CARLETON UNIVERSITY

FINAL
EXAMINATION
December 2004

DURATION: 3 HOURS

Department Name & Course Number: Mathematics and Statistics MATH 1009*ABC
Course Instructor(s) :Dr.E.Devdariani, Dr.G.Zelmer, Dr.S.Wang

AUTHORIZED MEMORANDA

Non programmable calculators are required

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This exam consists of **11** pages, including this title page.

This examination question paper **MAY NOT** be taken from the examination room.

This examination question paper **MAY NOT** be released to the library.

Family Name (print) : _____ **First Name :** _____

Student Number : _____

Question		Mark
Part A		
Part B	B1	
	B2	
	B3	
	B4	
	B5	
Exam Mark / 100		

PART A. MULTIPLE CHOICE QUESTIONS.

All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Each question is worth **2** marks.

A1. What is the equation of the tangent line to the curve $y = x^4 + x^3 + 4x + 3$ at the point $(1, 9)$?

a) $y = 11x + 20$ b) $y = 11x - 2$ c) $y = -11x + 20$ d) $y = -11x - 2$

A2. On which interval is the function $f(x) = (\frac{1}{2}x^2 + x - 1)e^{-x}$ increasing?

a) $(-2, 2)$ b) $(-\infty, -2)$ c) $(2, +\infty)$ d) $(-2, +\infty)$

A3. On which interval is the function $f(x) = x^3 - 9x^2 + 12x + 23$ concave down?

a) $(3, +\infty)$ b) $(-3, 3)$ c) $(-\infty, 3)$ d) $(-\infty, +\infty)$

A4. What is $f'(\frac{\pi}{4})$ if $f(x) = \cos^2 x$?

a) 1 b) -1 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

A5. Let $f(x) = \left(\frac{x+1}{x-1}\right)^5$. What is $f'(0)$?

a) -40 b) 10 c) -10 d) 40

A6. Let a curve $y = f(x)$ be given implicitly by $x^3 + 3xy + y^3 = 1$. What is $\frac{dy}{dx}$ at $x = 1,\ y = 0$?

a) -2 b) 1 c) -1 d) 2

A7. The second derivative of $f(x) = \ln(x^2 + 1)$ is

a) $\dfrac{2(1-x^2)}{x^2+1}$ b) $\dfrac{-2x}{(x^2+1)^2}$ c) $\dfrac{-2x}{x^2+1}$ d) $\dfrac{2(1-x^2)}{(x^2+1)^2}$

A8. A company estimates that its daily cost function is $C(x) = x^3 - 6x^2 + 13x$ million dollars, and its daily revenue function is $R(x) = 13x$ million dollars. What is the value of x that maximizes the profit?

a) 1 b) 2 c) 4 d) 5

A9. Let $f(x, y) = e^{3x+2y}$. What is $f_{yx}(0, 0)$, or, in another notation, $\dfrac{\partial^2 f}{\partial x \partial y}\,|_{x=0,y=0}$?

a) 12 b) 8 c) 9 d) 6

A10. Which of the following is one of the critical point of $f(x,y) = x^2 - 2xy + \frac{1}{8}y^4$?

a) $(0,1)$ b) $(1,0)$ c) $(2,-2)$ d) $(2,2)$

A11. Which of the following is equal to $\log_{\frac{1}{2}} \sqrt[3]{\frac{1}{16}}$?

a) $-\frac{4}{3}$ b) $\frac{4}{3}$ c) $-\frac{8}{3}$ d) $\frac{8}{3}$

A12. Which of the following is equal to $e^{-\frac{\ln 9}{2}}$?

a) $-\frac{1}{3}$ b) 3 c) -3 d) $\frac{1}{3}$

A13. Let $f(x) = \displaystyle\int_0^x \frac{\cos(t^3)}{1+t^2}dt$. What is $f'(x)$?

a) $\dfrac{3x^2\sin(x^3)}{1+x^2}$ b) $\dfrac{\cos(x^3)}{1+x^2}$ c) $\dfrac{3x^2\sin(x^3)}{(1+x^2)^2}$ d) $\dfrac{\cos(x^3)}{(1+x^2)^2}$

A14. Consider the Cobb-Douglas production function $f(x,y) = 16x^{1/4}y^{3/4}$, where x is the number of units of labor and y is the number of units of capital. What is the marginal productivity of labour at $x = 1, y = 16$?

a) 16 b) 24 c) 32 d) 64

A15. What is $\displaystyle\lim_{x\to\infty} \frac{3x+2}{x^2-5}$?

a) 1 b) 2 c) 3 d) 0

PART B.

B1. [15 Marks]

John and Rita have received an inheritance of $100,000. With the cash they are deciding whether they should invest it in a bank or buy a house.

[3] (a) If they invest in the bank offering 6 % per year over the 10 year period compounded quarterly, what would be the interest earned on their money?

[3] (b) What would be the maximum accumulated amount (or return) they could expect after 10 years from an investment giving a yearly interest of 6 % compounded continuously?

[4] (c) What would the interest rate be with continuous compounding in order for them to double their original amount of money after 10 years?

[5] (d) John and Rita assume that housing prices will double in 10 years. If they buy a house and are correct in their assumption, what rate of interest compounded annually over the 10 year period would give them the same gain?

B2. [18 Marks]

Find and classify all the critical point of the function

$$f(x, y) = 2x^3 + y^2 - 6x + 4y - 1.$$

B3. [15 Marks]

Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y) = 4x + 6y$$

subject to the constraint

$$x^2 + y^2 = 13.$$

B4. [7 Marks]

The Double D Company manufactures ovens and has a daily fixed cost of \$4000 per day. The marginal profit function for the company is known to be

$$P'(x) = -0.0009x^2 + 0.06x + 80$$

where x is the number of ovens produced daily.
[4] (a) Find an integral expression for the total profit realized from the sale of 400 ovens per day. Evaluate this total profit.
[3] (b) What additional profit is realized if the sale of ovens is increased to 450 per day?

B5. [15 Marks]
Evaluate the following integrals:

[5] (a) $\int \sqrt{3x-7}\,dx$

[5] (b) $\int xe^{x^2+3}\,dx$

[5] (c) $\int_3^{11} \frac{t+3}{t+5}\,dt$