

CARLETON UNIVERSITY

FINAL/DEFERRED
EXAMINATION
December 2005

DURATION: 3 HOURS

Department Name & Course Number: Mathematics and Statistics MATH 1009*ABCD
Course Instructor(s) : Dr Bumagin, Dr Devdariani, Ms Haley, Dr Zelmer

AUTHORIZED MEMORANDA

Non programmable, non graphing, non symbolic logic calculators are allowed.

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This exam consists of **13** pages, including this title page. There are two parts of the exam. Part A consists of 9 Multiple Choice questions worth 3 marks each. The answers to part A must be given on the **Multiple Choice Answer Sheet** which is attached to this exam paper.

This examination question paper **MAY NOT** be taken from the examination room.

This examination question paper **MAY** be released to the library.

Family Name (print) : _____ First Name : _____

Student Number : _____ Section : A (E. Devdariani, Mon-Wed 10 am)
B (C. Haley, Tue-Thur 2:30 pm)
C (I. Bumagin, Tue-Thur 2:30 pm)
D (G.Zelmer, Mon-Wed 7:30 pm)

Question	Mark
Part A	
Part B	B1
	B2
	B3
	B4
	B5
	B6
	B7
	B8
Exam Mark / 120 (60%)	
Term Mark/ 40%	
Final Mark / 100%	

Multiple Choice Answer Sheet

NAME (Please PRINT): _____

STUDENT NUMBER: _____

Please shade in your answer to each question in Part A.

1. (a) (b) (c) (d)

2. (a) (b) (c) (d)

3. (a) (b) (c) (d)

4. (a) (b) (c) (d)

5. (a) (b) (c) (d)

6. (a) (b) (c) (d)

7. (a) (b) (c) (d)

8. (a) (b) (c) (d)

9. (a) (b) (c) (d)

PART A. MULTIPLE CHOICE QUESTIONS.

All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Each question is worth **3** marks. No explanation required.

A1. The domain of the function $f(x) = \sqrt{(x+1)(x-3)}$ is

- (a) $[-1, 3]$, or equivalently, $\{x : -1 \leq x \leq 3\}$.
- (b) $(-1, 3)$, or equivalently, $\{x : -1 < x < 3\}$.
- (c) $(-\infty, -1) \cup (3, \infty)$, or equivalently, $\{x < -1\} \cup \{x > 3\}$.
- (d) $(-\infty, -1] \cup [3, \infty)$, or equivalently, $\{x \leq -1\} \cup \{x \geq 3\}$.

A2. What is $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2}$?

- a) 1.
- b) ∞ .
- c) -4.
- d) Does not exist.

A3. The graph of the function $y = \frac{2x + 7}{x - 5}$ has a vertical asymptote $x = 5$ and

- a) a horizontal asymptote $y = 2$.
- b) a horizontal asymptote $y = -\frac{7}{5}$.
- c) a horizontal asymptote $y = 0$.
- d) no horizontal asymptote.

A4. Which of the following is equal to $\log_3 \sqrt{\frac{1}{27}}$?

a) $-\frac{2}{3}$

b) $-\frac{3}{2}$

c) $\frac{2}{3}$

d) $\frac{3}{2}$

A5. If $e^{1-x} = 2$, what is x ?

a) $1 - \ln 2$

b) $3 - \ln 2$

c) $\ln 2 - 1$

d) $\ln 2 - 3$

A6. Let $f(x, y) = e^{5x+y^2}$. What is $f_{xx}(0, 0)$?

a) 1

b) 18

c) 25

d) 5

A7. Consider the Cobb-Douglas production function $f(x, y) = 9x^{2/3}y^{1/3}$, where x is the number of units of labour and y is the number of units of capital. What is the marginal productivity of **labour** at $x = 27$, $y = 64$?

- a) 16 b) 324 c) 8 d) 64

A8. A company estimates that its daily **cost** function is $C(x) = x^3 - 9x^2 + 27x$ million dollars, and its daily **revenue** function is $R(x) = 27x$ million dollars. What is the value of x that maximizes the **profit** $P(x)$, if $0 \leq x \leq 9$?

- a) 1 b) 2 c) 6 d) none of these

A9. Find the present value of \$40,000 due in 4 years at an interest rate of 8% compounded semiannually. Choose the closest answer:

- a) \$31,445 b) \$30,212 c) \$29,227 d) \$27,112

PART B. Please show all your work.**B1. [16 Marks]**

(a) Find the derivative of the functions below. You do NOT need to simplify. Show some intermediate steps of your computation.

[3] (i) $f(x) = \ln \sqrt{x^2 + 1}$

[4] (ii) $g(x) = \frac{e^{x^2}}{\sin x}$

[5] (iii) $h(x) = x^x$ (Hint: use logarithmic differentiation)

[4] (b) Consider the equation $xy^2 - 2xy + x = 1$, where $y = y(x)$ is defined implicitly as a function of x . Find $y'(x) = \frac{dy}{dx}$.

B2. [9 Marks] The quantity x demanded each week is related to the unit price p by the demand equation

$$x = \sqrt{169 - p^2} \quad (0 \leq p \leq 13).$$

- [5] (a) Find the elasticity of demand that corresponds to $p = 5$.
- [1] (b) Is the demand elastic or inelastic when $p = 5$?
- [2] (c) For what price p is the demand unitary?
- [1] (d) If the unit price is increased slightly from 5, will the revenue increase or decrease?

B3. [19 Marks] Consider the function $f(x) = 2x^3 - 3x^2 + \frac{1}{2}$.

[5] (a) Find the intervals where f is increasing and those where f is decreasing.

[2] (b) Find the relative extrema of f and the values of f at these points.

[4] (c) Determine the intervals where the graph of the function is concave up and those where it is concave down.

[2] (d) Find the inflection points of f . (Give both the x - and the y -coordinates).

[3] (e) Find the y -intercept of f . Show that there is an x -intercept on each of the intervals $[-1, 0]$ and $[1, 2]$.

[3] (f) Sketch the graph of f .

B4. [12 Marks]

Use the method of Lagrange multipliers to minimize the function

$$f(x, y) = x^2 + 2y^2 - xy$$

subject to the constraint

$$2x + y = 22.$$

B5. [9 Marks] A child's parents have received an inheritance of \$10,000. They wish to establish a trust for the child's college education.

[3] **(a)** If they invest in the bank offering 6 % per year over an 8 year period compounded quarterly, what would be the **interest** earned on their money?

[3] **(b)** What would be the maximum accumulated amount (or return) they could expect after 8 years from an investment giving a yearly interest of 6 % compounded continuously?

[3] **(c)** What would the interest rate be with continuous compounding in order for them to double their original amount of money after 8 years?

B6. [9 Marks]

Find and classify all the critical points of the function

$$f(x, y) = y^3 + 2x^2 + 4x - 27y + 1.$$

B7. [8 Marks]

A company manufactures and sells chairs and has a daily fixed cost of \$600 per day. The **marginal profit** function for the company is known to be

$$P'(x) = -0.0009x^2 + 30$$

where x is the number of chairs produced daily.

[5] (a) Find an integral expression for the total profit realized from the sale of 200 chairs per day. Evaluate this total profit.

[3] (b) What is the change in profit if the sale of chairs is increased to 250 per day? Does the company sustain a loss or a gain?

B8. [11 Marks]

Calculate the following integrals:

[3] (a) $\int \sin(2x) dx.$

[4] (b) $\int \sqrt{3x - 7} dx.$

[4] (c) $\int \frac{x}{x + 5} dx.$