MAT 1341 Assignment 1

Summer 2014

Total: 14 points; Due: May 26, **beginning of DGD**

Family Name:______________________

First Name:_______________________

Student Number: __________________

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY

1. Print out this assignment, and write your answers in the space provided.
2. For the multiple choice problems, circle the correct answer. You do not have to show your work.
3. For the long answer questions, explain how you arrive at your answer. You earn points by correctly using mathematical notation, using correct reasoning and logic, and by identifying the key aspects of the problems.
4. You are allowed to discuss the problems with your classmates, but the work you hand in should be your own. Copying other people's solutions counts as plagiarism and will be dealt with as such.

1. True or False? If true, show it. If false, give a counterexample.

   (a) If $\vec{u}$ and $\vec{v}$ are orthogonal, then $||2\vec{u} - 3\vec{v}||^2 = 4||\vec{u}||^2 + 9||\vec{v}||^2$.

   (b) If $\vec{u} \times \vec{v} = \vec{0}$, then either $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

   (c) If $\vec{u} + \vec{v} + \vec{w} = \vec{0}$, then $\vec{u} \times \vec{v} = \vec{v} \times \vec{w}$.

   Solution: (a) True.

   $$||2\vec{u}-3\vec{v}||^2 = (2\vec{u}-3\vec{v})\cdot(2\vec{u}-3\vec{v}) = (2\vec{u})\cdot(2\vec{u})-(2\vec{u})\cdot(3\vec{v})-(3\vec{v})\cdot(2\vec{u})+(3\vec{v})\cdot(3\vec{v})$$
   $$= 4||\vec{u}||^2 + 9||\vec{v}||^2.$$

   (b) False. For example, $\vec{u} = (1, 0, 0) \neq \vec{0}$, $\vec{v} = (2, 0, 0) \neq \vec{0}$. But $\vec{u} \times \vec{v} = \vec{0}$.

   (c) True. Since $\vec{u} = -\vec{v} - \vec{w}$,

   $$\vec{u} \times \vec{v} = (-\vec{v} - \vec{w}) \times \vec{v} = -\vec{v} \times \vec{v} - \vec{w} \times \vec{v} = -\vec{w} \times \vec{v} = \vec{v} \times \vec{w}.$$

   3points: 1point each.

2. Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(1, 3, 4)$, and $R(2, 3, 2)$.

   Solution: $\vec{PQ} = Q - P = (0, 1, 1)$, $\vec{PR} = R - P = (1, 1, -1)$.

   $$Area = \frac{1}{2}||\vec{PQ} \times \vec{PR}|| = \frac{1}{2}||(-2, 1, -1)|| = \frac{\sqrt{6}}{2}.$$

   2points .

3. Find the volume of the parallelepiped spanned by $\vec{u} = (1, 1, 1), \vec{v} = (1, 3, 2), \vec{w} = (1, 2, 1)$.

   Solution: Volume $= |\vec{u} \cdot (\vec{v} \times \vec{w})| = |(1, 1, 1) \cdot (-1, 1, -1)| = |-1| = 1.$

   2points.

4. Let $z^3 = -8i$.

   (a) Find all z in the form $a + bi$. (Hint: more than one answers!)

   (b) Find all z in polar form. (Hint: more than one answers!)

   Solution: (a) $z^3 = -8i = 8e^{i(\frac{3\pi}{2}+2k\pi)}, k = 0, 1, 2; \Rightarrow z = 2e^{i(\frac{\pi}{2}+\frac{2k\pi}{3})}, k = 0, 1, 2$. Thus

$$z = 2i, -\sqrt{3} - i, \sqrt{3} - i.$$

   (b)

$$z = 2e^{i\frac{\pi}{2}}, 2e^{i\frac{7\pi}{6}}, 2e^{i\frac{11\pi}{6}}.$$

   3points: 1.5points each.

5. Given two planes $x - 3y - 2z = -12$ and $2x + y - 3z = -1$.

   (a) Show that the intersection is a line L.

   (b) which of the following is a point on the line L:
   $A(1, 3, 5)$, $B(1, 3, 7)$, $C(1, 3, 6)$, $D(1, 3, 1)$, $E(1, 3, 2)$?

   (c) Find the parametric equation of the line L.

   Solution: (a): Proof: The two normal vectors are $\vec{n_1} = (1, -3, -2)$, $\vec{n_2} = (2, 1, -3)$, they are not parallel. Thus the intersection is a line.

   (b): E.

   (c) $\vec{n_1} \times \vec{n_2} =< 11, -1, 6 >$, which is a direction vector of L. So the parametric equation of the line is:

$$(x, y, z) = (1, 3, 2) + t(11, -1, 6), \quad t \in \mathbb{R}.$$

   4points1+1+2.