University of Ottawa
MAT 1332 Practice Midterm 2

Family Name:_____

First Name:_____

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number:_____, Total marks: _____ out of 32

Problem	1	2	3	4	5	6
Marks						

Question 1. [4 points] Consider the differential equation

$$\frac{dx}{dt} = 25x - x^3.$$

(a) Find all equilibria.
(b) Determine the stability of each equilibrium point.
(c) Draw the phase-line diagram for the differential equation.
(Do **not** solve the equation explicitly.)

Question 2. [6 points] Consider the equation

$$x^3 - 6x^2 + 13x - 10 = 0,$$

(a) Show that $x_1 = 2$ is a solution of the equation.
(b) Use long division to show that the other two roots are $x_2 = 2 - i$ and $x_3 = 2 + i$.
(c) Calculate $x_2 x_3$.
(d) Calculate $\frac{x_2}{x_3}$.
(e) Express $(3 + i)^2$ in the form $a + ib$.
(f) Express e^i in the form $a + ib$.

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Question 3. [5 points] (a) For which values of c is the matrix

$$B = \begin{pmatrix} 1 & 0 & c \\ 2 & 4 & 5 \\ 3 & 1 & 0 \end{pmatrix}$$

invertible?
(b) Consider the system

$$\begin{array}{rcrcrcl} 5x & + & 3y & + & z & = & 1 \\ x & + & 2y & + & z & = & 2 \\ 3x & - & y & + & a\,z & = & b \end{array}$$

(i) Determine the conditions on a and b to get a unique solution.
(ii) Determine the conditions on a and b to get infinitely many solutions.
(iii) Determine the conditions on a and b such that the system has no solutions.

Question 4. [6 points] Let A be the matrix

$$A = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of A are $4 + i$ and $4 - i$.
(b) Find an eigenvector corresponding to the eigenvalue $4 + i$.

Question 5. [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 + 5x_2 + x_3 &= 1. \end{aligned}$$

Be sure to show all your work.

Question 6. [8 points] Consider the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -5 & 0 \\ 4 & 0 & 5 \end{bmatrix}.$$

(a) Find A^{-1}.

(b) Solve the equation $Ax = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$.

(c) Show that $\lambda_1 = -5$ is an eigenvalue of A and find the other two eigenvalues.

(d) Find the eigenvectors corresponding to $\lambda_1 = -5$.

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