Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT1330 D: Calculus for life sciences I

Instructor: Aziz Khanchi

Test I-Blue
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Surname ________________________ First Name ______________________

Student # ______________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Make a note of the questions that you feel confident you can do, and then do those first: <u>you do not</u> have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam. You can use the back of the pages to write your solutions.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

585, av. King-Edward C.P. 450, Succ. A
Ottawa (Ontario) K1N 6N5 Canada

585 King Edward Ave., P.O. Box 450, Stn. A
Ottawa, Ontario K1N 6N5 Canada

(613) 562-5864 • Téléc./Fax (613) 562-5776
Courriel/Email: uomaths@science.uottawa.ca

Q1	Q2	Q3	Q4	Q5	~~Q6~~	~~Q7~~	Total
/4	/9	/10	/5	/4	~~/4~~	~~/4~~	/~~40~~ 32

Question 1. Solve the following equations:

a) $x^2 - x = 3\log_5 25$

$$x^2 - x = 3 \times 2 = 6 \longrightarrow x^2 - x - 6 = 0 \longrightarrow \Delta = 1 - 4(-6) = 1 + 24 = 25$$

$$x = \frac{1 \pm 5}{2} \begin{cases} 3 \\ -2 \end{cases}$$

$x = \boxed{-2, 3}$

b) $8e^{3x} = 32$

$$e^{3x} = \frac{32}{8} = 4 \longrightarrow \ln e^{3x} = \ln 4 \longrightarrow 3x = \ln 4$$

$$\longrightarrow x = \frac{\ln 4}{3}$$

$x = \boxed{\frac{\ln 4}{3}}$

Question 2. To make the birds happy, Monique deposits 40 grams of sunflower seeds at the beginning of each week in a conspicuous place. During the week, the birds eat 1/2 of seeds available. The dynamical system modeling the quantity of seeds is

$$G_{t+1} = 0.5G_t + 40$$

where t is in weeks.

a) Find the updating function of the DTDS. $\boxed{f(x) = 0.5x + 40}$

b) Find the equilibrium point of the DTDS.

Solve $x^* = 0.5x^* + 40 \longrightarrow 0.5x^* = 40$

$\longrightarrow x^* = \frac{40}{0.5} = 80$

$x^* = \boxed{80}$

c) Give the solution formula for the DTDS with initial condition $G_0 = 12$:

Solution is of the form $G_t = a(0.5)^t + b$. Evaluating at $t=0$ and $t=1$, $\begin{cases} a + b = 12 \\ 0.5a + b = 46 \end{cases} \Rightarrow 0.5a = -34 \Rightarrow a = -68$

and $-68 + b = 12 \longrightarrow b = 12 + 68 = 80$

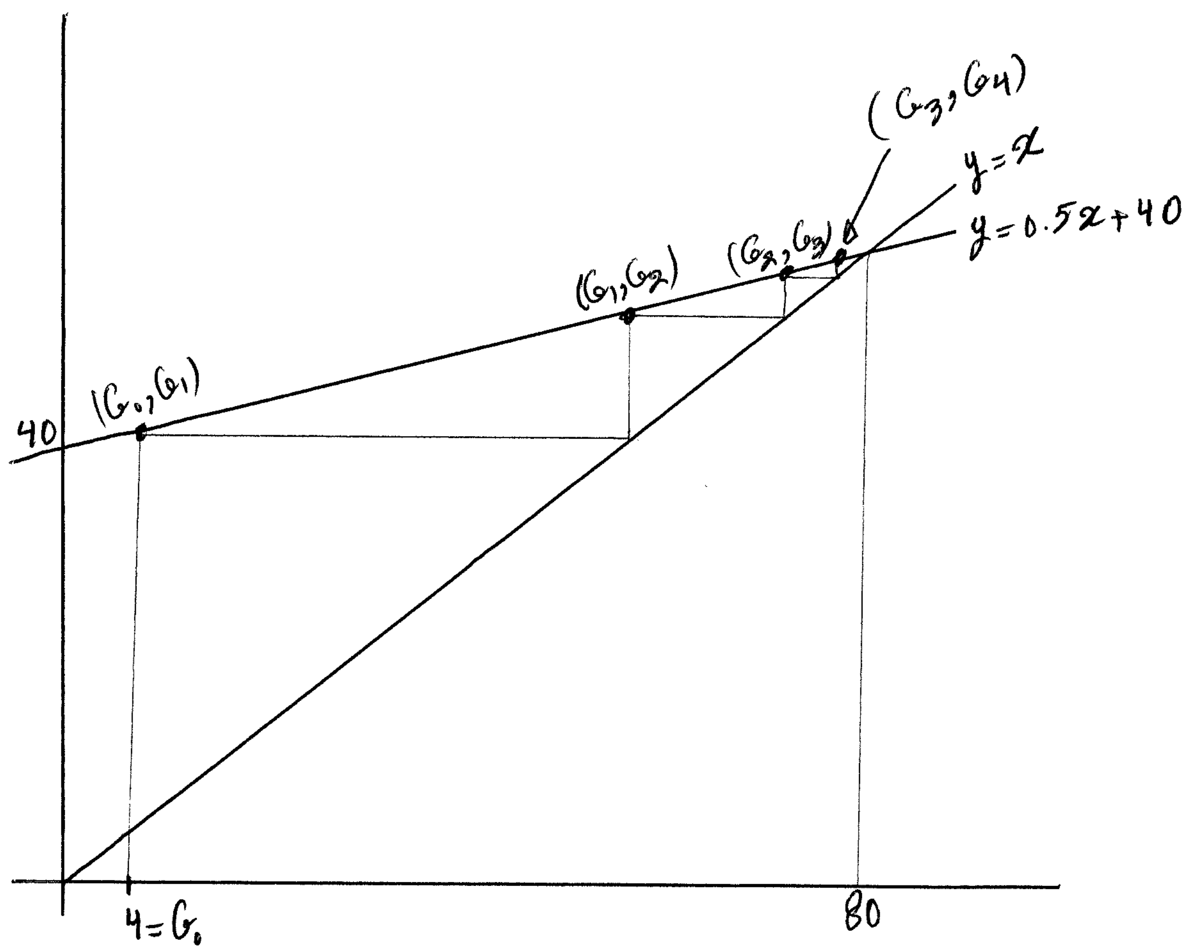
$G_t = \boxed{-68(0.5)^t + 80}$

d) Calculate G_{13} if $G_0 = 12$.

$$G_{13} = -68(0.5)^{13} + 80 \approx 79.9917 \approx 80$$

$G_{13} = \boxed{80}$

e) Graph the updating function and draw the cobweb diagram of the DTDS, starting from $G_0 = 4$ for 4 steps.



Question 3. Evaluate the following limits. If a limit does not exist determine if it is $-\infty, \infty$ or neither.

a) $\lim_{x\to 2} \frac{x-2}{x^2-4} \overset{0/0}{=}$ $\boxed{\lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}}$

b) $\lim_{x\to 0} \frac{-x^{2012}+2}{2x^2+10} =$ $\boxed{\frac{2}{10} = \frac{1}{5}}$

c) $\lim_{x\to 25} \frac{x-25}{\sqrt{x}-5} \overset{0/0}{=}$

$$\lim_{x\to 25} \frac{x-25}{\sqrt{x}-5} \times \frac{\sqrt{x}+5}{\sqrt{x}+5} = \lim_{x\to 25} \frac{\cancel{(x-25)}(\sqrt{x}+5)}{\cancel{x-25}}$$

$$= \lim_{x\to 25} (\sqrt{x}+5) = 10$$

d) $\lim_{x\to 3^+} \frac{3x}{(x-3)^5} =$ $\boxed{\frac{9}{0^+} = +\infty}$

e) $\lim_{x\to 3^-} \frac{3x}{(x-3)^5} =$ $\boxed{\frac{9}{0^-} = -\infty}$

f) $\lim_{x\to\infty} \frac{x^6-3x+5}{3x^6-9} =$

$$\lim_{x\to\infty} \frac{x^6(1-3/x^5+5/x^6)}{x^6(3-9/x^6)} = \frac{1-0+0}{3-0} = \frac{1}{3}$$

g) $\lim_{x\to 1^-} \frac{x^2-3x}{x-1} =$ $\boxed{\frac{-2}{0^-} = +\infty}$

Question 4. Consider the function $f(x) = \dfrac{\overbrace{|x+\frac{1}{2}| - |x-\frac{1}{2}|}^{A}}{x}$.

a) Determine the domain of f.

$D_f = \boxed{\{x \mid x \neq 0\}}$

b) Write f as a piecewise function without the absolute value.

if $x + \frac{1}{2} = 0$, then $x = \frac{-1}{2}$

if $x - \frac{1}{2} = 0$, then $x = \frac{1}{2}$

x		$-1/2$		$\frac{1}{2}$	
$x+\frac{1}{2}$	$-$	0	$+$		$+$
$x-\frac{1}{2}$	$-$		$-$	0	$+$

if $x \leqslant \frac{-1}{2}$, then $A = -x - \frac{1}{2} + x - \frac{1}{2} = -1$

if $\frac{-1}{2} < x \leqslant \frac{1}{2}$, $x \neq 0$, then $A = x + \frac{1}{2} + x - \frac{1}{2} = 2x$

if $x > \frac{1}{2}$, then $A = x + \frac{1}{2} - x + \frac{1}{2} = 1$

$$f(x) = \begin{cases} \frac{-1}{x} & \text{if } x \leqslant \frac{-1}{2} \\ 2 & \text{if } \frac{-1}{2} < x \leqslant \frac{1}{2},\ x \neq 0 \\ \frac{1}{x} & \text{if } x > \frac{1}{2} \end{cases}$$

Question 5. During one year, Ottawa receives maximum sunlight for 16 hours and a minimum of 8 hours sunshine. The maximum occurs in June, the minimum in the month of December. Assuming that the number of sunshine hours varies according to a sinusoidal function of the classical form:

$$f(t) = A + B\cos\left(\frac{2\pi}{T}(t-\Phi)\right),$$

Where t is the month of the year with t = 0 corresponding to the month of January.

Find the parameters in the standard cosine description.

$A = \boxed{12}$

$T = \boxed{12}$

$B = \boxed{4}$

$\Phi = \boxed{5}$

Average $= \frac{16+8}{2} = 12$

period = 12 months

$B = 16-12 = 4$

$\phi = 5$ months first max after January

$f(t) = 12 + 4\cos\left(\frac{2\pi}{12}(t-5)\right)$

Draw the graph of the function for at least one period and identify the above four parameters on the graph.