CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	**Date**	**Pages**
Final	April 2012	3

Instructors	Course Examiner
M. Brakocevic, J. Brannlund, H. Greenspan, T. Hughes, M. Rogers, B. Rhodes, F. Romanelli, U. Tiwari.	R. Raphael

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ **Only approved calculators are allowed.**

MARKS

[9] 1. (a) Let $f(x) = \frac{x^2+x-6}{2x^2-3x-2}$. Find

(A) $\lim_{x \to 2} f(x)$ (B) $\lim_{x \to -1} f(x)$

(b) Given that $\lim_{x \to 2} g(x) = -3$ and $\lim_{x \to 2} h(x) = 3$, find the

$\lim_{x \to 2} \sqrt{h(x) - g(x)}$

(c) If $\lim_{x \to 4} k(x) = 4$, then $\lim_{x \to 5} k^2(x) = 16$. True or False. Explain your answer.

[10] 2. (a) If $g(x) = 2x^3 - 3x^4 - x\sqrt{x}$, find $g'(x)$.

(b) If $f(x) = \frac{x+1}{x^3}$, find $f'(x)$.

(c) If $y = e^\pi$, find y'.

(d) Find y' if $xy^2 = e^y$.

(e) If $y = \sqrt[3]{x^4 + 2}$, then $y' =?$

[9] 3. Given the price-demand equation

$$x + 500p = 10,000$$

(a) Express the demand x as a function of the price p.

(b) Express the revenue R as a function of the price p.

(c) Find the elasticity of demand, $E(p)$.

[9] 4. Let $y = x^2 - 3x + 2$. Suppose $x = -3$, $\Delta x = dx = -0.1$.

(a) Find Δy and dy.

(b) Display Δy and dy on a graph of $f(x)$.

[9] 5. The total profit (in dollars) from the sale of x lawn mowers is

$$P(x) = 30x - 0.03x^2 - 750 \qquad 0 \le x \le 1,000$$

(a) Find the average profit per mower if 50 mowers are produced.

(b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.

(c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.

[9] 6. A point is moving on the graph of $x^2 + 2y^2 = 41$. When the point is at $(3, -4)$, its y co-ordinate is decreasing by 3 units per second. How fast is the x co-ordinate changing at that moment?

[6] 7. Evaluate the following integrals [accurate to 2 decimals].

(a) $\displaystyle\int_1^3 (3x + 2e^x - \frac{7}{x})\, dx$

(b) $\displaystyle\int_0^5 \frac{5x}{x^2 + 7}\, dx$

[12] 8. Compute the following:

(a) $\int e^{5x}\,dx$

(b) $\int (5x^6 - 2x^3)\,dx$

(c) $\int (x-8)^{-8}\,dx$

(d) $\int (x^3 - ex)\,dx$

(e) $\int x(x^2+3)^{-5}\,dx$

(f) $\int \frac{5x^3}{7-x^4}\,dx$

[9] 9. Find the area bounded by $f(x) = 2x^2$ and $g(x) = 4 - 2x$ for $-2 \le x \le 2$.

[9] 10. Use the graphing strategy to analyze the function

$h(x) = \frac{4x}{x-2}$

State all pertinent information and sketch the graph of h.

[4] 11. Suppose f is continuous at $x = 3$. Decide if f must be differentiable at $x = -3$.

[5] 12. Two countries have the same Lorenz Curve. What conclusion can you draw from this information?

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Final Exam Solutions - April 2012.

1. A. i.

$$\lim_{x\to 2} \frac{x^2+x-6}{2x^2-3x-2} = \frac{4+2-6}{8-6-2} = \frac{0}{0} \quad I.F.$$

$$\lim_{x\to 2} \frac{x^2+x-6}{2x^2-3x-2} = \lim_{x\to 2} \frac{(x-2)(x+3)}{(x-2)(2x+1)} =$$

$$\lim_{x\to 2} \frac{x+3}{2x+1} = \frac{5}{5} = 1.$$

ii.

$$\lim_{x\to -1} \frac{x^2+x-6}{2x^2-3x-2} = \frac{1-1-6}{2+3-2} = \frac{-6}{3} = -2.$$

B. $\lim_{x\to 2} g(x) = -3$; $\lim_{x\to 2} h(x) = 3$.

$$\lim_{x\to 2} \left(h(x) - g(x)\right)^{1/2} = \left(\lim_{x\to 2} h(x) - \lim_{x\to 2} g(x)\right)^{1/2}$$

$$= \left(3-(-3)\right)^{1/2} = \sqrt{6}.$$

C. False. It would depend on the function $h(x)$.

If $h(x) = 4$ then true; If $h(x) = -4x+20$ then false.

2. A.

$$g(x) = 2x^3 - 3x^4 - x\sqrt{x}$$

$$g(x) = 2x^3 - 3x^4 - x^{3/2}.$$

$$g'(x) = 6x^2 - 12x^3 - \frac{3}{2}x^{1/2}.$$

B. $$f(x) = \frac{x+1}{x^3}.$$

$$f'(x) = \frac{x^3(1) - (x+1)(3x^2)}{x^6} = \frac{-2x^3 - 3x^2}{x^6}$$

$$f'(x) = \frac{-2x-3}{x^4}.$$

C. $$y = e^{\pi}$$

$$y' = 0.$$

D. $$xy^2 = e^y$$ *Implicit Differentiation.*

$$x(2yy') + y^2(1) = e^y y'$$

$$2xyy' + y^2 = e^y y'$$

$$2xyy' - e^y y' = -y^2$$

$$y'(2xy - e^y) = -y^2$$

$$y' = \frac{-y^2}{2xy - e^y}.$$

E. $y = \sqrt[3]{x^4+2} = (x^4+2)^{1/3}.$

$$y' = \frac{1}{3}(x^4+2)^{-2/3}(4x^3)$$

$$y' = \frac{4}{3}x^3(x^4+2)^{-2/3}.$$

3. $x + 500p = 10{,}000.$

A. $x = -500p + 10{,}000.$

B. $R = xp = (10{,}000 - 500p)p = 10{,}000p - 500p^2.$

C.

$$E(p) = -\frac{p f'(p)}{f(p)}$$

$$= \frac{-p(-500)}{10{,}000 - 500p}$$

$$E(p) = \frac{500p}{10{,}000 - 500p}.$$

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4. $f(x) = x^2 - 3x + 2.$ $x = -3;\ \Delta x = dx = -.1.$

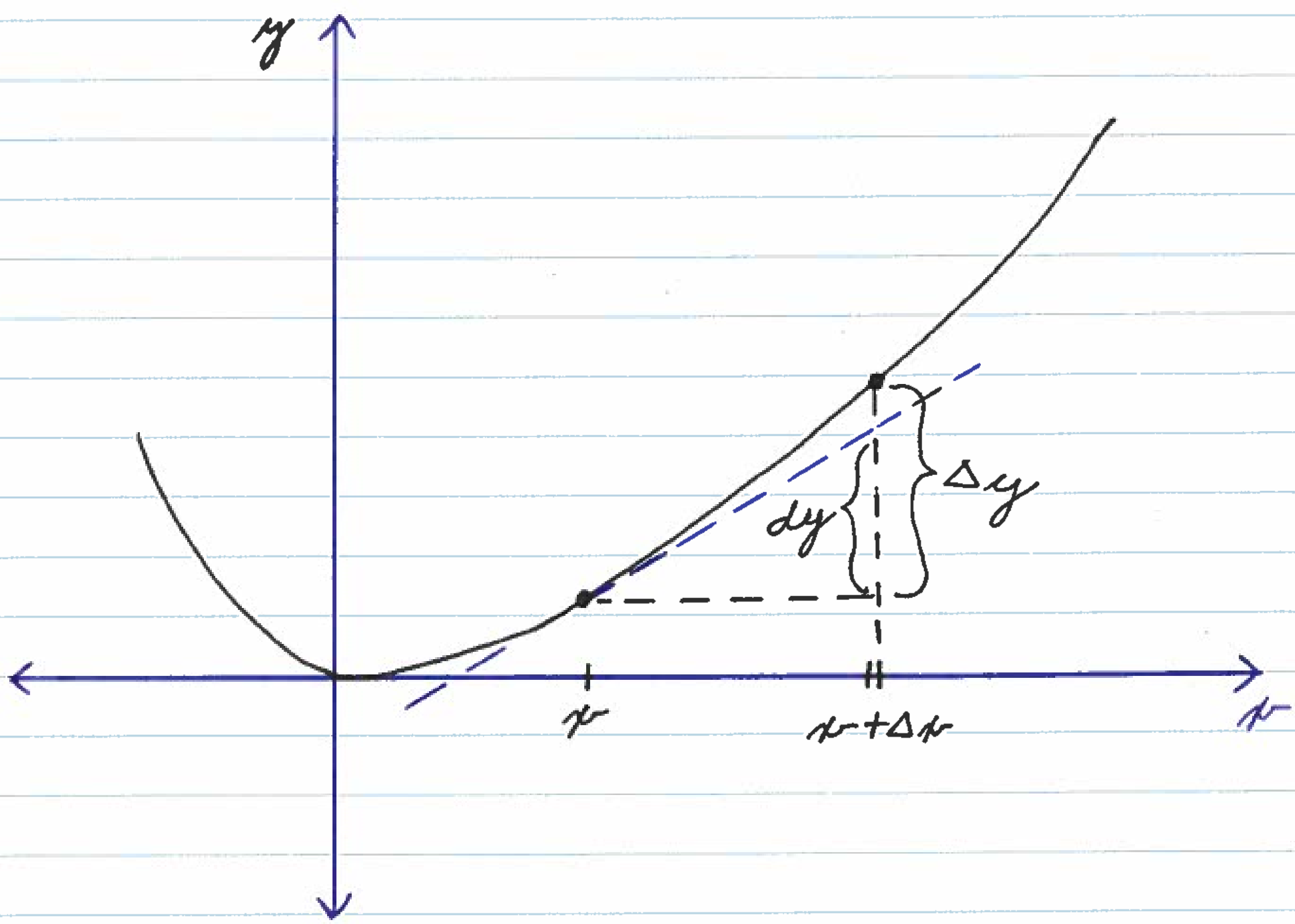
A. $\Delta y = f(-3.1) - f(-3) = \cancel{(2.21)}\ (20.91) - (20)$

$\Delta y = .91.$

$dy = f'(x)\,dx$

$dy = (2x - 3)\,dx = (2(-3) - 3)(-.1) = .9.$

B.



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5. $P(x) = 30x - .03x^2 - 750 \qquad 0 \le x \le 1{,}000$

A. $\bar{P} = \dfrac{P}{x} = \dfrac{30x - .03x^2 - 750}{x} = 30 - .03x - \dfrac{750}{x}.$

$\bar{P}(50) = 30 - .03(50) - \dfrac{750}{50} = \$13.5.$

B. $\bar{P}'(x) = -.03 + \dfrac{750}{x^2}.$

$\bar{P}'(50) = -.03 + \dfrac{750}{(50)^2} = .27.$

average profit is increasing at a rate of .27 \$ after 50 mowers produced.

C.
$\bar{P}(51) \approx \bar{P}(50) + \bar{P}'(50)$

$\approx 13.5 + .27$

$\bar{P}(51) \approx \$13.77.$

6. $x^2+2y^2=41.$ $(3,-4);\ \frac{dy}{dt}=-3;\ \frac{dx}{dt}?$

Related Rates Problem.

$$2x\frac{dx}{dt}+4y\frac{dy}{dt}=0$$

$$4y\frac{dy}{dt}=-2x\frac{dx}{dt}$$

$$\frac{dx}{dt}=\left(\frac{-2y}{x}\right)\frac{dy}{dt}=\left(\frac{-2(-4)}{3}\right)(-3)=-8 \text{ units/sec.}$$

7. A. $\int_1^3\left(3x+2e^x-\frac{7}{x}\right)dx$

$$=\left(\frac{3x^2}{2}+2e^x-7\ln|x|\right)\Big|_1^3$$

$$=\left(\frac{27}{2}+2e^3-7\ln(3)\right)-\left(\frac{3}{2}+2e-0\right)$$

$$=12+2e^3-7\ln(3)-2e$$

$$=39.04422$$

$$\approx 39.04.$$

B. $\int_0^5 \frac{5x}{x^2+7}\,dx.$

Let $u = x^2+7$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\therefore$

$$\int_0^5 \frac{5x}{u}\left(\frac{du}{2x}\right) = \frac{5}{2}\int_0^5 \frac{du}{u}$$

$$= \frac{5}{2}\ln|u|\Big|_0^5$$

$$= \frac{5}{2}\ln|x^2+7|\Big|_0^5$$

$$= \frac{5}{2}\left[\ln(32) - \ln(7)\right]$$

$$= 3.799564$$

$$\approx 3.80.$$

8\. A. $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C.$

B. $\int (5x^6 - 2x^3)\,dx = \frac{5}{7}x^7 - \frac{x^4}{2} + C.$

C. $\int (x-8)^{-8}\,dx$ Let $u = x-8$

$du = dx$

$$\int u^{-8}\,du = \frac{u^{-7}}{-7} = \frac{(x-8)^{-7}}{-7} + C.$$

D. $\int (x^3 - ex)\,dx = \frac{x^4}{4} - \frac{e}{2}x^2 + C.$

E. $\int x(x^2+3)^{-5}\,dx$ Let $u = x^2+3$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int x\,u^{-5}\left(\frac{du}{2x}\right) = \int \frac{1}{2}u^{-5}\,du = \frac{1}{2}\int u^{-5}\,du$$

$$= \left(\frac{1}{2}\right)\frac{u^{-4}}{-4} = \frac{-1}{8}(x^2+3)^{-4} + C.$$

F. $\int \frac{5x^3}{7-x^4}\,dx$ Let $u = 7-x^4$

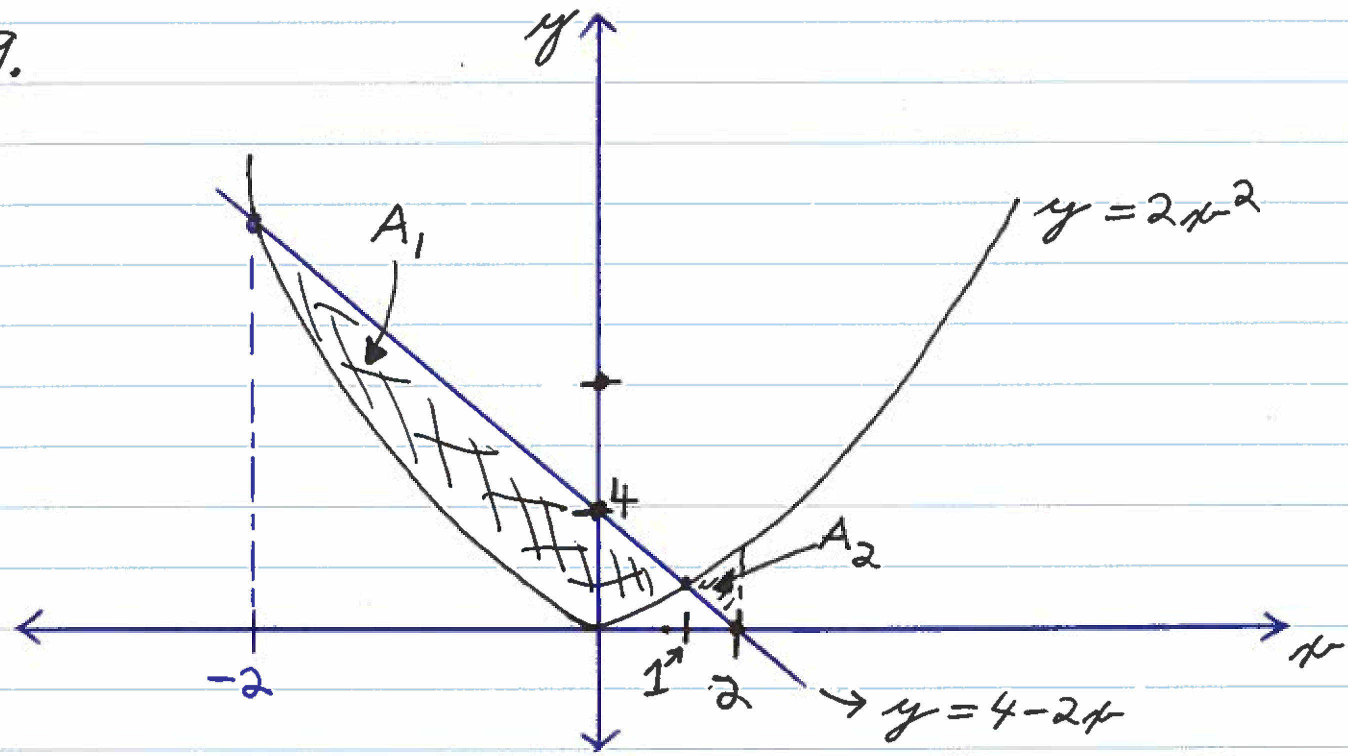
$du = -4x^3\,dx$

$dx = \frac{du}{-4x^3}$

$$\int \frac{5x^3}{u}\left(\frac{du}{-4x^3}\right) = \frac{-5}{4}\int \frac{du}{u} = \frac{-5}{4}\ln|u|$$

$$= \frac{-5}{4}\ln|7-x^4| + C.$$

9.



Total area: A.

$A = A_1 + A_2$.

$$A_1 = \int_{-2}^{1} (\text{T.C.} - \text{B.C.})\,dx = \int_{-2}^{1} [(4-2x)-(2x^2)]\,dx$$

$$= \int_{-2}^{1} (4-2x-2x^2)\,dx = \left(4x - x^2 - \frac{2}{3}x^3\right)\Big|_{-2}^{1}$$

$$A_1 = \left(4-1-\frac{2}{3}\right) - \left(-8-4+\frac{16}{3}\right) = 15-6 = 9.$$

$$A_2 = \int_1^2 [T.C. - B.C.]\,dx = \int_1^2 [(2x^2) - (4-2x)]\,dx$$

$$= \int_1^2 (2x^2 + 2x - 4)\,dx = \left(\frac{2}{3}x^3 + x^2 - 4x\right)\Big|_1^2$$

$$= \left(\frac{16}{3} + 4 - 8\right) - \left(\frac{2}{3} + 1 - 4\right) = \frac{14}{3} - 1 = \frac{11}{3}.$$

$\therefore$

$$A = A_1 + A_2 = 9 + \frac{11}{3} = \frac{38}{3}.$$

10.

$$f(x) = \frac{4x}{x-2}.$$

Domain : $\mathbb{R}/2$

x - intercept : 0

y - intercept : 0

V.A.: $x = 2$

H.A.: $y = 4$

C.P. : $\emptyset$

P.I. : $\emptyset$

C.U. $(2, \infty)$

C.D. $(-\infty, 2)$

Increasing : $\emptyset$

Decreasing : $(-\infty, 2)$; $(2, \infty)$

No absolute max. or min. points

No local max. or min. points.

$$f(x) = \frac{4x}{x-2}$$

C.P.?

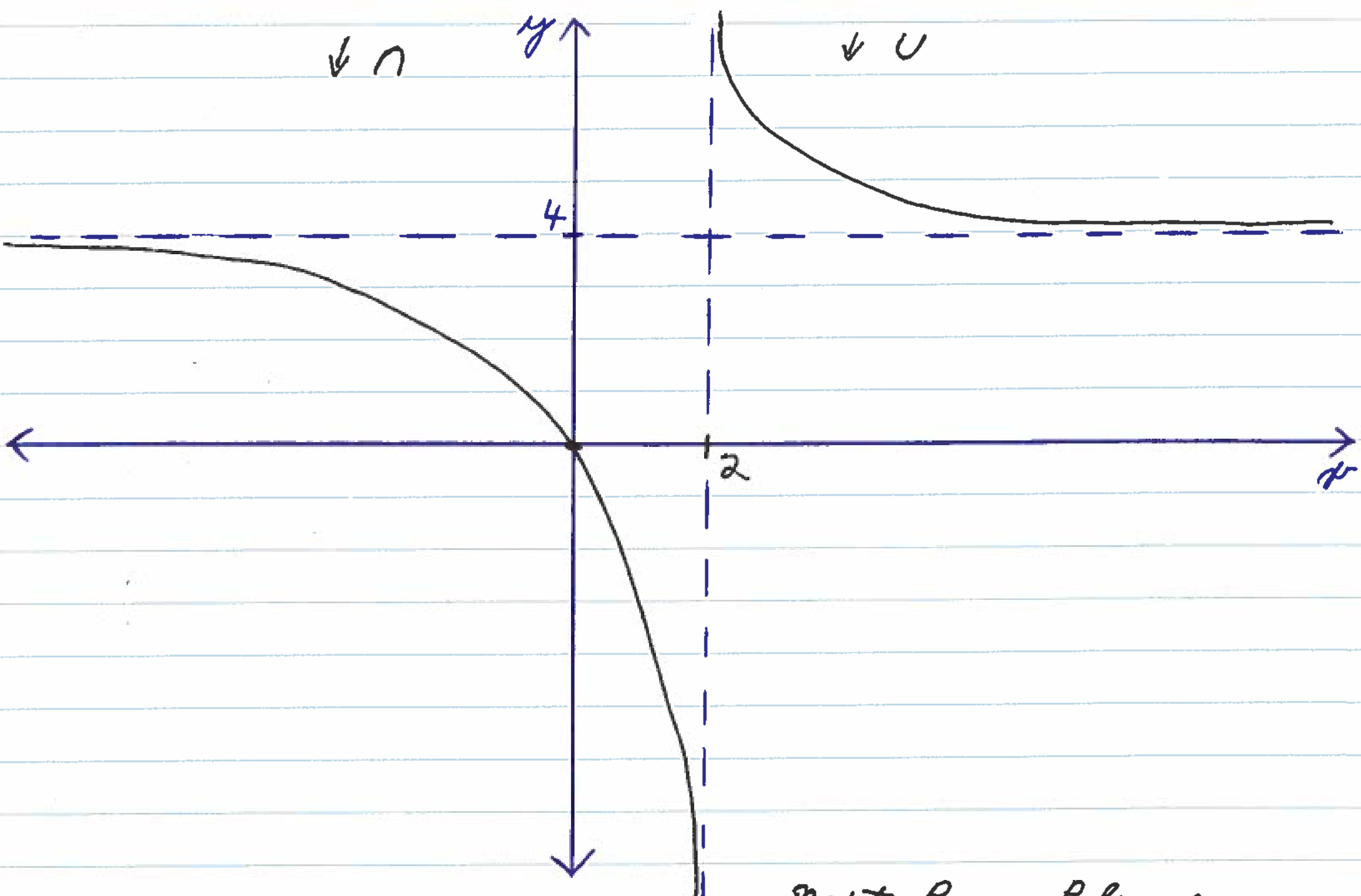
$$f'(x) = \frac{(x-2)(4)-(4x)(1)}{(x-2)^2} = \frac{-8}{(x-2)^2} = 0 \Rightarrow \text{no C.P.}$$

P.I.?

$$f'(x) = -8(x-2)^{-2}.$$

$$f''(x) = 16(x-2)^{-3}(1) = 0$$

$$\frac{16}{(x-2)^3} = 0 \Rightarrow \text{no P.I.}$$



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No C.P., can not use S.D.T. Must use Concavity Test and F.D.T.

F.D.T.

$f'(0) < 0$ ↓

$f'(3) <$ ↓

Concavity Test

$f''(0) < 0$ C.D. ∩

$f''(3) > 0$ C.U. ∪

11. Undecidable! Whether the function $f(x)$ is differentiable at $x=-3$, is independent of $f(x)$ being continuous at $x=3$.

12. The Conclusion being that the two countries have the same Income Distribution. The Gini Index's are the same.