MATH 1107 R - Winter 2014 Final Practice Problems

1. In this question, $z = 1 + i$, $w = 2 + i$. Give the answer to each part. Simplify your answer as much as possible. All complex numbers must be in **rectangular/standard form**.

   (a) $\overline{w} =$______________________.

   (b) $z - w =$______________________.

   (c) $w^{-1} =$______________________.

   (d) $|w| =$______________________.

   (e) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & i \end{bmatrix}$. Then $\det(A^{10}) =$______________________.

   (f) Let $B = \begin{bmatrix} 2 & -2 & 0 \\ 2 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$. The rank of B is______________________.

   (g) Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, and $C = A^{\mathsf{T}} + 2B^{-1}$.

   Then $C_{1,2} =$______________________.

   (h) The eigenvalues of $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ are ______________________.

   (i) Let W be a proper subspace of the vector space of 2×3 matrices with real entries.

   The largest possible value for the dimension of W is ______________________.

   (j) Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$ such that $T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = 1$ and $T\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix}\right) = -1$. Then $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) =$______________________.

2. Find all real values a such that the system of linear equations

$$\begin{array}{rcrcrcr} x & - & y & + & z & = & 1 \\ x & - & 2y & + & 2z & = & -1 \\ & & y & - & z & = & a \end{array}$$

has no solutions.

3. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ be matrices defined over the real numbers.

   (a) Find a basis for the nullspace of A.

   (b) Find a basis for the column space of A.

   (c) Determine if the nullspace of A is the same as the column space of B. Justify your answer.

4. Consider the matrix

$$A = \begin{bmatrix} k & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & k & 1 \\ 1 & -1 & 2 & -1 \end{bmatrix}.$$

   Determine all values of k such that A is singular.

5. Consider the linear transformation T from $\mathbb{R}^4$ to the vector space of 2×2 real matrices given by

$$T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} 2a + c & a - d \\ -2a + 2d & b - d \end{bmatrix}$$

   (a) Determine a basis for the range of T.

   (b) Is T surjective? injective? bijective? Justify your answer.

6. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

   (a) Determine A^{-1}.

   (b) Diagonalize A by finding a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.