PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 6, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.

2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers 1-3 in the space provided below.

3. Questions 4 – 6 and are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.**

4. Where it is possible to check your work, do so.

5. Good luck! Bonne chance!

Name and Student ID number: _____

Signature and date: _____

Question	1	2	3	4	5	6	Total
Answer							
Grade							

1. (1 point) If $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, and A is a $3 \times n$ matrix, then the matrix PA

   A. has the same rows as A.
   B. has the same columns as A.
   C. is the same as A, but with the second and third columns swapped.
   D. is the same as A, but with the second and third rows swapped.
   E. both A and D.
   F. both B and C.

2. (1 point) Suppose $A = \begin{bmatrix} 1 & -2 & 0 \\ 5 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Which is true?

   A. A is not invertible.
   B. The third row of A^{-1} is $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.
   C. The second row of A^{-1} is $\begin{bmatrix} 5 & -1 & 10 \end{bmatrix}$.
   D. The third row of A^{-1} is $\begin{bmatrix} -5 & 1 & -10 \end{bmatrix}$.
   E. The second column of A^{-1} is $\begin{bmatrix} 2 & 1 & 0 \end{bmatrix}^T$.
   F. The third column of A^{-1} is $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

3. (1 point) For what values of x are the vectors $(1, -1, 0), (x, 1, 0)$, and $(0, 2, 3)$ a basis for $\mathbb{R}^3$?

A. All $x \in \mathbb{R}$.
B. Any x not equal to 0 or 1.
C. No x.
D. For $x = -1$ only.
E. For any $x \neq -1$.
F. For $x = 0$ only.

4. (6 points) Let $u_1 = (2, 1, 0, 1), u_2 = (1, 0, 3, -2), u_3 = (0, 3, -2, -3)$, and $U = \text{span}\{u_1, u_2, u_3\}$.
   (a) Show that $\{u_1, u_2, u_3\}$ is an orthogonal set.
   (b) Give a basis for U.
   (c) Find the best approximation to $(2, 0, 3, 1)$ by vectors in U.
   (d) Extend $\{u_1, u_2, u_3\}$ to a basis of $\mathbb{R}^4$ and justify that it is a basis.

5. (6 points) Let

$$A = \begin{bmatrix} 1 & -1 & 4 & 9 \\ 1 & -1 & 4 & 7 \\ 6 & -6 & 24 & 54 \end{bmatrix}.$$

(a) Find a basis for $\mathrm{Col}(A)$.

(b) Find a basis for $\mathrm{Row}(A)$.

(c) Find a basis for $\mathrm{Ker}(A) (= \mathrm{Null}(A))$.

(d) What is the dimension of $\{Ax \mid x \in \mathbb{R}^4\}$?

6. (4 points) (a) If A and B are square matrices of the same size and both are invertible, prove $(A^{-1}B)^T$ is also invertible. *(Hint: can you figure out what the inverse should actually be?)*

(b) If A is an $n \times n$ matrix, state three properties equivalent to "The reduced row echelon form of A is I_n."

(i)

(ii)

(iii)