

Be sure this exam has 6 pages including the cover

The University of British Columbia

MATH 215/255

Midterm Exam II – November 2014

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Section Number \_\_\_\_\_

This exam consists of 4 questions worth 10 marks each. No notes nor calculators.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

(5 points) 1. (a) Find a particular solution to  $y'' + 3y = 9x^2 + 3x$ .

(5 points) (b) Find the inverse Laplace transform  $f(t)$  of  $F(s) = \frac{4}{s(s^2 + 4)}$ .

(5 points) 2. (a) Find a particular solution to  $y'' + 2y' + y = 2e^{-x}$  using the method of undetermined coefficients.

(5 points) (b) Find a particular solution to the same equation  $y'' + 2y' + y = 2e^{-x}$ , this time using the method of variation of parameters.

3. Suppose the displacement  $x(t)$  of a damped mass-spring system subject to sinusoidal forcing of amplitude  $F_0$  is modelled by:

$$x'' + 4x' + 13x = F_0 \sin(t), \quad x(0) = 0, \quad x'(0) = 1.$$

- (5 points) (a) Find the solution  $x(t)$  when  $F_0 = 0$  (no forcing).

- (5 points) (b) Now take  $F_0 = 40$ , and find the steady periodic solution (the part of the solution  $x(t)$  which remains as  $t \rightarrow \infty$ ). Do not find the transient part.

(5 points) 4. (a) Find the Laplace transform of the solution to the following initial value problem:

$$x'' - x' + x = f(t), \quad x(0) = 0, \quad \text{and} \quad x'(0) = 1,$$

where

$$f(t) = \begin{cases} 1, & \text{if } 0 \leq t < 1, \\ t, & \text{if } t \geq 1. \end{cases}$$

(5 points) (b) Solve the initial value problem:

$$x'' + x' = \delta(t - 1), \quad x(0) = x'(0) = 0.$$