University of Ottawa
MAT 1330A Final Exam

December 10, 2009. Duration: 3 hours. Instructor: Frithjof Lutscher

Family Name:______________________

First Name:________________________

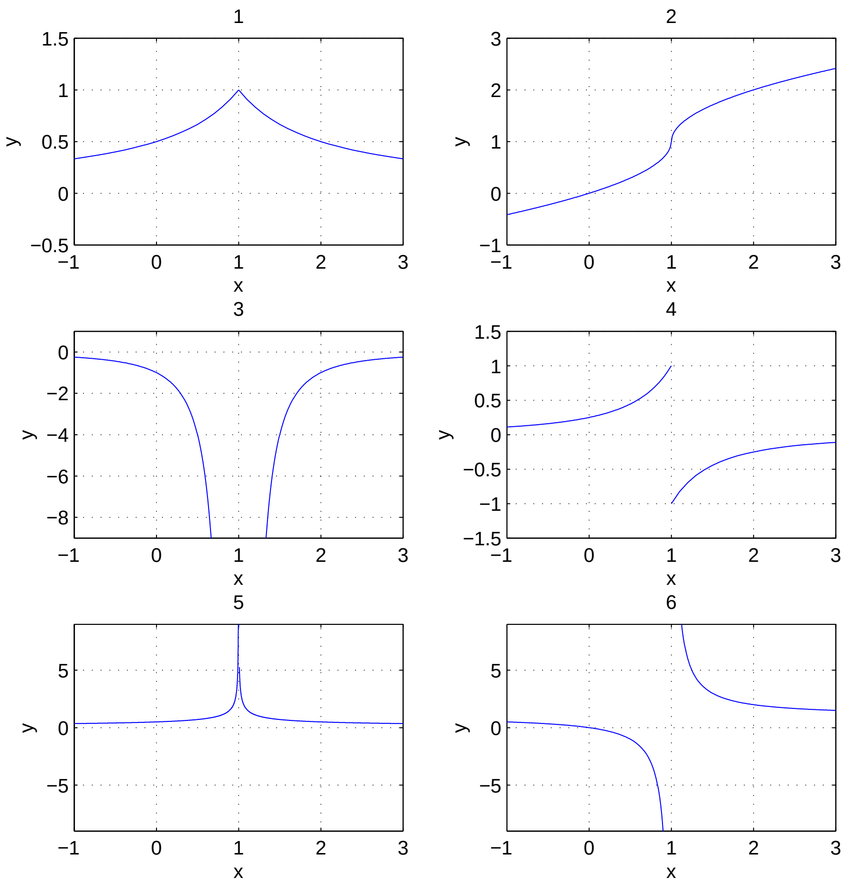
Student Number:________________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 3 hours to complete this exam. You can use the back of the pages to write our solutions.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Question	1	2	2	4	5	6	7	8	9	10	11	12	Total (60)
Points													

Question 1. [3 points] Three of the figures represent the graphs of functions and the other three represent the graphs of the derivatives of these functions. Write down the pairs (n_1, n_2), where n_1 is the number of the figure associated to the graph of a function f and n_2 is the number of the figure associated to the graph of the derivative f' of the function. **You cannot use a figure more than once**.



Answer:

n_1	n_2

Question 2. [1 point] Suppose that the Taylor polynomial of degree three with base point $a = 1$ for a certain function f is given by $P_3(x) = 1 + 2(x-1) + 3(x-1)^2 + 4(x-1)^3$. Find the curvature of the function f at $x = 1$, i.e., $f''(1) =$ ☐

Question 3. [6 points] Determine if the following limits exist. If the limit exists, compute the limit without using table of values.

(a) $\lim_{x \to 2} \dfrac{|x-2|}{x^3 - 4x} =$

(b) $\lim_{x \to 0} \dfrac{\ln(x^2+1)}{\sqrt{x}} =$

(c) $\lim_{x \to \infty} \dfrac{e^x}{x \ln(x)} =$

Question 4. [4 points] Compute the derivatives of the following functions

(a) $f(x) = \ln\left(5e^{x^2}(x^2+1)^2\right)$. You must simplify your answer.

$f'(x) =$

(b) $g(x) = \dfrac{1}{\sin^2(x^2)}$.

$g'(x) =$

Question 5. [4 points] Consider the function $f(x) = \dfrac{x^2+2x}{e^x}$ on the interval $x \in [0,4]$.

(a) The global maximum occurs at $x =$

(b) The global minimum occurs at $x =$

Question 6. [4 points] Consider the function $f(x) = 1 + \sqrt{2}\sin(\sqrt{2}(x-1))$.

(a) Use the linear approximation of f to estimate the value of $f(0.9)$.

The linear approximation is [ ]

and $f(0.9)$ is approximately [ ]

(b) Use a Taylor polynomial of degree 3 to approximate $f(0.9)$.

The Taylor polynomial is

[ ]

and $f(0.9)$ is approximately [ ]

Question 7. [12 points] Compute the following indefinite integrals:

(a) $\displaystyle\int \frac{t^{5/7}+t}{t^{1/7}}\,dt =$

(b) $\displaystyle\int x^2 \ln(5x)\,dx =$

(c) $\displaystyle\int \sin^3(2x)\cos^2(2x)\,dx =$

(Hint: $\sin^2(\theta) = 1 - \cos^2(\theta)$.)

Question 8. [3 points] The height of a tree is increasing according to the equation

$$\frac{dH}{dt} = \frac{1}{1+2t}$$

Initially, the tree has hight zero. Find the time T at which the tree will have height 2.

$H(t) =$ [ ]

$T=$ [ ]

Question 9. [4 points] A patient receives each day one dose of 5μg of a drug that is initially absent from the body. Over the course of 24 hours half the drug gets cleared from the body. Denote by x_t the amount of drug in the patient's body in μg right after drug administration on day t.

(a) The linear DTDS for the amount of drug is $x_{t+1} =$ [ ]

(b) The steady state for this DTDS is $x^* =$ [ ]

(c) The explicit solution of the DTDS is given by

$x_t =$ [ ]

Question 10. [7 points] Consider a population that grows according to the logistic updating function and is harvested according to a linear rate $h \geq 0$. The number of individuals of the species satisfies the DTDS

$$x_{t+1} = x_t(4 - x_t) - hx_t, \qquad t = 0, 1, 2, \ldots.$$

(a) The steady states of this DTDS are

(b) The steady states are biologically relevant only when h is in the interval

(c) The positive steady state is stable precisely if h is in the interval

(d) The positive steady state is unstable precisely if h is in the interval

(e) The maximal yield occurs for a harvesting rate of $h =$.

The maximum yield at this value of h is given by

Question 11. [4 points] Consider the function $f(x) = \dfrac{x}{3x-1}$. Calculate $f'(x)$ using the **definition of the derivative** (i.e. compute the derivative from first principles).
(A lot of importance will be given to the mathematical notation.)

Question 12. [8 points] Use the first and second order derivatives to sketch the graph of

$$f(x) = \frac{x^2 + x - 2}{x^2}$$

You have to find the zeros, critical points, inflexion points, intervals where the function is increasing and decreasing, and the vertical and horizontal asymptotes if any.

(a) Zeros and signs of f

x	
$f(x)$	

(b) Calculate f' and find critical points

$f'(x) =$

x	
$f'(x)$	

(c) Calculate f'' and find inflexion points

$f''(x) =$

x	
$f''(x)$	

(d) The vertical asymptote(s) is (are) given by (if applicable)

(e) The horizontal asymptote(s) is (are) given by (if applicable)

(f) Draw the graph of f

Additional space for calculations