Carleton University
Department of Civil and Environmental Engineering
Engineering Economics (ECOR 3800 A)
ASSIGNMENT # 1

Issued Oct 01, 2013 Due Date: Oct 09, 2013 at 6:00 pm
Drop off your location: Filing cabinet near the entrance to the Civil and Environmental Engineering office. The cabinet located to the right of room 3424 ME.
==========

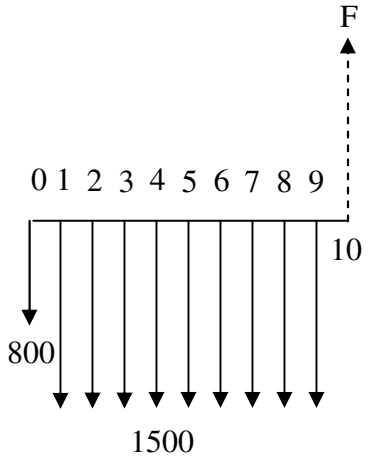
Q.1 (1.5 Marks)

(A)
If you make the following series of deposits at an interest rate of 10%, compounded annually, what would be the total balance at the end of 10 years?
(Support your answer with C.F.D.)

End of Period	Amount of Deposit
0	$800
1-9	$1500
10	0

Solution

F = P (F/P, 10%, 10)+A(F/A,10%, 9)*(F/P, 10%, 1)
F = 800 (F/P, 10%, 10)+1500(F/A,10%, 9)*(F/P, 10%, 1)
F = 800*2.5937 + 1500* 13.5795 *1.1
F = 24481.135



OR

$$F = P(1+i)^N + A\left[\frac{(1+i)^N - 1}{i}\right] * (1+i)^N$$

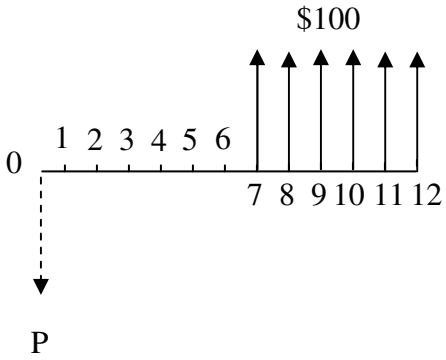
$$F = 800(1+0.1)^{10} + 1500\left[\frac{(1+0.1)^9 - 1}{0.1}\right] * (1+0.1)^1$$

$$F = 800 * 2.5937 + 1500 * 13.5794 * 1.1$$

$$F = 2{,}074.96 + 22{,}406.01$$

$$F = 24{,}480.97$$

(B)
Compute the equivalent present worth of the following cash flow series at period 0, i = 6%.



Solution
P=100 (P/A, 0.06, 6)(P/F, 0.06, 6)
P= 100*4.9173*0.7050

P= 346.66

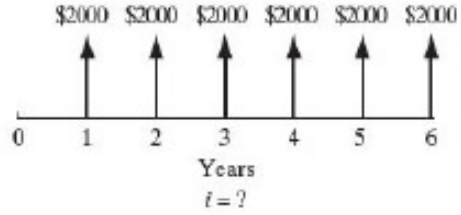
$$P = 100\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]\left[\frac{1}{(1+i)^N}\right]$$

$$P = 100\left[\frac{(1+0.06)^6 - 1}{0.06(1+0.06)^6}\right]\left[\frac{1}{(1+0.06)^6}\right]$$

$$= 100 * 4.91732 * 0.7049$$

$$= 346.65$$

(C)
Determine the interest rate *(i)* that makes the pairs of cash flows shown economically equivalent.



Solution
Establishing equivalence at $n = 0$ (find the present value)

$$P_1 = \frac{A\left[(1+i)^N - 1\right]}{i(1+i)^N}$$

$$P_1 = \frac{2{,}000\left[(1+i)^6 - 1\right]}{i(1+i)^6}$$

OR

$$P_1 = \$2{,}000(P/A, i, 6)$$

g = -25%

$$For \quad i \neq g \; P_2 = \frac{A'\left[1-(1+g)^N(1+i)^{-N}\right]}{(1-g)}$$

$$P_2 = \frac{2500\left[1-(1-0.25)^6(1+i)^{-6}\right]}{(1-0.25)}$$

OR

$$P_2 = \$2{,}500(P/A', -25\%, i, 6)$$

Solving for i with Excel or trial and error, we obtain $i = 92.35\%$

Q.2 (1 Mark)

Kersey Manufacturuing Co. a small fabrication of plastic, needs to purchasee an erxtrusion moulding machine for $120,000. Kersey will borrow money from a bank at an interest rate of 9% over five years. Kersey expects its product sales to be slow during the first year, but to increase subsequently at an annual rate of 10%. Kersey therefore arranges with the bank to pay off the loan on a"balloon scale", which results in the lowest payment at the end of the first year and each subsequent payment being just 10% over the previous one . Determine the five annual payments.

Solution

Using the geometric gradient series present worth factor, we can establish the equivalence between the loan amount $120,000 and the balloon payment series as

1)
$$\$120,000 = A_1(P/A_1, 10\%, 9\%, 5) = 4.6721A_1$$
$$A_1 = \$25,684.38$$

2) Payment series

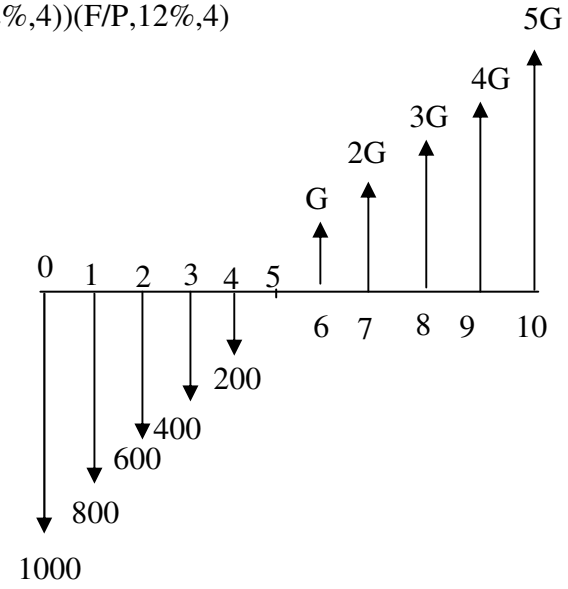
n	Payment
1	$25,684.38
2	$28,252.82
3	$31,078.10
4	$34,185.91
5	$37,604.50

Q.3 (1 Mark)
For the following transactions, draw the C.F.D and find the value of G that makes the deposit series equivalent to the withdrawal series at interest rate of 12%, compounded annually.

End of period	Deposit	Withdrawal
0	$1000	
1	800	
2	600	
3	400	
4	200	
5		
6		G
7		2G
8		3G
9		4G
10		5G

Solution

G(P/G,12%,6) = 800(F/A,12%,4)+(1,000-200(P/G,12%,4))(F/P,12%,4)
G=458.90



Q.4 (1 Mark)

Fairmont Textile has a plant in which employees have been having trouble with carpal tunnel syndrome (CTS. an inflammation of the nerves that pass through the carpal tunnel, a tight space at the base of palm), resulting from long-term repetitive activities, such as years of sewing. It seems as if 15 of the employees working in this facility developed signs of CTS over the last five years. Healthco, the company's insurance firm has been increasing Fairmont's liability insurance steadily because of this problem. Healthco is willing to lower the insurance premiums to $16,000 a year (from the current $30,000 a year) for the next five years if Fairmont implements an acceptable CTS-prevention program that includes making the employees aware of CTS and how to reduce the chances of it developing. What would be the maximum amount that Fairmont should invest in the program to make it worthwhile? The firm's interest rate is 12% compounded annually.

Solution

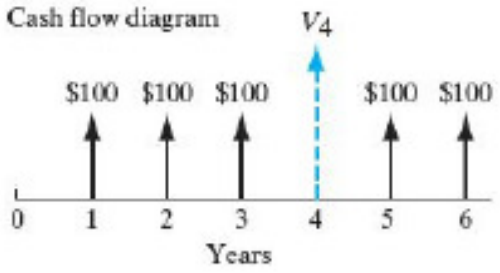
The maximum amount to invest in the prevention program is

$$P = \$14{,}000(P/A,12\%,5) = \$50{,}467$$

Q. 5 (1.5 Marks)

(A)

Consider the cash flow series given. In computing the equivalent worth at n=4, which of the following equations is incorrect?



(a) $V_4 = [\$100(P/A, i, 6) - \$100(P/F, i, 4)](F/P, i, 4)$

(b) $V_4 = \$100(F/A, i, 3) + \$100(P/A, i, 2)$

(c) $V_4 = \$100(F/A, i, 4) - \$100 + \$100(P/A, i, 2)$

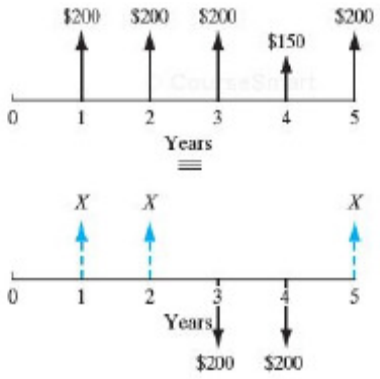
(d) $V_4 = [\$100(F/A, i, 6) - \$100(F/P, i, 2)](P/F, i, 2)$

Solution

b is incorrect, the correct equation➔ ***V_4 = $100(F/A, i,3)(P/F, i, 1)+ 100(P/A, i, 2)***

(B)

Find the value of X so that the two cash flows shown in the diagram are equivalent for an interest rate of 8%.



Solution

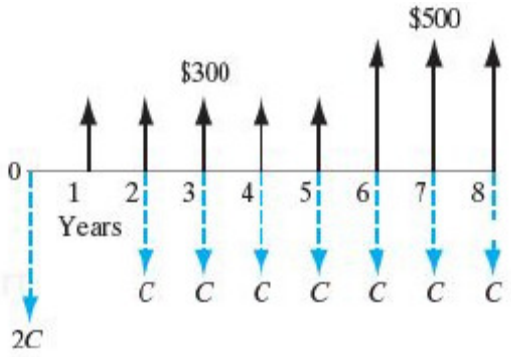
Establishing equivalence at $n = 5$

$$\$200(F/A,8\%,5) - \$50(F/P,8\%,1)$$
$$= X(F/A,8\%,5) - (\$200 + X)[(F/P,8\%,2) + (F/P,8\%,1)]$$
$$\$1{,}119.32 = X(5.8666) - (\$200 + X)(2.2464)$$

X=$433.29

(C)

Consider the cash flow shown in the accompanying diagram. What value of C makes the inflow series equivalent to the outflow series at an interest rate of 10%.



Solution

Establishing equivalence at $n = 8$, we find

$$\$300(F/A,10\%,8) + \$200(F/A,10\%,3) = 2C(F/P,10\%,8) + C(F/A,10\%,7)$$
$$\$4{,}092.77 = 2C(2.1436) + C(9.4872)$$
$$C = \$297.13$$

Q.7 (1 Mark)
Suppose that an oil well is expected to produce 100,000 barrels of oil during its first year in production. However, its subsequent production (yield) is expected to decrease by 10% over the previous year's production. The oil well has a proven reserve of 1,000,000 barrels.

(a) Suppose that the price of oil is expected to be \$60 per barrel for the next several years. What would be the present worth of the anticipated revenue stream at an interest rate of 12% compounded annually over the next seven years?
(b) Suppose that the price of oil is expected to start at \$60 per barrel during the first year, but to increase at the rate of 5% over the previous years' price. What would be the present worth of the anticipated revenue stream at an interest rate of 12% compounded annually over the next seven years?
(c) Consider part (b) again. After three years' production, you decide to sell the oil well. Assume that the well's remaining useful life is four more years. What would be a fair price?

Solution

(a) $P = \$6{,}000{,}000(P/A_1, -10\%, 12\%, 7) = \$21{,}372{,}076$

(b) Note that the oil price increases at the annual rate of 5% while the oil production decreases at the annual rate of 10%. Therefore, the annual revenue can be expressed as follows:

$$A_n = \$60(1+0.05)^{n-1}1000{,}000(1-0.1)^{n-1}$$
$$= \$6{,}000{,}000(0.945)^{n-1}$$
$$= \$6{,}000{,}000(1-0.055)^{n-1}$$

This revenue series is equivalent to a decreasing geometric gradient series with g = –5.5%.
So $P = \$6{,}000{,}000(P/A_1, -5.5\%, 12\%, 7) = \$23{,}847{,}896$

(c) Computing the present worth of the remaining series (A_4, A_5, A_6, A_7) at the end of period 3 gives

$$P = \$5{,}063{,}460(P/A_1, -5.5\%, 12\%, 4) = \$14{,}269{,}652$$

Q.8 (1 Mark)

(A)
What is the future worth of a series of equal deposits of $1000 for 10 years in a saving account that earns 7% annual interest if

(a) All deposits are made at the end of each year?
(b) All deposits are made at the beginning of each year?

Solution

(a) With deposits made at the end of each year
$F = \$1{,}000(F/A, 7\%, 10) = \$13{,}816$

(b) With deposits made at the beginning of each year
$F = \$1{,}000(F/A, 7\%, 10)(1.07) = \$14{,}783$

(B)
Consider the following cash flow;

Year End	Payment ($)
0	500
1-5	1000

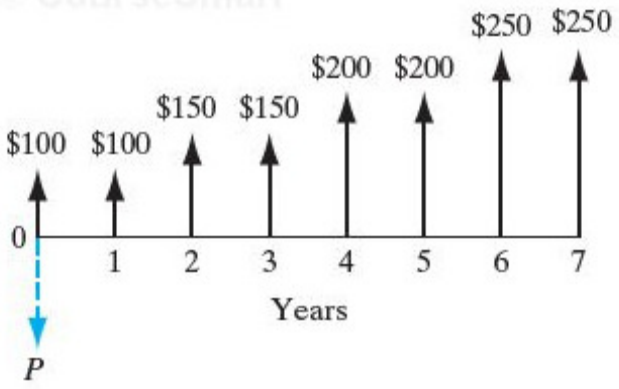
In computing F at the end of year 5 at an interest rate of 12%, which of the following equations in incorrect?

(a) $F = \$1000(F/A, 12\%, 5) - \$500(F/P, 12\%, 5)$
(b) $F = \$500(F/A, 12\%, 6) + \$500(F/A, 12\%, 5)$
(c) $F = [\$500 + \$1000(P/A, 12\%, 5)] \times (F/P, 12\%, 5)$
(d) $F = [\$500(A/P, 12\%, 5) + \$1000] \times (F/A, 12\%, 5)$

Solution

a is incorrect, the correct equation➔ ***F = $1000(F/A, 12%, 5)+ 500(F/P, 12%, 5)***

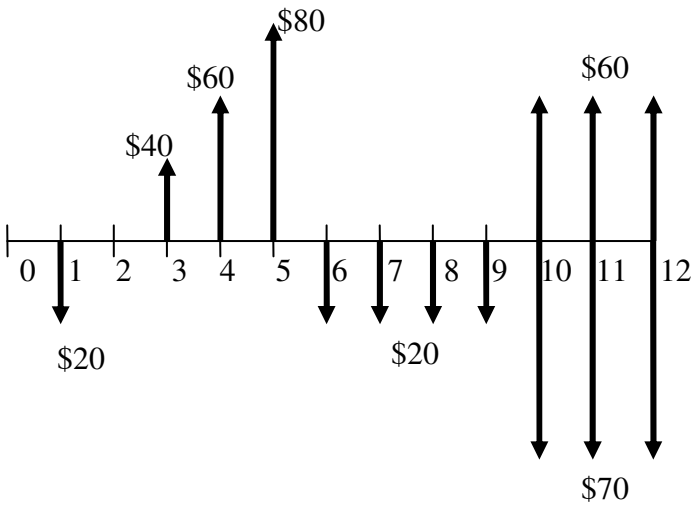
Q.9 (1 Mark)
Compute the value of P in the accompanying cash flow diagram, assuming i=9%.



Solution

$$A = \$15{,}000 - \$1{,}000(A/G,8\%,12)$$
$$= \$10{,}404.30$$

Q.10 (1 Mark)
Calculate the present worth of the cash flow shown in the accompanying diagram, using at most three kinds of interest factors at 10% interest compounded annually.



Solution

There are multiple ways to interpret the cash flows, below is one way to solve the cash flow diagram.

$$P = -20\left(P/F,0.10,1\right) + 40\left(P/A,0.10,3\right)\left(P/F,0.10,2\right)\left\{1 + (0.5)\left(A/G,0.10,3\right)\right\}$$
$$- 20\left(P/A,0.10,4\right)\left(P/F,0.10,5\right) - 10\left(P/A,0.10,3\right)\left(P/F,0.10,9\right)$$
$$= \$52.61$$