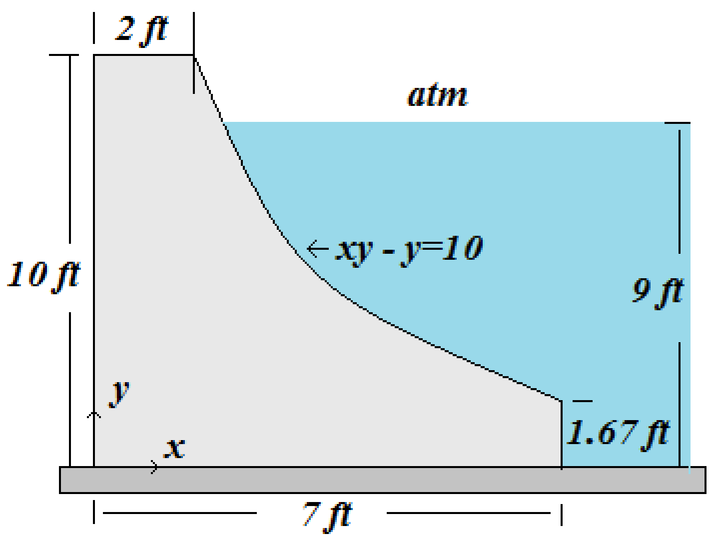
CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
FINAL EXAMINATION
FLUID MECHANICS I ENGR 361/T
WINTER 2013

**Instructor: Dr. Nabil Esmail**

**Question No. 1.** (25 marks). A dam is to be constructed across a river using the cross-section shown, where the curved section follows the equation $xy - Ay = B, A = 1\ ft, B = 10\ ft^2$. The dam's width is $w = 160\ ft$. For water height $9\ ft$, (a) calculate the magnitude and lines of action of the vertical and horizontal forces of water on the dam face. (b) Provide proof whether it is possible that the water forces could overturn the dam? Water density 1.94 $slugs/ft^3$, gravity 32.2 $ft/s^2$ and 1 $slug = 1\ Ib_f.s^2/ft$.




$$F_H = \rho g h_c A = 1.94\frac{slug}{ft^3}X32.2\frac{ft}{s^2}X\frac{9ft}{2}(9ftX160ft)\frac{Ib_f.s^2/ft}{slug} = 4.05X10^5 Ib_f \ \ Answer$$

$$y_H = \frac{2}{3}X9ft = 6\ ft\ from\ water\ surface\ ANSWER$$

$$F_{H1} = \rho g h_c A = 1.94\frac{slug}{ft^3}X32.2\frac{ft}{s^2}X\frac{(9-1.67)ft}{2}((9-1.67)ftX160ft)\frac{Ib_f.s^2/ft}{slug} = 2.69X10^5 Ib_f$$

$$F_{H2} = \rho g(9-1.67)(1.67X160)ft^3 + \rho g h_c A$$

$$y_{H1} = \frac{2}{3}X(9-1.67)ft = 4.89\ ft$$

$$= 1.94\frac{slug}{ft^3}X32.2\frac{ft}{s^2}\left\{(9-1.67)(1.67X160)ft^3 + \frac{(9+1.67/2)ft}{2}(1.67ftX160ft)\right\}\frac{Ib_f.s^2/ft}{slug}$$
$$= 1.36X10^5 Ib_f$$

$$y_{H2} = ??$$

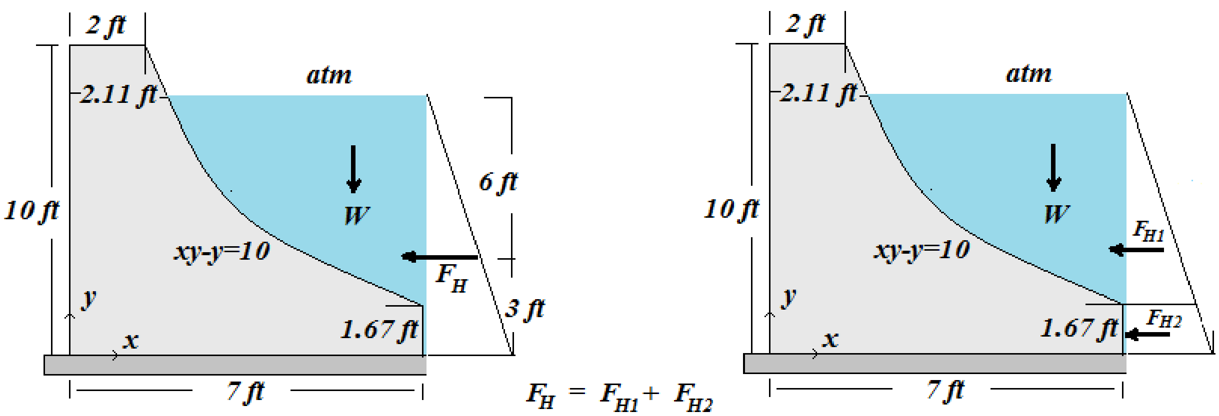
$$xy - y = 10, for\ y = 9, x = \frac{19}{9} = 2.11\ ft$$

$$Element\ of\ volume\ 160ft\ XdA = 160ft\ X(9 - y)dx = 160ft\ X\left(9 - \frac{10}{x-1}\right)dx$$

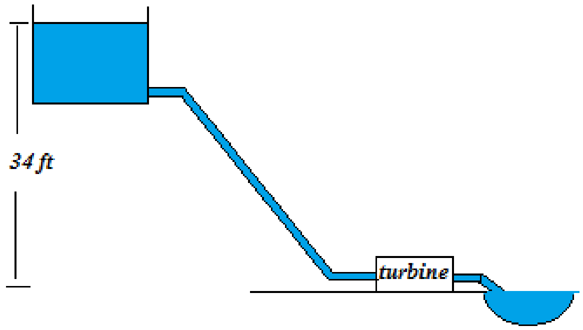
$$Weight = W = \rho g Volume$$

$$W = \rho g(160\ ft)\int_{2.11}^{7}\left(9 - \frac{10}{x-1}\right)dx = \rho g(160\ ft)[9x - 10\ln(x-1)]_{2.11}^{7} = 2.71X10^5 Ib_f \quad Answer$$

$$W.x_W = \rho g(160\ ft)\int_{2.11}^{7}\left(9 - \frac{10}{x-1}\right)xdx, \qquad x_W = 4.96\ ft \quad Answer$$




**Question No. 2.** (20 marks) A small stream of water of $Q = 2\ cfs$ (cubic feet per second) at temperature 40° F is falling from an elevation of $H = 34\ ft$ above ground. It is planned to divert this stream into a commercial-grade plastic pipe (absolute roughness $e = 1.2\ 10^{-5}\ ft$) that will supply an electric-generating turbine located on the ground. The plastic supply pipe is 87 $ft$ long. The maximum acceptable head loss in the supply pipe is 3 $ft$. Neglecting minor losses find the minimum diameter that can be used for the supply pipe. Assume pipes available in diameters 5, 6, 7, 8 and 9 in., and a friction factor in the range $f = 0.02 - 0.01$. Water density $1.94\ slugs/ft^3$, $viscosity$ $2.344\ 10^{-5}\ Ib_f.s/ft^2$.




$$Flow\ Q = 2\ ft^3/s\ through\ L = 87\ ft\ of\ plastic\ tubing$$

$$Diameter\ D_1 = D_2, V_1 = V_2, Therefore\ \frac{\Delta p}{\rho} + gh = h_L$$

$$Maximum\ allowable\ head\ loss\ h_L = 3g = 96.51\ ft^2/s^2,$$

$$Q = VA = V\left(\frac{\pi D^2}{4}\right),\ \ V = \frac{4Q}{\pi}\left(\frac{1}{D^2}\right),$$

$$Re = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu}\left(\frac{1}{D}\right),$$

$$e = 0.000012\ ft, \frac{e}{D} = 0.000012\left(\frac{1}{D}\right)$$

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{e}{3.7065D} + \frac{2.5226}{Re\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2} \leq 96.51\ ft^2/s^2$$

$$Algorithm\ of\ solution$$

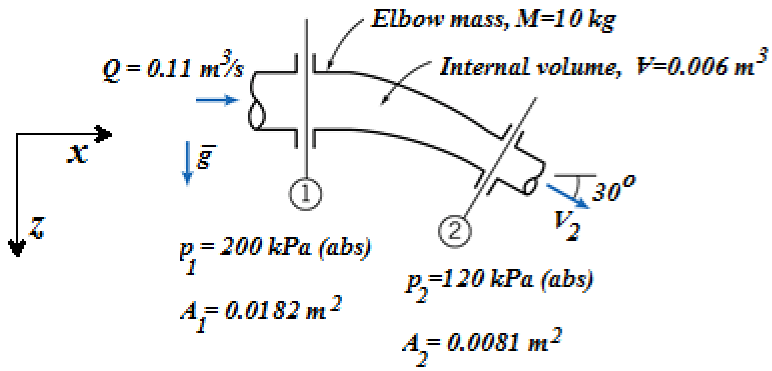
$$1.\ Assume\ a\ Diameter, \quad 2.\ Calculate\ V\ and\ Re\ and\ e/D,\ \ 3.\ Find\ f$$

$$4.\ Check\ \ h_L = f\frac{L}{D}\frac{V^2}{2} \leq 96.51\ ft^2/s^2,$$

$$Results:\ 5\ in\ h_L = 304.29, 6\ in\ h_L = 125.39, 7\ in\ h_L = 59.38, 8\ in\ h_L = 31.12, 9\ in\ h_L = 17.61$$

$$Answer\ 7\ in\ pipe$$

**Question No. 3.** (20 Marks) A 30° reducing elbow is a part in a water (ρ=999 kg/m$^3$, g=9.81 m/s$^2$) pipeline. The information given is marked on the Figure below. Compute the components of the force required to hold the elbow from moving,




$$(\rho_{in}\bar{V}_{in}.\bar{A}_{in})\bar{\boldsymbol{V}}_{\boldsymbol{in}} - (\rho_{out}\bar{V}_{out}.\bar{A}_{out})\bar{\boldsymbol{V}}_{\boldsymbol{out}} + \sum \bar{\boldsymbol{F}}_{cv} = 0$$

$$\boldsymbol{V_1(\rho V_1 A_1) - [V_2 cos30(\rho V_2 A_2)] + p_1 A_1 - p_2 A_2 cos30 + F_{Xanchor} = 0}$$

$$\boldsymbol{Mass\ balance\ \ \rho V_1 A_1 = \rho V_2 A_2 = \rho Q,\ \ V = \frac{Q}{A},\ \ V(\rho VA) = \frac{Q}{A}(\rho Q) = \rho\frac{Q^2}{A}}$$

$$\boldsymbol{-F_{Xanchor} = \rho\frac{Q^2}{A_1} - \rho\frac{Q^2}{A_2}cos30 + p_1 A_1 - p_2 A_2 cos30}$$

***We subtract atmospheric because the CV is the entire elbow***

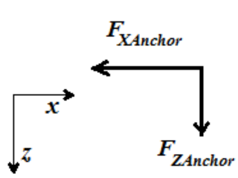
$$\boldsymbol{-F_{Xanchor} = 999\,\frac{kg}{m^3}\left(0.11\,\frac{m^3}{s}\right)^2\left(\frac{1}{0.0182\,m^2} - \frac{cos30}{0.0081\,m^2}\right) + (200-101)kPa.\,0.0182\,m^2 - (120-101)kPa.\,0.0081\,m^2 cos30 = 1040\,N,\ \ F_{Xanchor} = -1040\,N\ Answer}$$

$$\boldsymbol{= 664.17(Mom)_{in} - 1292.4(Mom)_{out} + 1801.8(P)_{in} - 133.28(P)_{out}}$$

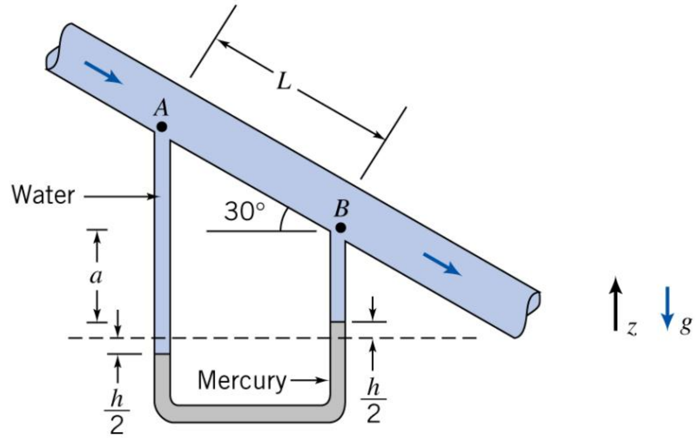
$$\boldsymbol{-F_{Zanchor} = 999\,\frac{kg}{m^3}\left(0.11\,\frac{m^3}{s}\right)^2\left(-\frac{sin30}{0.0081\,m^2}\right) - (120-101)kPa.\,0.0081m^2 sin30 + \left(10kg\left(9.81\frac{m}{s^2}\right) + 999\,\frac{kg}{m^3}\left(9.81\frac{m}{s^2}\right)0.006m^2\right) = -666.2\,N,}$$

$$\boldsymbol{F_{Zanchor} = 666.2\,N\quad Answer}$$

$$\boldsymbol{= -746.17(Mom)_{out} - 76.95(P)_{out} + 156.90(Weight)}$$




**Question No. 4 (**20 Marks**)** Water flows downward along a pipe that is inclined at 30° below the horizontal. Drive an expression for the pressure difference $p_A - p_B$ in terms of the distances $L\ and\ h$. Water density $\rho = 1.94\ slug/ft^3$, $g = 32.2\ ft/s^2$, mercury specific gravity $SG = 13.55$, $slug = lb_f.s^2/ft$. For two different pipe lengths $L = 5\ ft, and\ 10\ ft$ the corresponding mercury readings are $h = 6\ in\ and\ 12\ in\ respectively$, find the pressure drop relationship $\frac{\Delta p}{L}$ for this pipe flow.




$$p_A = p_B + \rho_w g a + \rho_m g h - \rho_w g(h + a + L.\sin(30)) =$$

$$p_A - p_B = 13.55X\rho_w g h - \rho_w g(h + L.\sin(30))$$

$$p_A - p_B = \rho_w g\left(13.55h - h - \frac{L}{2}\right)$$

$$p_A - p_B = \rho_w g\left(h(13.55 - 1) - \frac{L}{2}\right)$$

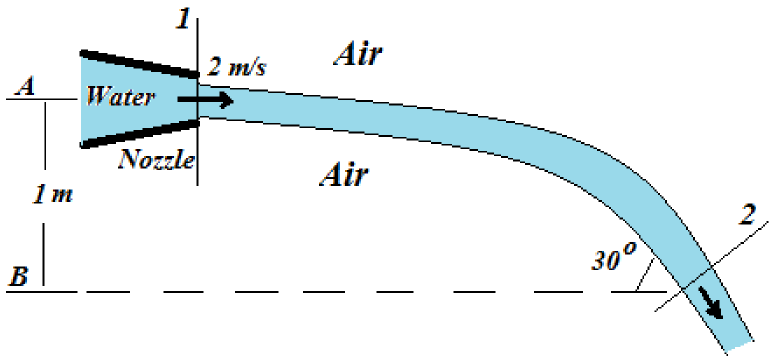
$$p_A - p_B = \rho_w g\left(12.55h - \frac{L}{2}\right) \quad Answer$$

$$p_A - p_B = 1.94\frac{slug}{ft^3}32.2\frac{ft}{s^2}\left(12.55\left(6\ in\frac{ft}{12\ in}\right) - \frac{5ft}{2}\right)\frac{Ib_f.s^2}{slug.ft}\left[\frac{1ft^2}{144in^2}\right] = 1.6376\ psi$$

$$p_A - p_B = 1.94\frac{slug}{ft^3}32.2\frac{ft}{s^2}\left(12.55\left(12\ in\frac{ft}{12\ in}\right) - \frac{10ft}{2}\right)\frac{Ib_f.s^2}{slug.ft}\left[\frac{1ft^2}{144in^2}\right] = 3.2752\ psi$$

$$\frac{\Delta p}{L} = 2 \quad Answer$$

**Question No. 5 (**15 Marks**)** A round nozzle 2 cm diameter at the exit horizontally issues a water jet into air with a speed 2 m/s. The jet trajectory turns towards the earth. What should be the average speed of the jet at a height B, 1 m below the nozzle level, A? What should be the jet diameter at height B? Neglect the jet friction with air, assuming that the water jet remains laminar.




$$p + \frac{\rho V^2}{2} + \rho g z = const = C$$

$$p_1 + \frac{\rho_1 V_1^2}{2} + \rho_1 g z_1 = p_2 + \frac{\rho_2 V_2^2}{2} + \rho_2 g z_2$$

$$p_1 = p_2, \qquad \rho = const, \qquad \frac{\rho V_1^2}{2} + \rho g z_1 = \frac{\rho V_2^2}{2} + \rho g z_2$$

$$V_2^2 = 2g\Delta z - V_1^2 = 2X9.81\frac{m}{s^2}X1m - \left(2\frac{m}{s}\right)^2 = 19.62\frac{m^2}{s^2} - 4\frac{m^2}{s^2} = 15.62\frac{m^2}{s^2}$$

$$V_2 = 3.95\ m/s \qquad \frac{\pi d_1^2}{4}V_1 = \frac{\pi d_2^2}{4}V_2,\ d_2^2 = \frac{d_1^2 V_1}{V_2} = \frac{(0.02\ m)^2\left(2\frac{m}{s}\right)}{3.95\frac{m}{s}}$$

$d_2 = 1\ cm$ *Answer*