Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

MAT 1330D Final Exam

April, 2011. Duration: 3 hours. Instructor: Aziz Khanchi

Family Name:_____ First Name:_____

Student Number:_____ Seat Number:_____

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 3 hours to complete this exam. You can use the back of the pages to write our solutions.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Question	1	2	3	4	5	6	7	8	9	10	Total (60)
Points											

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Question 1. [4 points] Compute the derivatives of the following functions. Do NOT simplify.

(a) $f(x) = (\sin x) \ln\left((x+2)^3\right)$.

$f'(x) =$

(b) $g(x) = \sqrt{\dfrac{e^{2x}}{4x+1}}$.

$g'(x) =$

Question 2. [8 points] Determine if the following limits exist. If the limit exists, compute the limit without using table of values. If the limit does not exist determine if it is $\infty, -\infty$ or neither.

(a) $\lim_{x\to -2} \dfrac{x^2+x-6}{x^2-2x+4} =$ $\boxed{}$

(b) $\lim_{x\to 0} \dfrac{1-\sqrt{1+x}}{x} =$ $\boxed{}$

(c) $\lim_{x\to 0^+} \sqrt{x}\ln x =$ $\boxed{}$

(d) $\lim_{x\to \infty} \dfrac{e^x}{x^2} =$ $\boxed{}$

Question 3. [4 points] Determine all continuity points of the function

$$g(x) = \begin{cases} \frac{x-2}{x^2-4} & \text{if } x > 2 \\ x - \frac{7}{4} & \text{if } x \leq 2. \end{cases}$$

Answer:

Question 4. [4 points] Find the Taylor polynomial of degree 3 for $f(x) = \ln(2 - x)$ centered at $a = 1$.

(a) $P_3(x) =$

(b) Use this polynomial to approximate $f(0.9)$. $P_3(0.9) =$

Question 5. [4 points]

(a) [1 points] Let f be a function. Give the definition for the derivative $f'(x)$.

(b) [3 points] Consider the function $f(x) = \dfrac{5}{x-3}$. Use the definition in part (a) to find $f'(x)$.

Question 6. [5 points] Consider the following variant of the logistic model:

$$N_{t+1} = 2.5N_t(1 - N_t) - hN_t \qquad t = 0, 1, 2, \ldots$$

where N_t is the population at time t and h is a non-negative constant.

(a) The equilibrium point of the system that depends on h is $N^* =$ [ ]

(b) Knowing that $h \geq 0$, find the closed interval for h on which N^* is non-negative.

Answer: [ ]

(c) Define the equilibrium harvest to be $P(h) = hN^*$ on the interval in part (b). Find h that maximizes the equilibrium harvest.

$h=$ [ ]

Question 7. [12 points] Compute the following indefinite integrals:

(a) $\int (\sin(x) - 3x^4)dx =$

(b) $\int \frac{1 - e^{-2x}}{e^{-2x}}dx =$

(c) $\int 9x \ln(3x)dx =$

(d) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx =$

Question 8. [4 points] Consider the function $f(x) = x^2 + x - 1$.
(a) [2 points] Use the Intermediate Value Theorem to explain why this function has a root in the interval $[0, 1]$.

(b) [2 points] Use Newton's method with $x_0 = 0.5$ to calculate the root to 3 decimal places.

$x^* =$

Question 9. [4 points] A bacterial colony has population $P(t)$ at time t where P is given in millions (m) and time is in seconds. Suppose that the function P satisfies the differential equation $\dfrac{dP(t)}{dt} = 10e^{2t}$ with initial condition $P(0) = 10$. Answer the following questions:

(a) Find the equation for the population as a function of time.

$P(t) =$ [ ]

(b) How long will it take the population to reach 25 m?

$t=$ [ ]

Question 10. [10 points] Consider the function $f(x) = \frac{1}{x^2-1}$. Follow these steps to graph the function.

(a) The domain of f is

(b) The y-intercept(s) of f are

(c) Show that the derivative of f is $f'(x) = \dfrac{-2x}{(x^2-1)^2}$.

(d) The critical point(s) of f are

(e) The second derivative of f is $f''(x) =$ [ ]

(f) The point(s) of inflection are [ ]

(g) The vertical asymptotes of f are [ ]
You should justify your answer.

(h) $\lim_{x\to\infty} f(x) =$ [ ]

(g) $\lim_{x\to-\infty} f(x) =$ [ ]

(i) f is increasing on

f is decreasing on

f is concave upward on

f is concave downward on

(j) The graph of f is