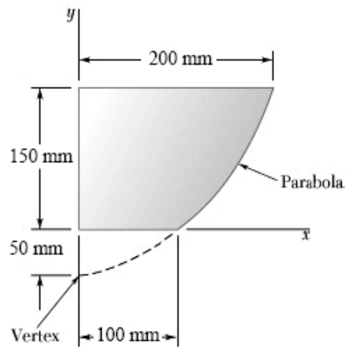
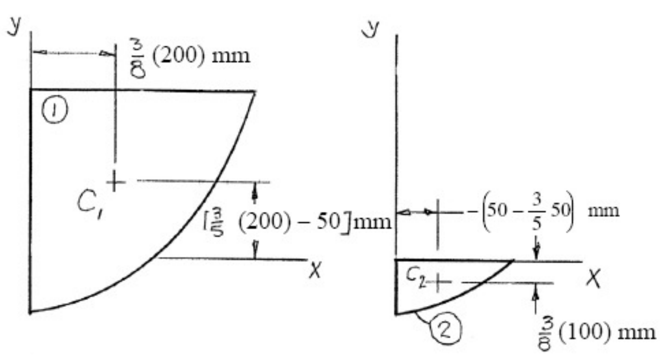
CVG2149 – Civil Engineering Mechanics
Fall 2012

SOLUTION ASSIGNMENT# 1



PROBLEM 5.16

Locate the centroid of the plane area shown.



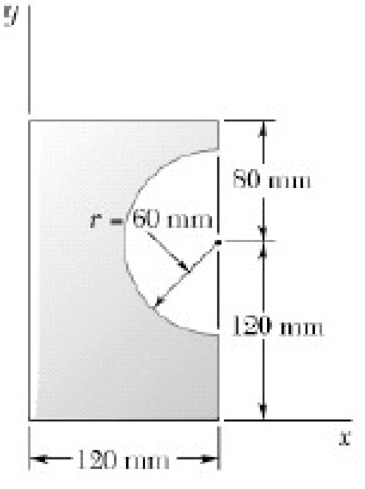
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm$^3 \times 10^4$	$\bar{y}A$, mm$^3 \times 10^4$
1	$\frac{2}{3}$ (200) (200) = 26666.67	75	70	200.0	186.67
2	$-\frac{2}{3}$ (100)50 = –3333.33	37.5	– 20	– 12.5	6.67
Σ	23333.34			187.5	193.34

$$\bar{X}\Sigma A = \Sigma\bar{x}A$$

$$\bar{X}\ (23333.34\ \text{mm}^2) = 187.5\times 10^4\ \text{mm}^3 \qquad \text{or} \qquad \bar{X} = 80.4\ \text{mm}$$

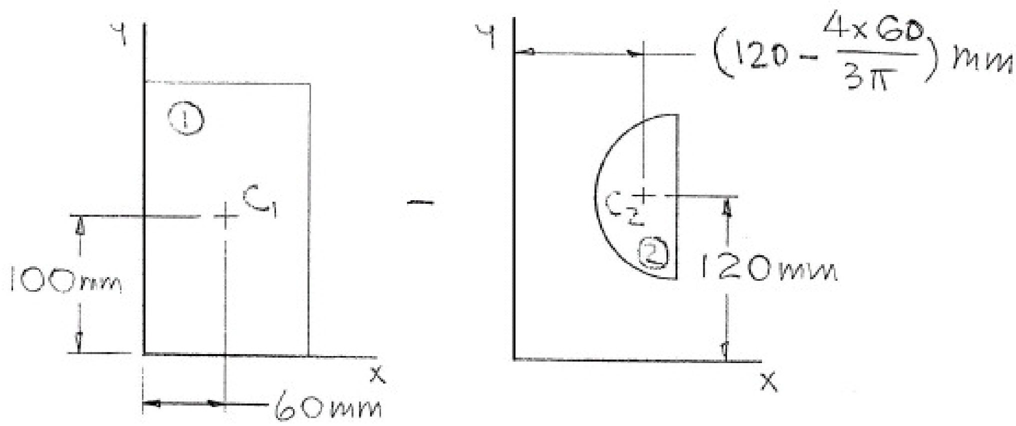
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\ (23333.33\ \text{mm}^2) = 193.34\times 10^4\ \text{mm}^3 \qquad \text{or} \qquad \bar{Y} = 82.9\ \text{mm}$$



PROBLEM 5.16

Locate the centroid of the plane area shown.



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$120 \times 200 = 24\ 000$	60	120	1 440 000	2 880 000
2	$-\dfrac{\pi(60)^2}{2} = -5654.9$	94.5	120	−534 600	−678 600
Σ	18 345			905 400	2 201 400

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{905\ 400\ \text{mm}^3}{18\ 345\ \text{mm}^2}$$

$$\bar{X} = 49.4\ \text{mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2\ 201\ 400\ \text{mm}^3}{18\ 345\ \text{mm}^2}$$

$$\bar{Y} = 93.8\ \text{mm}$$

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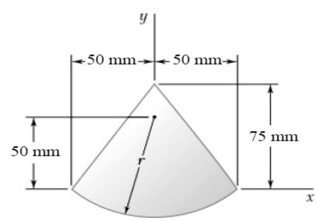
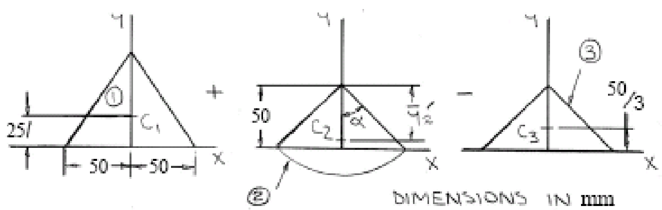


PROBLEM 5.11

Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies $\overline{X} = 0$ ◀



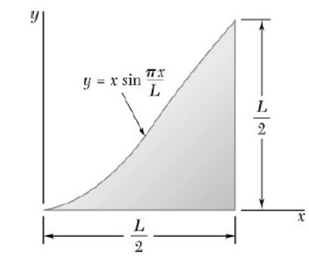
$$r_2 = 50\sqrt{2}\ \text{mm},\ \alpha = 45°$$

$$\bar{y}_2' = \frac{2r\sin\alpha}{3\alpha} = \frac{2\left(50\sqrt{2}\right)\sin\left(\frac{\pi}{4}\right)}{3\left(\frac{\pi}{4}\right)} = 42.44\ \text{mm}$$

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{1}{2}$ (100) (75) = 3750	25	93,750
2	$\frac{\pi}{4}\left(50\sqrt{2}\right)^2$ = 3926.99	$50 - \bar{y}'$ = 7.56	29688.04
3	$-\frac{1}{2}$ (100) (50) = − 2500	16.67	− 41675
Σ	5176.99		81763.04

Then $\overline{Y}\Sigma A = \Sigma\,\bar{y}A$

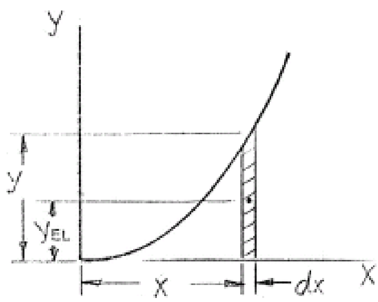
$\overline{Y}$ (5176.99 mm^2) = 81763.04 mm^3 or $\overline{Y}$ = 15.8 mm ◀



PROBLEM 5.46

Determine by direct integration the centroid of the area shown.

SOLUTION



Have $\qquad \bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2}x\sin\frac{\pi x}{L}$

and $\qquad dA = ydx$

$$A = \int dA = \int_0^{L/2} x\sin\frac{\pi x}{L}dx = \left[\frac{L^2}{\pi^2}\sin\frac{\pi x}{L} - \frac{L}{\pi}x\cos\frac{\pi x}{L}\right]_0^{L/2} = \frac{L^2}{\pi^2}$$

and $\qquad \bar{x} = \int \bar{x}_{EL}dA = \int_0^{L/2} x\left(x\sin\frac{\pi x}{L}dx\right)$

$$= \left[\frac{2L^2}{\pi^2}x\sin\left(\frac{\pi x}{L}\right) + \frac{2L^3}{\pi^3}\cos\left(\frac{\pi x}{L}\right) - \frac{L}{\pi}x^2\sin\left(\frac{\pi x}{L}\right)\right]_0^{L/2} = \frac{L^3}{\pi^2} - 2\frac{L^3}{\pi^3}$$

Also $\qquad \bar{y} = \int \bar{y}_{EL}dA = \int_0^{L/2} \frac{1}{2}x\sin\frac{\pi x}{L}\left(x\sin\frac{\pi x}{L}dx\right)$

$$= \frac{1}{2}\left[\frac{2L^2}{\pi^2}x\sin\frac{\pi x}{L} - \left(\frac{L}{\pi}x - \frac{2L^3}{\pi^3}\right)\cos\frac{\pi x}{L}\right]_0^{L/2}$$

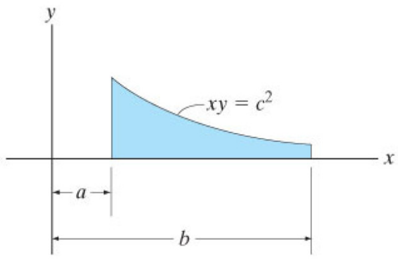
$$= \frac{1}{2}\left[\frac{1}{6}\left(\frac{L^3}{8}\right) - \frac{L^2}{4\pi^2}\left(\frac{L}{2}\right)(-1)\right] = \frac{L^3}{96\pi^2}\left(6 + \pi^2\right)$$

$$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{L^2}{\pi^2}\right) = L^3\left(\frac{1}{\pi^2} - \frac{z}{\pi^3}\right)$$

or $\bar{x} = 0.363L$ ◀

$$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{L^2}{\pi^2}\right) = \frac{L^3}{96\pi^2}\left(\frac{1}{\pi^2} - \frac{2}{\pi^3}\right)$$

or $\bar{y} = 0.1653L$ ◀



PROBLEM 5.31

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{c^2}{x}\,dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2} = \frac{c^2}{2x}$.

Area: Integrating,

$$A = \int_A dA = \int_a^b \frac{c^2}{x}\,dx = c^2 \ln x \Big|_a^b = c^2 \ln\frac{b}{a} \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_a^b x\left(\frac{c^2}{x}\,dx\right)}{c^2\ln\frac{b}{a}} = \frac{\int_a^b c^2\,dx}{c^2\ln\frac{b}{a}} = \frac{c^2 x\Big|_a^b}{c^2\ln\frac{b}{a}} = \frac{b-a}{\ln\frac{b}{a}} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_a^b \left(\frac{c^2}{2x}\right)\left(\frac{c^2}{x}\,dx\right)}{c^2\ln\frac{b}{a}} = \frac{\int_a^b \frac{c^4}{2x^2}\,dx}{c^2\ln\frac{b}{a}} = \frac{-\frac{c^4}{2x}\Big|_a^b}{c^2\ln\frac{b}{a}} = \frac{c^2(b-a)}{2ab\ln\frac{b}{a}} \qquad \textbf{Ans.}$$

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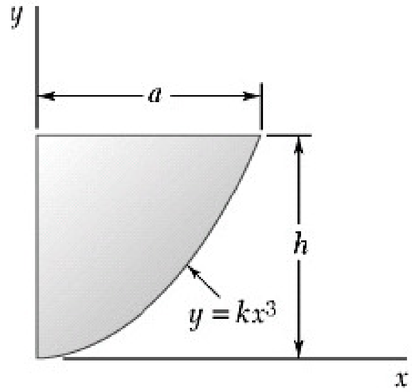
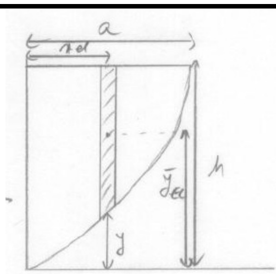


Determine by direct integration the centroid of the area shown. Choose a vertical strip. Express your answer in terms of a and h



$y = kx^3 \Rightarrow$ FOR $\begin{cases} x = a \\ y = h \end{cases} \Rightarrow h = ka^3 \Rightarrow k = \frac{h}{a^3} \Rightarrow$

$$y = \frac{h}{a^3} x^3$$

$$dA = (h - y)\,dx = \left(h - \frac{h}{a^3} x^3\right) dx = h\left(1 - \frac{x^3}{a^3}\right) dx$$

$$\begin{cases} \bar{x}_{EL} = x \\ \bar{y}_{EL} = \frac{h+y}{2} = \frac{1}{2}\left(h + \frac{h}{a^3} x^3\right) = \frac{h}{2}\left(1 + \frac{x^3}{a^3}\right) \end{cases}$$

$$A = \int dA = \int_0^a (h - y)\,dx = \int_0^a h\,dx - \int_0^a \frac{h}{a^3} x^3\,dx = hx\Big|_0^a - \frac{h}{a^3} \cdot \frac{x^4}{4}\Big|_0^a =$$

$$= ha - \frac{h a^4}{a^3 4} = ha - \frac{ha}{4} = \frac{3}{4} ha \Rightarrow \boxed{A = \frac{3}{4} ah}$$

$$Q_y = \int \tilde{x}_{EL}\, dA = \int_0^a x \cdot h\left(1 - \frac{x^3}{a^3}\right) dx = \int_0^a x h\, dx - \int_0^a \frac{h x^4}{a^3}\, dx =$$

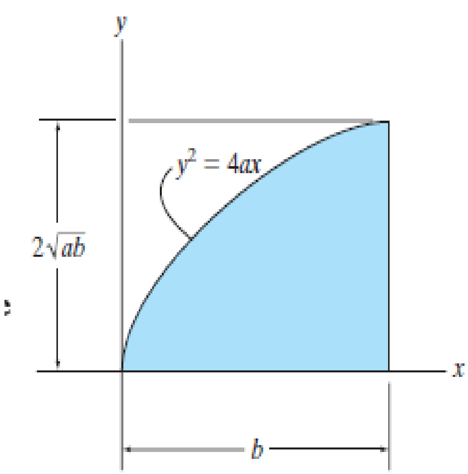
$$= \frac{h x^2}{2}\Big|_0^a - \frac{h}{a^3}\,\frac{x^5}{5}\Big|_0^a = \frac{h a^2}{2} - \frac{h a^5}{5 a^3} = \frac{3}{10} h a^2 \qquad \boxed{Q_y = \frac{3}{10} h a^2}$$

$$Q_x = \int \tilde{y}_{EL}\, dA = \int_0^a \frac{h}{2}\left(1 + \frac{x^3}{a^3}\right) \cdot h\left(1 - \frac{x^3}{a^3}\right) dx = \int_0^a \frac{h^2}{2}\left(1 - \frac{x^6}{a^6}\right) dx =$$

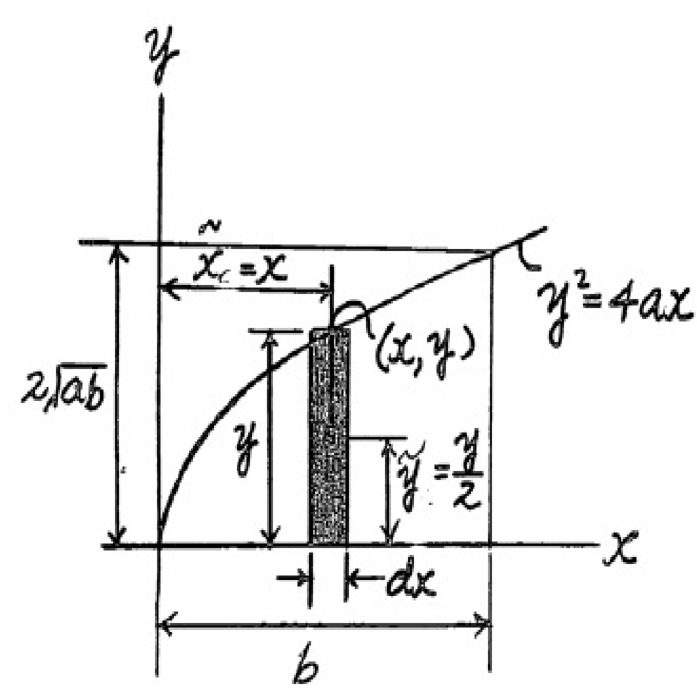
$$= \frac{h^2}{2}\left(x\Big|_0^a - \frac{x^7}{7a^6}\Big|_0^a\right) = \frac{h^2}{2}\left(a - \frac{a^7}{7a^6}\right) = \frac{3}{7} h^2 a \Rightarrow \boxed{Q_x = \frac{3}{7} h^2 a}$$

$$Q_y = \bar{x} A = \int \tilde{x}_{EL} A \Rightarrow \bar{x} \cdot \left(\frac{3}{4} a h\right) = \frac{3}{10} h a^2 \Rightarrow \boxed{\bar{x} = \frac{2}{5} a}$$

$$Q_x = \bar{y} A = \int \tilde{y}_{EL} A \Rightarrow \bar{y} \left(\frac{3}{4} a h\right) = \frac{3}{7} h^2 a \Rightarrow \boxed{\bar{y} = \frac{4}{7} h}$$



Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



$$dA = y\,dx = 2a^{1/2} x^{1/2}\, dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = a^{1/2}x^{1/2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^b 2a^{1/2}x^{1/2}\,dx = \frac{4}{3}a^{1/2}x^{3/2}\Big|_0^b = \frac{4}{3}a^{1/2}b^{3/2}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b x\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2a^{1/2}x^{3/2}\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\frac{4}{5}a^{1/2}x^{5/2}\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{5}b$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \left(a^{1/2}x^{1/2}\right)\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2ax\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{ax^2\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{4}\sqrt{ab}$$