

This test was written in room: _____

Total marks: 40 marks

This test is closed book. No calculators or electronic aids are permitted. Please supply your answers on this sheet.

PLEASE PRINT

First name _____

Last name _____

Student number _____

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. Fill in the blanks:

- a. [1] $\arcsin(-1/2) = \underline{-\pi/6}$
- b. [1] Let $f(x) = 2 \arccos x$. The range of f is $\underline{[0, 2\pi]}$
- c. [1] Let $f(x) = \arctan(x-2)$. The domain of f is: $\underline{\mathbb{R}}$
- d. [1] Let $f(x) = \ln(x+1)$. The domain of f is $\underline{(-1, \infty)}$
- e. [1] The function $f(x) = \log_{1/3} x$ is decreasing on its domain. TRUE FALSE
- f. [1] $\arctan x = \frac{\arcsin x}{\arccos x}$. TRUE FALSE

2. Let $f(x) = \frac{2x^2 - 3x - 5}{x+1}$. Determine:

a. [1] $\lim_{x \rightarrow 2} f(x) \xrightarrow{D.S.} \frac{2(2)^2 - 3(2) - 5}{2+1} = \frac{8-11}{3} = \underline{-1}$

b. [1] $\lim_{x \rightarrow 0} f(x) \xrightarrow{D.S.} \underline{-5}$

c. [3] $\lim_{x \rightarrow -1} f(x) \xrightarrow{D.S.} \frac{2(-1)^2 - 3(-1) - 5}{-1+1} = \frac{5-5}{0} = \frac{0}{0}$

... = $\lim_{x \rightarrow -1} \left[\frac{\cancel{x+1}(2x-5)}{\cancel{x+1}} \right] = \lim_{x \rightarrow -1} [2x-5] \xrightarrow{D.S.} 2(-1) - 5 = \underline{-7}$
($x \neq -1$)

3. Determine the following limits if they exist. If a limit does not exist, say so.

a. [3] $\lim_{x \rightarrow 0} \frac{-5(x-1)^2 + 5}{x} \xrightarrow{D.S.} \frac{-5(0-1)^2 + 5}{0} = \frac{0}{0}$

... = $\lim_{x \rightarrow 0} \left[\frac{-5(x^2 - 2x + 1) + 5}{x} \right] = \lim_{x \rightarrow 0} \left[\frac{-5x^2 + 10x - 5 + 5}{x} \right] =$

... = $\lim_{x \rightarrow 0} \left[\frac{\cancel{x}(-5x + 10)}{\cancel{x}} \right] = \lim_{x \rightarrow 0} [-5x + 10] \xrightarrow{D.S.} \underline{10}$
($x \neq 0$)

b. [4] $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+9} - 3}{x^2} \xrightarrow{D.S.} \frac{\sqrt{0^2+9} - 3}{0^2} = \frac{0}{0}$

(remember the trick used in class)

... = $\lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+9} - 3}{x^2} \cdot \frac{\sqrt{x^2+9} + 3}{\sqrt{x^2+9} + 3} \right] = \lim_{x \rightarrow 0} \left[\frac{\cancel{x^2+9} - 9}{x^2} \cdot \frac{1}{\sqrt{x^2+9} + 3} \right]$

... = $\lim_{x \rightarrow 0} \left[\frac{1}{\sqrt{x^2+9} + 3} \right] \xrightarrow{D.S.} \underline{\frac{1}{6}}$
($x \neq 0$)

c. [3] $\lim_{x \rightarrow 4^-} \frac{x^2 - 16}{|x-4|} \xrightarrow{D.S.} \frac{4^2 - 16}{4-4} = \frac{0}{0}$

= $\lim_{x \rightarrow 4} \frac{(x-4)(x+4)}{-(x-4)} = \lim_{x \rightarrow 4} -(x+4) \xrightarrow{D.S.} -(4+4) = \underline{-8}$

d. [3] $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2-1} \xrightarrow{D.S.} \frac{\sin(1-1)}{1^2-1} = \frac{0}{0}$
 $= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \left(\frac{1}{x+1} \right) \cdot \lim_{\substack{x \rightarrow 1 \\ (x-1) \rightarrow 0}} \left[\frac{\sin(x-1)}{x-1} \right]$
 $\dots = \frac{1}{1+1} \cdot 1 = \boxed{\frac{1}{2}}$

e. [3] $\lim_{x \rightarrow 3} \frac{\frac{5}{x} - \frac{5}{3}}{x-3} \xrightarrow{D.S.} \frac{\frac{5}{3} - \frac{5}{3}}{3-3} = \frac{0}{0}$
 $= \lim_{x \rightarrow 3} \left[\frac{\frac{15-5x}{3x}}{x-3} \right] = \lim_{x \rightarrow 3} \left[\frac{5 \overset{-1}{(3-x)}}{3x \underset{(x \neq 3)}{(x-3)}} \right] = \lim_{x \rightarrow 3} \left[\frac{-5}{3x} \right] \xrightarrow{D.S.}$
 $\frac{-5}{3(3)} = \boxed{\frac{-5}{9}}$

4. Simplify as much as possible.

a. [3] $\tan(\arccos(-3/4)) = \frac{\sin(\arccos(-3/4))}{\cos(\arccos(-3/4))} = \frac{\sqrt{1 - \cos^2(\arccos(-3/4))}}{-3/4}$
 $\dots = \frac{\sqrt{1 - 9/16}}{-3/4} = \frac{\sqrt{7}/4}{-3/4} = \boxed{\frac{-\sqrt{7}}{3}}$

b. [3] $(\sqrt[3]{25})^{\log_5 27} = (5^{2/3})^{\log_5 27} = 5^{(2/3 \cdot \log_5 27)} =$
 $\dots = 5^{\log_5 27^{2/3}} = 27^{2/3} = \boxed{9}$

5. [3] Let $\log_2(y-1) - 2\log_2 5 = x^2 - \log_2 x$. Express y as a function of x .

$\log_2(y-1) - \log_2 25 + \log_2 x = x^2 \rightarrow y-1 = \frac{25 \cdot 2^{x^2}}{x}$
 $\log_2 \left[\frac{x(y-1)}{25} \right] = x^2$
 $\frac{x(y-1)}{25} = 2^{x^2}$
 $y = \frac{25 \cdot 2^{x^2}}{x} + 1$

6. Let $f(x) = \begin{cases} \sqrt{6-x} + 3, & \text{if } x \leq 2 \\ \frac{x^2 - 2x}{x-2}, & \text{if } x > 2 \end{cases}$

a. [3] Determine $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$

$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \sqrt{6-x} + 3 \xrightarrow{D.S.} \sqrt{6-2} + 3 = \boxed{5}$

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{x^2 - 2x}{x-2} \xrightarrow{D.S.} \frac{0}{0} = \lim_{x \rightarrow 2^+} \left[\frac{x \overset{+}{(x-2)}}{\cancel{x-2}} \right] =$

$\lim_{x \rightarrow 2^+} [x] = \boxed{2}$

b. [1] What is $\lim_{x \rightarrow 2} f(x)$? Justify your answer.

$\lim_{x \rightarrow 2} f(x)$ d.n.e. because $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$