MAT 1341 Assignment 2 - DGD#_____

Summer 2014

Total: 14 points; Due: June 9, **beginning of DGD**

Family Name:______________________

First Name:_______________________

Student Number: __________________

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY

1. For the multiple choice problems, circle the correct answer. You do not have to show your work.

2. For the long answer questions, explain how you arrive at your answer. You earn points by correctly using mathematical notation, using correct reasoning and logic, and by identifying the key aspects of the problems.

3. You are allowed to discuss the problems with your classmates, but the work you hand in should be your own. Copying other people's solutions counts as plagiarism and will be dealt with as such.

1. (1.5 points) Look at the following matrix:

$$\begin{bmatrix} 1 & h-1 & 1 & 0 & 1 \\ 0 & h & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

1) Find h such that the matrix is in reduced row echelon form;

2) Find h such that the matrix is in echelon form but not in reduced row echelon form.

3) Find h such that the matrix is not in row echelon form.

Solution: 1) $h = 1$ 2) $h = 0$; 3) $h \neq 0, 1$.

2. (1.5 points)

$$\begin{aligned} x_1 + 4x_2 &= 1 \\ 3x_1 + hx_2 &= k \end{aligned}$$

For what values of h and k does the system have

1) infinitely many solutions?

2) Only one solution?

3) No solution?

Solution: The augmented matrix

$$[A|\vec{b}] = \begin{bmatrix} 1 & 4 & 1 \\ 3 & h & k \end{bmatrix} \xrightarrow{R_2 - 3R_1} \begin{bmatrix} 1 & 4 & 1 \\ 0 & h-12 & k-3 \end{bmatrix}$$

Solutions are: 1) $h = 12, k = 3$; 2) $h \neq 12$; 3)$h = 12, k \neq 3$.

3. (3 points) Consider the following system of linear equations

$$\begin{aligned} x_1 + 4x_2 - 8x_3 &= 0 \\ 2x_1 + 5x_2 - 7x_3 &= 0 \\ -3x_1 - 7x_2 + kx_3 &= 0 \end{aligned}$$

(i) Find value(s) of k such that the system has only trivial solution.

(ii) Find value(s) of k such that the system has non-trivial solutions.

(iii) For the value(s) of k in (ii), find the general solution.

Solution: (i)

$$\text{augmented matrix} = \begin{bmatrix} 1 & 4 & -8 & 0 \\ 2 & 5 & -7 & 0 \\ -3 & -7 & k & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -8 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 5 & k-24 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & -8 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 5 & k-24 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & k-9 & 0 \end{bmatrix}$$

Hence, for $k \neq 9$, the system has only trivial solution.

(ii) Solution: $k = 9$.

(iii) When $k = 9$, from the discussion in (i), we have

$$\text{augmented matrix} \sim \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus

$$\begin{aligned} x_1 + 4x_3 &= 0 \\ x_2 - 3x_3 &= 0 \end{aligned}$$

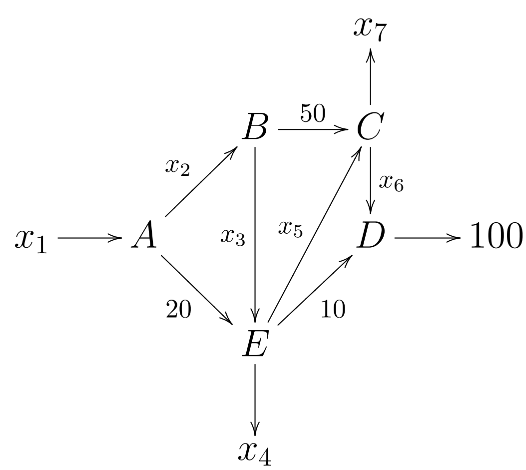
i.e.,

$$\begin{aligned} x_1 &= -4x_3 \\ x_2 &= 3x_3 \end{aligned}$$

Let $x_3 = s$, then general solution is:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = s \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

4. (3 points) The network below shows the traffic flow (vehicles/hour) over several one-way streets in downtown Ottawa.



(a) Construct the linear system, and write down all constraints.

(b) Construct the augmented matrix and carry it to the reduced row-echelon form.

(c) Determine the general flow pattern for the network.

Solution: (a) Set "flow in" = "flow out" at the intersections A, B, C, D, E, and Total Flow In=Total Flow Out, we have the following linear system:

$$
\begin{aligned}
x_1 &= x_2 + 20 \\
x_2 &= 50 + x_3 \\
50 + x_5 &= x_6 + x_7 \\
x_6 + 10 &= 100 \\
20 + x_3 &= x_4 + x_5 + 10 \\
x_1 &= x_4 + x_7 + 100
\end{aligned}
$$

where all $x_i \geq 0$, $i = 1, ..., 7$ and are integers.

(b) The augmented matrix is:

$$
A = \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 20 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 50 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & -50 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 90 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & -10 \\
1 & 0 & 0 & -1 & 0 & 0 & -1 & 100
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & -1 & 100 \\
0 & 1 & 0 & -1 & 0 & 0 & -1 & 80 \\
0 & 0 & 1 & -1 & 0 & 0 & -1 & 30 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 40 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 90 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(c) We get the general solution:

$$
\begin{aligned}
x_1 &= 100 + x_4 + x_7 \\
x_2 &= 80 + x_4 + x_7 \\
x_3 &= 30 + x_4 + x_7 \\
x_4 &= free \\
x_5 &= 40 + x_7 \\
x_6 &= 90 \\
x_7 &= free.
\end{aligned}
$$

Let $x_4 = s$, $x_7 = t$, then general solution is:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \\ 30 \\ 0 \\ 40 \\ 90 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

5. (2 points) Find the matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfying $2M^2 + 3M = 9I_2$, where I_2 is the 2×2 identity matrix, a, b, c, d are non-negative numbers.

Solution: Substitute M into the expression we have

$$\begin{bmatrix} 2a^2 + 2bc + 3a & b(2a + 2d + 3) \\ c(2a + 2d + 3) & 2bc + 2d^2 + 3d \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

Thus

$2a^2+2bc+3a = 9$; $2d^2+2bc+3d = 9$; $b(2a+2d+3) = 0$; $c(2a+2d+3) = 0$.

Since $2a + 2d + 3 \neq 0$, then $b = 0$, $c = 0$;

$2a^2 + 3a = 9$, $2d^2 + 3d = 9$, thus $a = 1.5$, $d = 1.5$.

6. (3 points) True or False? If true, explain in one sentence why. If false, give a counterexample.

   (a) If A is a 3x3 matrix for which the row echelon form has 2 leading 1s, then $A\vec{x} = \vec{b}$ has infinitely many solutions for any $\vec{b}$.

   (b) If A is a 3x3 matrix for which the row echelon form has 2 leading 1s, then $A\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$.

(c) Suppose A is a matrix with $A^2 = 0$. Then the row echelon form of A has no leading 1s.

Solution: (a) False. For example,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(b) False. For example,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(c) False. For example,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A^2 = 0.$$