable)

$$VAR(X) = \sum [x - E(X)]^2 \cdot p(x)$$

$$VAR(X) = E(X^2) - E(X)^2$$

$$\sigma^2(\alpha X) = \alpha^2 \cdot \sigma^2(X)$$

If A and B are independent,

$$\sigma^2(A+B) = \sigma^2(A) + \sigma^2(B)$$