Concordia University
Department of Economics

ECON 303: Intermediate Macroeconomic Theory I, Winter 2014,
Midterm exam #2, March 24, 2014
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LAST NAME___________________________________FIRST NAME____________________________

STUDENT ID ___________________________

SECTION_________**4B**_________

Instructions: There is a total of 25 points. The exam has only ONE problem. Your answers must be concise and clearly derived. Good luck!

Single problem

This question examines your knowledge and understanding of Chapter 9 and 10, from the 4th Edition of the textbook. In particular, this question was part of the sample problems posted on Moodle.

Consider an endowment economy populated by *2* identical consumers whose preferences are represented by the following utility function: $U(c,c')=\ln(c)+\frac{1}{\beta}\ln(c')$ where *c* and *c′* are present and future consumption, respectively, while the parameter *β*, is a scalar. For example, if *β>1*, then consumers would be putting more weight on present relative to future consumption and vice versa. If *β = 1*, then consumers weigh equally present and future consumption in their utility functions. Consumers also have present and future income of y and y', and they face a real interest rate of *r*. Finally, each consumer pays present and future lump-sum taxes *t* and *t′*.

(a) Show that the marginal rate of substitution between current and future consumption is $\beta c'/c$. What does it measure?

$$MRS_{cc'}=\frac{MUc}{MUc'}=\frac{\frac{\beta}{c}}{\frac{1}{c'}}=\frac{\beta c'}{c}$$

It measures the amount of future consumption that each consumer has to give up if he/she wants to increase his/her current consumption by 1 unit

(b) Define a competitive equilibrium for this economy.

In this two-period economy, an Competitive Equilibrium is an consumption allocation *(c, c′)*, an saving decision *s* for 2 consumers, and a real interest rate *r* such that, given y, y′, G, G′:

- **Each consumer chooses *c, c′* and *s* optimally given *r***
- **The government present-value Budget Constraint holds.**
- **The credit market clears: B=Sp**
- **National income identity holds: Y=C+G**

(c) Consumers in this economy have present income y of 110 and future income y' of 120. The real rate of interest is 10%. The distribution of taxes over time is given by $t=t'=10$. Finally, assume that $\beta = 1$. Find the levels of present and future consumption as well as the level of savings. Are our consumers borrowers or lenders. Illustrate your results on a well-labeled graph.

$$\textit{According to the optimality condition of the consumer's decision}, MRS_{cc'} = 1 + r$$

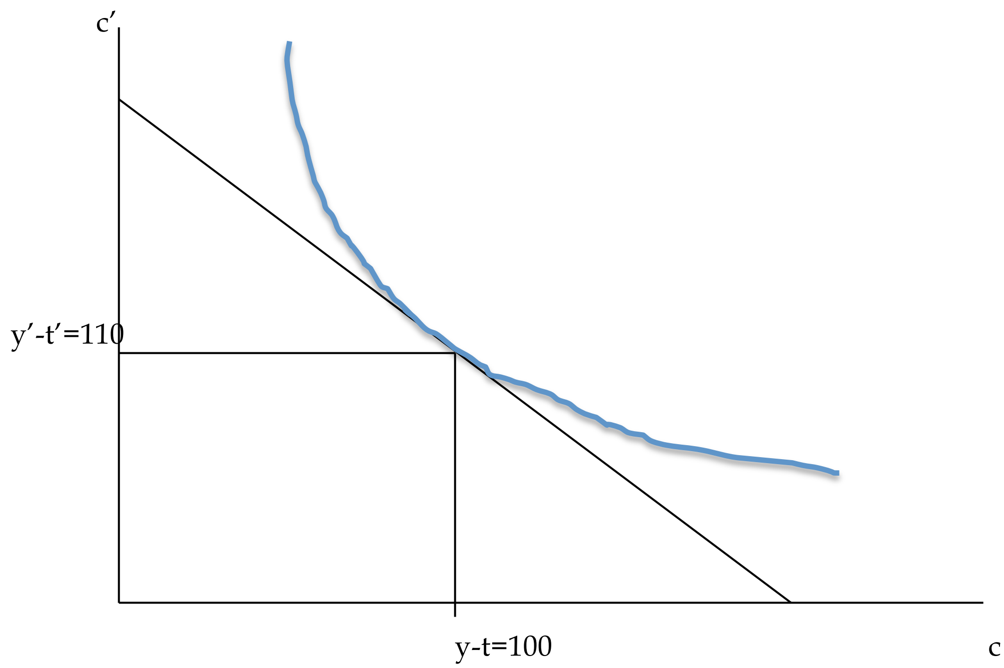
$$\Rightarrow \frac{\beta c'}{c} = 1 + r \quad \rightarrow \quad \frac{c'}{c} = 1 + 0.1 = 1.1, \ \ so\ c' = 1.1c \ \ (1)$$

$$Concumer's\ BC: c + \frac{c'}{1.1} = we = y - t + \frac{y' - t'}{1.1} = 110 - 10 + \frac{120 - 10}{1.1} = 200$$

$$\rightarrow c + \frac{1.1c}{1.1} = 200 \rightarrow 2c = 200 \ \ \rightarrow c^* = 100, so\ c'^* = 1.1x100 = 110,$$
$$for\ both\ concumers. So\ each\ consumer\ consumption\ bundle\ is\ c = 50, and\ c' = 55,$$

$$and\ s^* = y - t - c = 100 - 100 = 0$$

The consumers are neither borrowers, nor lenders. So the optimal point is at the endowment point.



(d) Find the value of the real GDP. Start by specifying the approach to be used.

$$Income\ Approac, Real\ GDP: Y = 2y = 2x55 = 110;\ \ Y' = 2y' = 2x60 = 120$$

$$\boldsymbol{Expenditure\ Approach, Real\ GDP: Y = C + G = 2x50 + 2x5 = 110, Y' = 2x55 + 2x5 = 120}$$

This is by setting G=T at any period, because we found that there is no saving, so government bond is zero

(e) Assume that consumers are informed about the existence of a national heritage. After the partition, each consumer receives an increase in his/her real income of 10 in current period. Assume that they can borrow and lend at the going rate of interest and this allows them to use financial markets to smooth their consumption in line with their preferences. How much of present consumption, if any, is financed by changing income

$$\boldsymbol{we' = y - t + 10 + \frac{y' - t'}{1.1} = 210,}$$

$$\boldsymbol{so\ new\ c = 105, which\ corresponds\ to\ c\ of\ 52,5\ for\ each\ consumer.}$$

$$\boldsymbol{New\ c' = 1.1(52.5) = 57.75\ \text{ for each consumer}}$$

$$\boldsymbol{The\ amount\ of\ c\ financed\ by\ changing\ wealth\ is\ 105 - 100 = 5\ for\ both\ consumers}$$

(f) Referring to part (e), assume that the rate of interest rises to 20%. What happens to present consumption and saving? What can we say about the relative strengths of the substitution and income effects? What has happened to saving compared with your part (e) results? Again, illustrate your results on a well-labeled graph.

$$\boldsymbol{2c = y - t + 10 + \frac{y' - t'}{1.2} = 201.66,}$$

$$\boldsymbol{so\ new\ c^* = 100.84, that\ is\ 50.42\ per\ consumer,}$$

$$\boldsymbol{total\ saving: 110 - 100.84 = 9.16}$$

The rise in the interest rate causes present consumption to fall, suggesting that the substitution effect is dominating the income effect.

(g) Referring to part (e), assume that the rate of interest is back at 10%. The government is worried both about the economy, which it feels is still fragile, but also about its fiscal position, which it wishes to address by raising lump sum taxes. It decides to leave taxes unchanged in the current period, so as to not harm the economy but to raise them by 5 in the future period. Use the model to calculate the effect of this policy on present consumption. How is this different/similar to the Ricardian Equivalence Theorem?

$$2c = y - t + 10 + \frac{y' - t' - 5}{1.1} = 196.45,$$

$$so\ new\ c = 98.27, that\ is\ c = 49.14\ and\ c' = 54.1\ per\ consumer$$

This is different from the Ricardian Equivalence Theorem, because the change in tax is made in the future. In such case, current and future consumption are affected (crowding out)

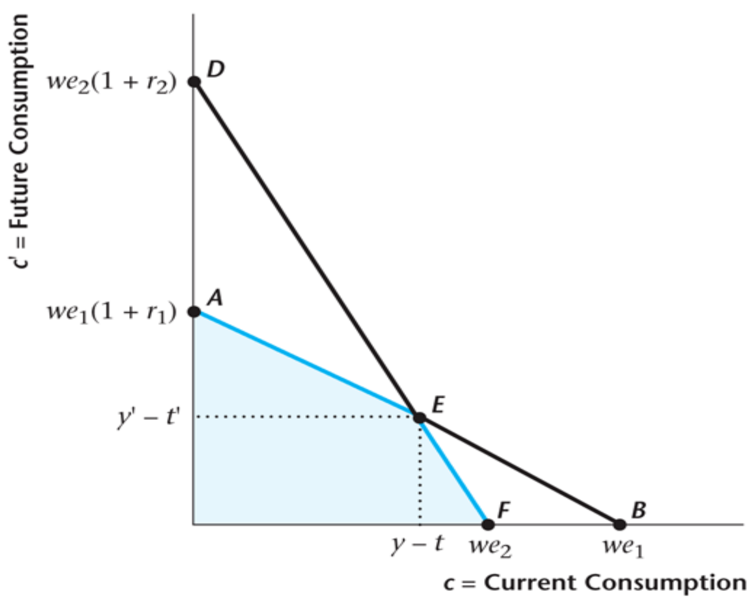
(h) Referring to part (g), assume all financial transactions are carried out through an independent bank. The bank charges the borrower an interest rate r_2 , and pays interest rate, r_1 to lenders. Write down the lifetime budget constraints for a borrower and a lender. Draw the corresponding budget curve.

For a Lender

$$c + \frac{c'}{1+r_1} = y + \frac{y'}{1+r_1} - t - \frac{t'}{1+r_1} = we_1$$

For a borrower

$$c + \frac{c'}{1+r_2} = y + \frac{y'}{1+r_2} - t - \frac{t'}{1+r_2} = we_2$$



(i) Referring to part (g), assume that $\alpha\%$ of the consumers are good borrowers and that the interest rate on deposit r_1 is 5%. What is the Bank's expected number of good borrowers if it sets the lending rate of interest at twice the interest rate on deposit?

$$r_2 = \frac{1 + r_1}{\alpha} - 1$$

$$\alpha r_2 + \alpha = 1 + r_1$$

$$\alpha = \frac{1 + r_1}{1 + r_2} = \frac{1 + 0.05}{1 + 0.1} = \frac{1.05}{1.1} = 0.95 = 95\%$$

Number of good borrowers=0.95x50=47.5=47

(j) Referring to part (g), assume the government can always change the rate of interest to offset any negative effects of its policy. Calculate by how much interest rates would have to fall to offset any negative effects on present consumption coming from the increase in tax.

$$2c = 210 = y - t + 10 + \frac{y' - t' - 5}{1 + r} = 110 + \frac{105}{1 + r}$$

$$210 = 110 + \frac{105}{1 + r}$$

$$210(1 + r) = 110(1 + r) + 105$$

$$100(1 + r) = 105$$

$$100(1 + r) = 105$$

$$r = 0.05 = 5\%, \textit{so interest rate would have to fall by } 10 - 5 = 5\%$$

Bonus question (2points)

In lecture, we assume zero initial real government bond, that is B_0=0, in deriving the Ricardian Equivalence Theorem. Now we relax this assumption by assuming that B_0 = 1: That is, households start with one unit of government bond in period 0. Suppose the government announces a tax cut in period 0, and fills the gap in budget by issuing one unit of debt in that period. In period 1, the government restores the balanced budget. Show **in algebra** whether the Ricardian Equivalence Theorem still holds in this case; explain the intuition of your result.

$$\textit{In period } 0, \textit{the budget constraint of the government is } G_0 = T_0 + B_0 = T_0 + 1$$

$$\textit{In period } 1, \textit{the Gvt BC is } G_1 + (1 + r) = T_1$$

Thus, we can conclude that the Ricardian Equivalence Theorem still holds when initial government debt is positive. The intuition is the same as the simple case. In face of a tax cut, households anticipate that for given government spendings and balanced budget strategy of the government, they would have to pay higher taxes T_1 in future period. In response, the households spend the extra income on purchase of the bonds which yield them (1 + r) units of income in future period, exactly the amount of extra taxes they are required to pay in the future period. Reacting in this way leaves no changes in the time path of consumption. More generally, since the present value of changes in taxes is zero, which implies there is no change in the disposable incomes faced by households, therefore, no income effect on consumption.