CARLETON UNIVERSITY

FINAL/DEFERRED
EXAMINATION
December 2007

DURATION: 3 HOURS

Department Name & Course Number: Mathematics and Statistics MATH 1009
Course Instructor(s) Dr.Devdariani, Dr.Horn, Dr.Mallick

AUTHORIZED MEMORANDA

Non programmable, non graphing, non symbolic logic calculators are allowed.

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This exam consists of **11** pages, including this title page. There are two parts of the exam. Part A consists of 10 Multiple Choice questions worth 2 marks each. The answers to part A must be given on the **Multiple Choice Answer Sheet** which is attached to this exam paper.

This examination question paper **MAY NOT** be taken from the examination room.

This examination question paper **MAY** be released to the library.

Family Name (print) : _____ **First Name :** _____

Student Number : _____ **Section : A (Dr.W. Horn, WF 4 pm)**
B (Dr.E. Devdariani, TR 1 pm)
C (Dr.E. Devdariani, WF 10 am)
D (Dr.R. Mallick, MW 8 pm)

Question		**Mark**
Part A (20)		
Part B (60)	B1 (10)	
	B2 (15)	
	B3 (10)	
	B4 (8)	
	B5 (10)	
	B6 (7)	
Exam Mark / (80) 55%		
Term Mark / 45%		
Final Mark / 100%		

Multiple Choice Answer Sheet

NAME (Please PRINT): _____

STUDENT NUMBER: _____

Please shade in your answer to each question in Part A.

1. (a) (b) (c) (d)

2. (a) (b) (c) (d)

3. (a) (b) (c) (d)

4. (a) (b) (c) (d)

5. (a) (b) (c) (d)

6. (a) (b) (c) (d)

7. (a) (b) (c) (d)

8. (a) (b) (c) (d)

9. (a) (b) (c) (d)

10. (a) (b) (c) (d)

PART A. MULTIPLE CHOICE QUESTIONS.

All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Correct answers are worth **2** marks. Incorrect and blank answers are worth **0**. Clearly circle only one answer. No explanation required.

A1. Let $f(x) = \dfrac{2}{x}$ and $g(x) = x^3 + 2$. Then the value of the composition $f(g(1))$ is

a) $\dfrac{2}{3}$. b) 2. c) 3. d) 11.

A2. What is $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$?

a) 0. b) 1. c) 2. d) Does not exist.

A3. The graph of the function $y = \dfrac{-x + 3}{x^2 + 5}$ has no vertical asymptote and

a) a horizontal asymptote $y = -1$.

b) a horizontal asymptote $y = 1$.

c) a horizontal asymptote $y = 0$.

d) no horizontal asymptote.

A4. An economy's consumer price index (CPI) between 1997 and 2007 was described by the function

$$I = -0.1t^3 + 2t^2 + 100, 0 < t < 10,$$

where t is measured in years, with $t = 0$ corresponding to 1997. At what rate was the CPI changing in 2000?

a) 1.2. b) 14.4. c) 88.5. d) 9.3.

A5. What are the critical numbers of the function $f(x) = \dfrac{1}{1+x}$?

a) $x = 0$. b) $x = 1$. c) $x = -1$. d) none.

A6. Which of the following is equal to $\log_5 \dfrac{1}{25}$?

a) $\dfrac{1}{2}$. b) $-\dfrac{1}{2}$. c) 2. d) -2.

A7. If $e^{x+2} = 5$, what is x ?

a) $\ln 5 - 2$. b) $5 - \ln 2$. c) $2 - \ln 5$. d) $\ln 2 - 5$.

A8. The expression $\dfrac{a^{1.6} \cdot (a^2)^{-0.3}}{a^{-2}}$ simplifies to

(a) a. (b) $a^{1.9}$. (c) $a^{-1.92}$. (d) a^3.

A9. Let $f(x, y) = e^{3xy}$. What is $f_{yy}(1, 0)$?

a) 0. b) 9. c) 3. d) e.

A10. Consider the Cobb-Douglas production function $f(x, y) = 3x^{2/3}y^{1/3}$, where x is the number of units of labour and y is the number of units of capital. What is the marginal productivity of **labour** at $x = 8,\ y = 27$?

a) 36. b) 64. c) 3. d) -18.

PART B. For full marks, be sure to show all of your steps.

B1. [10 Marks]

(a) Find the derivative of the functions below. Show some intermediate steps of your computation.

[2] (i) $f(x) = \ln \sqrt{3x^2 + x + 1}$.

[4] (ii) $y = x(x-3)^2(7x^2+2)^5$. Use logarithmic differentiation.

[4] **(b)** Consider the equation $x^3 - 2xy + y^4 = 5$, where $y = y(x)$ is defined implicitly as a function of x. Find $\dfrac{dy}{dx}$.

B2. [**15 Marks**] Consider the function $f(x) = x^4 - 2x^2 + 1$.

[1] **(a)** State the domain of f.

[2] **(b)** Find the y-intercept. Find the asymptotes, if any.

[5] **(c)** Determine the intervals where f is increasing and those where f is decreasing. Find the relative minima and maxima of f and the value of f at these points.

[5] **(d)** Determine the intervals where the graph of the function is concave up and those where it is concave down. Find the inflection points of f. (Give both the x- and the y-coordinates).

[2] **(e)** Sketch the graph of f.

B3. [10 Marks]

Use the method of Lagrange multipliers to find the maximum and the minimum values of the function

$$f(x, y) = xy$$

subject to the constraint

$$x^2 + y^2 = 8.$$

B4. [**8 Marks**] A child's parents have received an inheritance of \$5,000. They wish to establish a trust for the child's university education.

[3] **(a)** If they invest in the bank offering 4 % per year over a 10 year period compounded quarterly, what would be the **interest** earned on their money? (two decimals)

[2] **(b)** What would be the accumulated amount (or return) they could expect after 10 years from an investment giving a yearly interest of 4 % compounded continuously? (two decimals)

[3] **(c)** What should the interest rate be with continuous compounding in order for them to double their original amount of money after 10 years? (two decimals)

B5. [10 Marks]

[4] **(a)** Find all the critical points of the function $f(x, y) = x^2 - 4xy + 2y^2 + 4x + 8y - 1$. (Do NOT classify them).

[6] **(b)** The function $f(x, y) = 2x^3 - 2xy + y^2 + 7$ has two critical points $(0, 0)$ and $(\frac{1}{3}, \frac{1}{3})$. Use the Second Derivative Test to classify the nature of each point, if possible.

B6. [7 Marks]
Evaluate the following integrals:

[1] (a) $\int 4x^3 - 3x\,dx$

[3] (b) $\int_0^1 (2x+1)^5\,dx$

[3] (c) $\int \frac{x}{x+5}\,dx$