MATH 209/4 all sections except EC: - Fundamental Mathematics II

Midterm - Sunday February 24, 2013, 2pm (1h30min)
Only approved calculators are permitted.

MARKS

[7] **1.** (a) Find $\lim_{x\to 0} \dfrac{x}{\sqrt{x+25}-5} = 0.$

[7] (b) Give functions $f(x)$ and $g(x)$ with the following properties:

(i) $\lim_{x\to 3} f(x) = 0$ (ii) $\lim_{x\to 3} g(x) = 0$ (iii) $\lim_{x\to 3} \dfrac{f(x)}{g(x)} = 5$

[7] **2.** Let $h(x) = 4 - x^3$. Work out the following in detail:

$$\lim_{s\to 0} \frac{[h(x+s) - h(x)]}{s}$$

[12] **3.** (a) If $f(x) = -3x^{27} + 4$, find $f'(1)$. Do not simplify.

(b) If $g(x) = (2x^3 - 4)[ln(x^2) + 3]$, find $g'(2)$. Do not simplify.

(c) Find $h'(x)$ if $h(x) = \frac{x^3 - e^x}{[e^{2x} + ln(x)]}$.

(d) Find the value of dy if $y = x^3 + 2$, $x = 2$ and the change in x is .5.

[7] **4.** A stock portfolio grows from ten thousand dollars to thirty thousand dollars in ten years. Find the associated annual rate of growth assuming that it is compounded continuously.

[10] **5.** The cost of printing x books is given by the function $C(x) = 20,000 + 10x$.

(a) Find the average cost per unit if 1000 books are produced.

(b) Find the marginal average cost when 1000 books are produced and interpret the results.

(c) Use (a) and (b) to estimate the average cost of producing 1001 books.

[10] **6.** Find x' for the function $x(t)$ defined implicitly below. Evaluate x' at the indicated point.

$$x^2 - t^2x + t^3 + 11 = 0 \ (-2, 1).$$

[10] **7.** Boyle's law for enclosed gases states that if the volume is kept constant, the pressure P and the temperature T are related by the equation $P/T = k$, where k is a constant. Suppose that the rate of change of the temperature is 3 degrees per hour. What is the rate of change of pressure when T is 250 and P is 500?